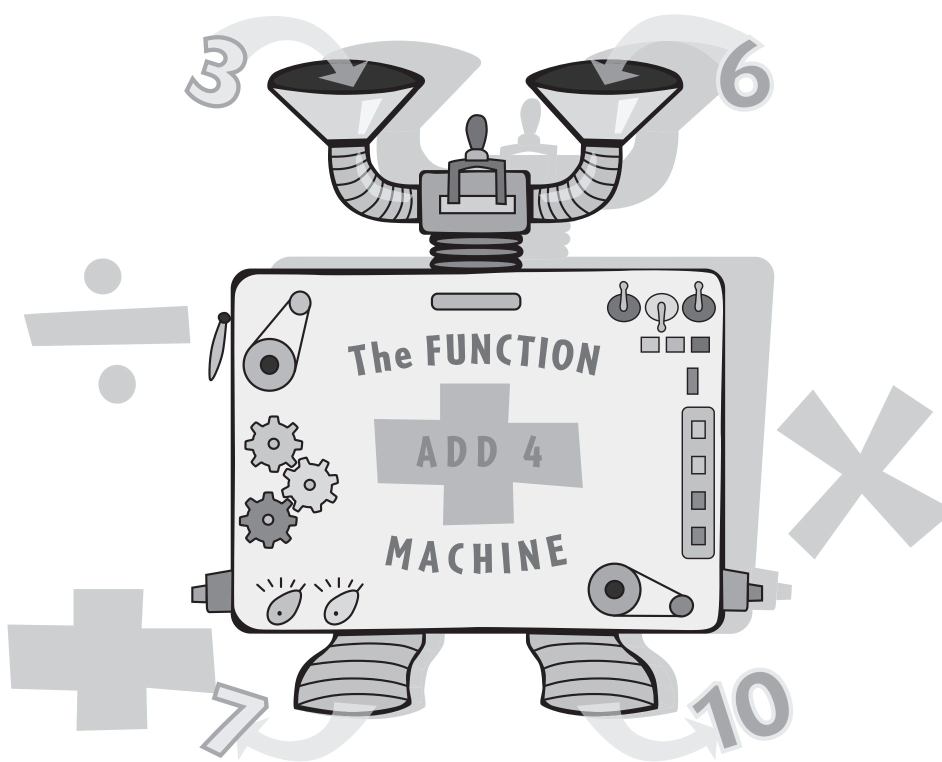
PRIMARY ALGEBRA

Developing Algebraic Reasoning

Hope Martin



GOOD YEAR BOOKS

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Dedication

My heartfelt thanks to Jill Martin, my daughter-in-law, for generously sharing her knowledge of the primary-grade mathematics curriculum and her experiences as a first-grade teacher.

Educational Standards

Primary Algebra contains lessons and activities that reinforce and develop skills as defined by the National Council of Teachers of Mathematics and the Common Core State Standards as appropriate for students in Grades K to 4. These include understanding patterns and functions; problem solving using numbers, pictures, and symbols; understanding equivalency and the meaning of equations; and making mathematical connections.

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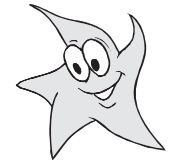


Introduction

Journalist Cokie Roberts once said, "As long as algebra is taught in school, there will be prayer in school." What is it about algebra that has so many people saying such unkind things about it? Perhaps the problem lies in when it is introduced into the mathematics curriculum and how it is taught.

Algebra has been called "generalized" arithmetic. We are all familiar with arithmetic and, although we may not like long division, it is not a fearful thing. However, when numbers become letters (or variables) and the language of mathematics changes from the concrete to the abstract, the change occurs very quickly. Educators need to begin teaching algebra concepts in kindergarten and then spiral it through the grade levels.

The concepts most appropriately addressed at the primary level are (1) the generalization of arithmetic and the meaning and use of the variable, (2) patterns and functions (in a more concrete way), and (3) equivalence and equations (seen as scales and balances). Using hands-on activities and concrete manipulatives to teach these significant concepts helps young students make the important connection between concrete arithmetic skills and abstract algebra.

The Algebra standard defined by the National Council of Teachers of Mathematics (NCTM, 2000) proposes that algebra be taught at pre-kindergarten through grade 12. It is important that primary-level students have an understanding of patterns, relations, and functions, analyzing patterns, using symbols to represent mathematical ideas and solve problems, and representing change in various contexts. The table on page 2 shows a detailed description of the recommendation of the National Council of Teachers of Mathematics. Many states have used this model to define their own mathematics education goals.

The most recent initiative to improve mathematics education for all of America's students are **the Common Core State Standards (CCSS).** This document is a state-led effort coordinated by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). The standards were developed in collaboration with teachers, school administrators, and experts, to provide a framework to prepare children for college and the workforce. The discussion of how the activities in *Primary Algebra* tie to the proposals in the CCSS follows.

Working with addition, subtraction, multiplication, and division equations.

The importance of students being able to find the unknown whole number in an equation is repeated throughout the primary grades in the Common Core State Standards and, depending on the grade, the equations require students to add, subtract, multiply or divide. In these equations, two of the numbers are known and one is unknown. *Primary Algebra* contains a variety of motivating activities to help students develop these algebra skills. They can solve puzzles (*Number Riddles*), make connections with historical events (*Find the Year Puzzles*), solve math jokes (*Math Jokes 1, 2, 3, and 4*), use visual models (*Domino Equations*), and use a linear model to find the missing variable (*Frog Jumps*). Students are introduced to the "balance scale model" associated with algebra in the *Balance the Scales* activities. They can also play stimulating games such as *Spinner Math* and the *Domino Algebra Game* while developing their reasoning skills.

Introduction (continued)

NCTM Algebra Standard

	Expectations	
Instructional programs from pre-kindergarten through grade 12 should enable all students to:	**Grades Pre-K to 2**	**Grades 3 to 5**
Understand patterns, relations, and functions	✓ sort, classify, and order objects by size, number, and other properties ✓ recognize, describe, and extend patterns, such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another ✓ analyze how both repeating and growing patterns are generated	✓ describe, extend, and make generalizations about geometric and numeric patterns ✓ represent and analyze patterns and functions, using words, tables, and graphs
Represent and analyze mathematical situations and structures using algebraic symbols	✓ illustrate general principles and properties of operations, such as commutativity, using specific numbers ✓ use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations	✓ identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers ✓ represent the idea of a variable as an unknown quantity using a letter or a symbol ✓ express mathematical relationships using equations
Use mathematical models to represent and understand quantitative relationships	✓ model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols	✓ model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions
Analyze change in various contexts	✓ describe qualitative change, such as a student's growing taller ✓ describe quantitative change, such as a student's growing two inches in one year	✓ investigate how a change in one variable relates to a change in a second variable ✓ identify and describe situations with constant and varying rates of change and compare them

Generating and analyzing patterns.

Mathematics is the study of patterns. The CCSS asserts that students should be able to generate a number or shape pattern that follows a given rule. The activities in *Primary Math* that encourage students to understand and analyze patterns are *Find the Missing Number*, *What's My Rule*, *The Function Machine*, *Patterns in the 100-Table*, *What Comes Next* and *Pattern Block Patterns*.

Working with money

Primary Math affords students the opportunity to solve problems involving dollar bills, quarters, dimes, nickels, and pennies while giving them additional practice in solving equations. *Equations with Money* uses pictures of money and shows the sum or difference using appropriate symbols ($ and ¢) and numerical values, and when students solve a riddle in *Math Joke 5* they, are indeed, **working with money**. In *Find the Missing Toy* activity, students problem-solve the value of the missing toy that will correctly solve the equation.

Solving Geometric Measurement Problems

The CCSS proposes that at the primary level students need non-traditional ways to solve real world and mathematical problems involving finding the perimeters of polygons when they are given the lengths of the sides. Students will improve their equation-solving and measurement skills while they enjoy working *Math Joke 6* and the *Rectangular Riddle*.

Written with the proposals of the NCTM and the CCSS in mind, *Primary Algebra* can be used as supplementary material in the mathematics classroom or by home schoolers who wish to make the abstract ideas of algebra more meaningful to their students. Students learn by doing! A vast majority of mathematics materials require students to solve problems that do not motivate them and in which they have no interest. *Primary Algebra* encourages students to first solve a problem and then make up their own to share with each other. These design-your-own activities require higher-level thinking. Students must have a deeper understanding of the concept in order to develop their own problems.

The book is organized into three chapters:

- Chapter 1, "Understanding Patterns and Functions," gives students the opportunity to work with different types of patterns and experience functions both kinesthetically and visually. Pattern block and number patterns, hundreds tables, and amusing "Function Machines" are used to help students learn about input and output.
- Chapter 2, "Problem Solving Using Numbers, Pictures, and Symbols," contains many motivating activities to encourage students to develop an algebraic, problem-solving frame of mind. Some of the activities take advantage of the current interest in Sudoku puzzles. These encourage logical thinking and an understanding of number sequence. Students can enjoy missing numbers and math riddles, use their newly developed algebra skills to discover the birthdays of famous Americans, and make use of number charts to solve word problems. Students learn to have fun doing math and working with variables.
- Chapter 3, "Understanding Equivalence and Equations," has students examine the similarities between solving equations and balancing scales, use number lines to solve equations, work with money and missing addends, and learn the meaning of the "equals sign" using dominoes as a visual model.

Resources included at the back of the book are:

- "Making Connections to Children's Literature" is an extensive list of books of stories and poems that relate literature to important mathematical concepts.
- The Bibliography and list of interesting Web sites offer additional resources to help you make algebra an exciting and interesting venture for young children.
- The Appendix has a collection of materials that can be used as part of the activities in this book or as enrichment and extension activities. Here you'll find spinners, a 100-Chart, pattern block pieces, sets of dominoes, and line-art illustrations of coins and dollar bills.

Each activity in *Primary Algebra* begins with a teacher's page that contains the following information:

What is the algebra?

This section defines the big algebraic skills and concepts that are addressed in the lesson.

What do you need?

Here you'll find a list of materials needed for each activity. At times, you'll be asked to provide overhead transparencies, which are an efficient way for you and students to work on the same problem. At other times, you'll be asked to provide a particular manipulative, such as pattern blocks. If these are not available, see the Appendix for pages of pattern blocks you can duplicate on card stock and laminate for durability. You'll find any other materials that are needed, such as scissors, glue, or calculators, listed here as well.

Some classroom procedures

This section will vary depending on the lesson. Some lessons suggest that you ask specific questions, and you'll also find additional examples, suggestions for presenting the lesson, and background information or needed mathematical background. Directions are provided, but, depending on your students' reading level, you may wish to read the directions and problems to the class.

How to extend the lesson

Here you will find additional activities or projects to use with students. Often you'll be asked to have students create their own books or lessons to share with others. This is an important extension to any lesson. While some students may be able to solve a particular problem, it is possible that their understanding is superficial and tenuous. It is always much more difficult to write a problem than to solve one!

Activity answers

Answers to the activity's problems are provided in this section along with any notes about there being more than one possible answer for any problem.

Now's the time to take those scissors, crayons, manipulatives, and glue out of the closet and have fun studying algebra. Who was it that said that algebra was a *prayerful endeavor*?

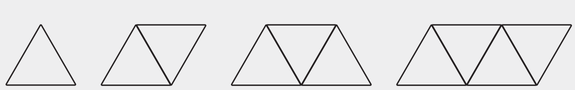
Chapter 1

Understanding Patterns and Functions

To understand is to perceive patterns.
Isaiah Berlin, 1909–1997

Very young children explore patterns in the world around them. Some of these patterns are displays of colors, combinations of shapes and geometric designs, and repetition of musical sounds. In the early elementary grades, patterns are described, classified, and created to help youngsters develop an understanding of why mathematics has been described as the study of patterns. Children analyze what *came before* to predict what *comes next*.

Patterns can be repeated or they can grow and change in a predictable way. Repeated patterns can be sounds (such as tap, tap, clink, tap, tap, clink), or colors (blue, red, red, green, blue, red, red, green), or geometric shapes (triangle, square, square, triangle, square, square) and so on. Growing patterns show an arithmetic change between what *came before* and what follows. These can be as simple as counting—1, 2, 3, 4, . . . —or recognizing the changes between the pairs of elements. This is an example of a geometric growing pattern:



Functions consider the relationship between the members of one set and members of another set. Early elementary students use "function machines" to find the rule that was used to calculate the *output number* from a given *input number*. An interactive Internet site that uses the idea of function machines to help students understand input, output, and function rules is http://www.shodor.org/interactivate/activities/FunctionMachine.

This chapter begins with the activity "Over and Over Again—Patterns That Repeat." These are examples of geometric patterns and use pattern block shapes. Students are asked to identify the pattern and then draw one that has the same name or is the same type of pattern. If these manipulatives are not available, see the book's Appendix for copies of pattern blocks. Duplicate the patterns on a heavy cardstock and then laminate them, and they will be sturdy enough for students to use as pattern pieces to design their own.

The activities in "Pattern Block Patterns That Grow" show students the first three or four arrangement of shapes and then ask them to predict what the next arrangement will look like based upon what came before in the pattern. Students are asked to explain their reasoning and then design their own growing pattern. The interactive Web site http://www.arcytech.org/java/patterns/patterns_j.shtml gives students an opportunity to use pattern block pieces to design their own repeating and growing patterns using these geometric shapes.

In the activity “What Comes Next,” students begin to investigate number patterns. These are formed by a sequence of numbers that depend on previous numbers for their pattern. Some students may need calculators to complete some of the computation in the problems.

“Patterns in the 100-Table” encourages students who have strong visual skills to see number patterns as a design on a grid. After students find the pattern formed by multiples of 3, they are given the opportunity to choose their own number and discover what the multiple pattern for their choice of number looks like.

“Using Number Patterns to Help Us Count” asks students to analyze the position of the objects in a design, looking for patterns that will help them find the total number of objects in the picture. Of course, they can find the total by counting, but it is much more challenging and exciting to find all of the possible ways patterns can be used to make the job less tedious.

The final activities, “The Function Machine” and “What’s My Rule?”, introduce students to the important concept of functions by having students analyze the relationship between input and output numbers. The first lesson of “The Function Machine” gives students the input numbers and a rule and asks them to find the output numbers. Then students are asked to find the rule when they know both the input and output numbers. Finally, students are given the opportunity to design their own functions using blank function machines. “What’s My Rule?” makes the concept of functions a bit more abstract by using tables. These problems also require students to add, subtract, or multiply to find the output.

Over and Over Again—Patterns That Repeat

What is the algebra?

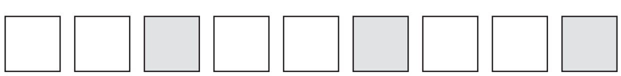
Describing, extending, and analyzing patterns

What do you need?

- "Over and Over Again—Patterns That Repeat" activity sheets (pp. 8–9) for each child
- "Design Your Own Patterns That Repeat" activity sheet (p. 10) for each pair of students.
- Pattern blocks (available in the Appendix)
- Crayons

Some classroom procedures

Introduce patterns to students. Patterns that repeat usually begin as sound patterns, such as clap, clap, snap, snap, clap, clap, snap, snap. Students then work with visual patterns, such as:



They can also be patterns formed using repeating geometry shapes such as the ones in this lesson. The sound pattern above is an A A B B pattern—clap, clap is represented by the letter A and snap, snap by the letter B. The visual or color pattern is an A A B pattern: Two white squares (A) and one dark square (B) make up the pattern. It is important that students understand how the letters used help define the pattern. To help students become comfortable with this model, have them design a sound pattern that is an A A B B B pattern. Ask, "How do you know that it can be defined in this way?" Once students are comfortable with patterns, hand out the "Over and Over Again—Patterns That Repeat" activity sheet and the pattern blocks. Then:

1. Have students use their pattern blocks to form patterns and define them using the AB model.
2. Design an original pattern block pattern that has the same model as the one shown above.

Give students a chance to describe and name their patterns based on the symbols used. Not all students will understand that a collection of symbols can only be described as a pattern if we can predict what will come next. Discussing student examples will help clarify, using this hands-on activity, the meaning of a repeating pattern.

How to extend the lesson

After students have completed the pattern worksheet, they can make their own unique repeating patterns working with a partner and using templates of pattern blocks or other geometric shapes. These can be shared with the rest of the class who will then be asked to "name" the patterns using letters from the alphabet.

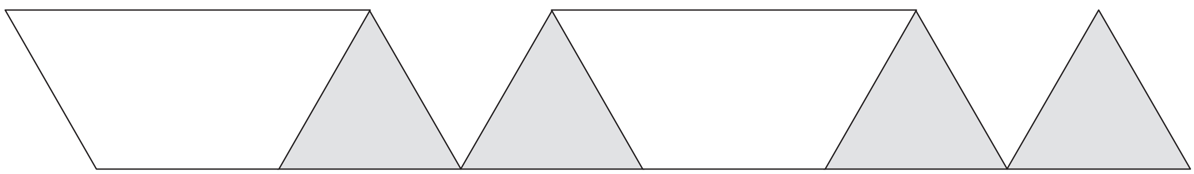
Activity answers

1. ABB, ABB
2. AB, AB, AB

Name .... Date ....

Over and Over Again— Patterns That Repeat

Directions: Use your pattern blocks to make this pattern.



1. This is called an ___ ___ ___, ___ ___ ___ pattern. Explain why you named this pattern the way you did.

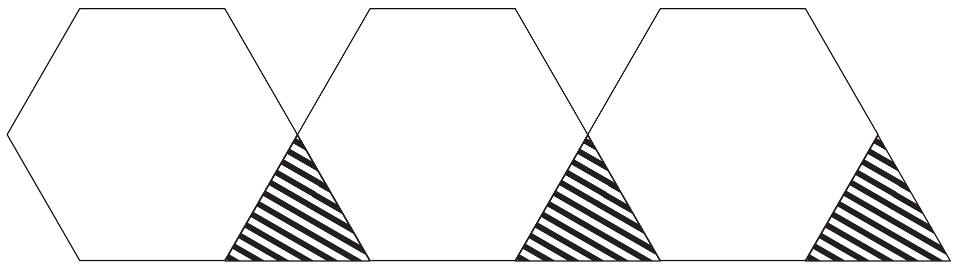
Use your pattern blocks to design a pattern that is also an ABB pattern. Draw it in this space.

Name .... Date ....

Over and Over Again—Patterns That Repeat

(continued)

Directions: Use your pattern blocks to make this pattern.



2. This is called an ___ ___, ___ ___, ___ ___, pattern. Explain why you named this pattern the way you did.

Use your pattern blocks to design a pattern that has the same name as the one above. Draw it in this space.

Name .... Date ....

Design Your Own Patterns That Repeat

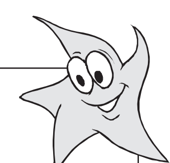
Activity

Directions: Use your pattern blocks to design your own original pattern. It can use two or three different pattern blocks . . . but remember that if it is a **pattern,** your partner must be able to predict what will come next.

My pattern is named an ________________________________

because:

This is a picture of my pattern:



Pattern Block Patterns That Grow

What is the algebra?

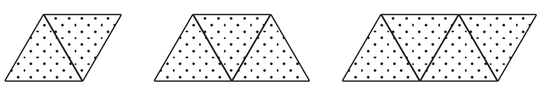
Describing, extending, and analyzing patterns that grow

What do you need?

- "Pattern Block Patterns That Grow" activity sheet (p. 12) for each child
- "Design Your Own Pattern Block Patterns That Grow" activity sheet (p. 13) for each child
- Pattern blocks (available in the Appendix)
- A set of overhead pattern blocks, if available

Some classroom procedures

Using overhead pattern blocks, demonstrate how a pattern grows. You might use this example:



Explain to students that the first pattern (containing two triangles) is the first "train" of the pattern. Have them use their own pattern blocks to make this starting train. Ask, "What must we do to form the next train of this growing pattern?" Students should recognize that they added one triangle and that they rotated it to fit into the design. You can use the type of table shown below with older students so they begin to associate the number of the term (the "train" or input) with the pattern (or output).

Ask students to predict how many triangles there will be in the fourth train, the fifth, the sixth, and so on. Have them develop a "word formula" to describe the pattern. For example, a possible answer might be, "There will always be one more triangle than the number of the train." Students may see other patterns; give them the opportunity to discuss those.

Hand out the "Pattern Block Patterns That Grow" activity sheet. Have students work the first problem and predict what the fourth "train" will look like. Give students the opportunity to discuss how they solved the problem.

The second problem is more difficult. Two different shapes are used and the pattern can be described in a number of ways.

How to extend the lesson

Have students work with a partner and design an original pattern block growing pattern that is accompanied by a table showing the relationship between the train and the number of each pattern block.

Activity answers

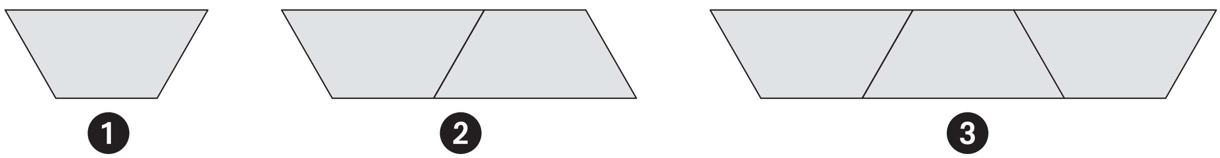
Students' answers may vary depending on how they view the pattern.

Train	1	2	3	4	5	6	7	8
Number of triangles	2	3	4					

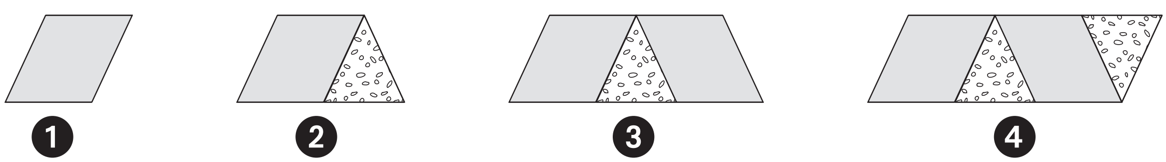
Name .... Date ....

Pattern Block Patterns That Grow

Directions: Use your pattern blocks to make each of these growing patterns.



Each pattern above is called a "train." What will the next, or "fourth train," in this pattern look like? Draw a picture of it below. Explain how you solved this problem.



What will the "fifth train" in this pattern look like? How many triangles will there be? _____ How many trapezoids? _____ Draw a picture of the fifth train below. Explain how you solved this problem.

Work with your partner to find a "rule" that will help you figure out how many trapezoids and how many triangles there will be in the tenth train of this pattern. Explain how you solved this problem.

Name .... Date ....

Design Your Own Pattern Block Patterns That Grow

Directions: Use pattern blocks to design an original growing pattern. You can use any blocks that you like, but remember: For it to be a **pattern,** you must be able to predict what will come next.

The first "train" of my pattern has: ______________________________

The second "train" of my pattern has added ______________ to the first.

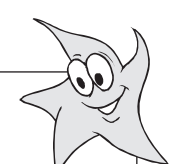
The third "train" of my pattern has added ______________ to the second.

Describe how your pattern grows.

____

____

This is a picture of the trains of my growing pattern:



What Comes Next?

What is the algebra?

Describing, extending, and analyzing patterns that grow

What do you need?

- "What Comes Next?" activity sheets (pp. 16–17) for each student
- "Design Your Own Number Sequences" activity sheet (p. 18) for each student
- Overhead transparencies of the student sheets

Some classroom procedures

Primary-level students are introduced to a variety of patterns that help them recognize order and make predictions. There are physical patterns, such as sound patterns (snap, snap, stomp). There are also patterns that occur in songs, such as the "Hokey, Pokey" or "The Farmer in the Dell." Geometric patterns, using pattern blocks, allow students to not only see but also experience, tactilely, the patterns formed by using polygons. This activity encourages students to experience patterns formed by a sequence of numbers.

Hand out the "What Comes Next?" activity pages. Before students begin work, offer examples of patterns using the overhead projector or a blackboard. Some examples:

✓ a pattern formed using even numbers: 8, 10, 12, 14, . . .

✓ a pattern formed using odd numbers: 7, 9, 11, 13, . . . (This pattern is like the example used in the first problem of the activity sheet.)

✓ adding on a constant difference: 2, 5, 8, 11, . . . (add on 3)

✓ subtracting away a constant difference: 20, 18, 16, 14, . . . (subtracting 2)

✓ adding on using a pattern: 2, 3, 5, 8, 12, . . . (add on 1, then 2, then 3, then 4, . . .). Problem 4 on the activity sheet includes a pattern that appears to be similar to this one, but it follows a different pattern: Add on the number that precedes it. This is called the Fibonacci Sequence. For example, 1 + 2 = 3, 2 + 3 = 5, 3 + 5 = 8, and so on.

Be sure that students understand how important it is to have a sufficient number of examples in the original sequence so that one and only one pattern can be predicted.

While it is important for students to be able to find the pattern in a sequence, it is also important for them to be able to problem solve the inverse—using numbers to develop their own sequences. The second activity sheet is designed for this purpose. Students are asked to create their own sequences. They are asked to place the first four numbers of the sequence in the rectangles and *then check to see that they have indeed created a number pattern*. Then they do not complete their own sheets; they exchange with a partner and solve each other's sheets. The "Design Your Own" activity page can also be used as an additional extension to the lesson.

When all of the groups have completed their work, discuss the students' number patterns. See if some of the examples could have formed different patterns than the one described.

What Comes Next? (continued)

How to extend the lesson

Students can explore many interesting number patterns. For example, they can subtract a constant difference, resulting in negative numbers. This is a wonderful way to have youngsters discover that there are, in fact, numbers that are less than zero. An example of this type of sequence might be: 8, 6, 4, 2, 0, –2, –4, ...

Another interesting problem—some number sequences can be correctly described using different sequences. For example, look at this sequence: 1, 2, 4, ... *There are not enough numbers to predict one particular sequence*. We could be doubling the numbers. Then the sequence would be 1, 2, 4, 8, 16, 32, . . . , *or* we could be adding on 1, then 2, then 3. In this case, our sequence would look like this: 1, 2, 4, 7, 11, ... Give this problem to students and see how many other possible sequences they can predict.

Activity answers

1. 9, 11, 13, 15

2. 15, 21, 28, 36

3. 14, 10, 6, 2

4. 21, 34, 55, 89

5. 25, 20, 15, 10

6. 32, 64, 128, 256

7. 48, 54, 60, 66

8. 41, 54, 69, 86

Accept any reasonable explanation for the development of each pattern.

Name .... Date ....

What Comes Next?

Directions: Let's take a look at some number patterns. These are called sequences because there is a relationship among the numbers. Examine each of these sequences. Then problem solve . . . **"What comes next?"**

❶ **"What comes next?" 1, 3, 5, 7, ____, ____, ____, ____,**

In your own words, describe how the pattern is formed:

____

____

❷ **"What comes next?" 1, 3, 6, 10 ____, ____, ____, ____,**

This pattern grows in a different way. In your own words, describe how the pattern is formed:

____

____

❸ **"What comes next?" 30, 26, 22, 18, ____, ____, ____, ____,**

This pattern is different from the others. In your own words, describe how the pattern is formed:

____

____

❹ **"What comes next?" 1, 2, 3, 5, 8, 13, ___, ___, ___, ___,**

This pattern is very different! In your own words, describe how the pattern is formed:

____

____

Name .... Date ....

What Comes Next (continued)

Activity

Examine each of these sequences. Then problem-solve . . .
"What comes next?

5 "What comes next?" **45, 40, 35, 30, _____, _____, _____, _____,**

In your own words, describe how the pattern is formed:

6 "What comes next?" **1, 2, 4, 8, 16, _____, _____, _____, _____,**

This pattern grows in a different way. In your own words, describe how the pattern is formed:

7 "What comes next?" **24, 30, 36, 42, _____, _____, _____, _____,**

This pattern is different from the others. In your own words, describe how the pattern is formed:

8 "What comes next?" **5, 6, 9, 14, 21, 30 ____, ____, ____, ____,**

This pattern is very different! In your own words, describe how the pattern is formed:

Name .... Date ....

Design Your Own Number Sequences

Activity

Directions: Now it's your turn to make up your own number sequences. Be as creative as you wish. Place your first four numbers in the boxes—check carefully to make sure that your numbers form a sequence. Then share your sequences with your partner—you solve his or hers and your partner will solve yours.

☐, ☐, ☐, ☐, ____, ____, ____, ____

☐, ☐, ☐, ☐, ____, ____, ____, ____

☐, ☐, ☐, ☐, ____, ____, ____, ____

☐, ☐, ☐, ☐, ____, ____, ____, ____

☐, ☐, ☐, ☐, ____, ____, ____, ____

☐, ☐, ☐, ☐, ____, ____, ____, ____

☐, ☐, ☐, ☐, ____, ____, ____, ____

Patterns in the 100-Table

What is the algebra?

Describing, extending, and analyzing patterns

What do you need?

- "Patterns in the 100-Table—Counting by 3s" activity sheet (p. 21) for each child
- "Patterns in the 100-Table—My Own Number" activity sheet (p. 22) for each child
- Overhead transparency of "Patterns in the 100-Table—Counting by 3s," if available
- Overhead markers (non-permanent) or colored chalk
- 1-inch graph paper
- Markers or crayons

Some classroom procedures

Hand out the "Patterns in the 100-Table" activity sheet to students and place the overhead transparency of the sheet on the projector. Ask students: "Predict what you think will happen if you were to count to three and shade in the third number all the way down the chart. Do you think a pattern will develop? What do you think the pattern might look like?"

Demonstrate for the class how to count and shade in the numbers and have them color in the correct squares during the demonstration. It might be necessary to shade in the first four or five multiples of three for all students to understand the procedures. Once students begin to work, walk around to make sure that every child understands how to complete the pattern design.

When students have finished, ask:

1. How did you describe the pattern on your activity sheet?
2. When you wrote the numbers that you shaded in, what type of pattern do you see in the numbers? (Students might respond that the pattern falls on the diagonals or that there are two uncolored spaces and then one is colored, etc.) There are many different patterns that students might see and they should be given the opportunity to describe them. There is no one right answer.
3. What do you think the pattern might be if we were to count "one, two" and shade in the second number? Would this design look like the "Count by 3" design? (Ask students to explain their responses. Shaded columns form this pattern only.)

The design created by "Count by 3" looks like this:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Patterns in the 100-Table (continued)

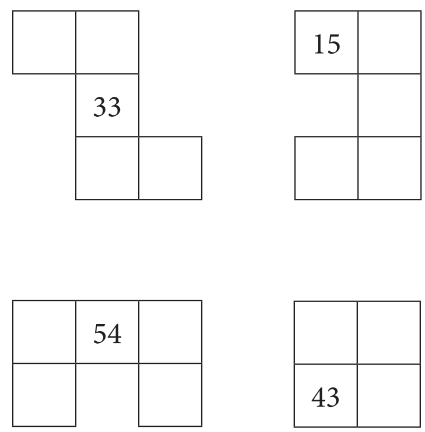
The design created by "Count by 2" looks like this:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Hand out the "Patterns in the 100-Table—My Own Number" activity sheet. This sheet asks students to choose any number and design a pattern. Encourage them to choose smaller numbers (2 through 6) because the chart only extends to 100.

How to extend the lesson

By using only parts of the 100-chart, students can problem solve which numbers are missing. These are some examples:



Give students 1-inch graph paper and ask them to design their own "Missing Number Puzzles" from the 100-Chart.

Name .... Date ....

Patterns in the 100-Table— Counting by 3s

Activity

Directions: Count by 3 in the table below, and shade in each third number. For example, count 1, 2, 3, and color in the number 3. Continue counting 1, 2, 3, …, and what number do you come to? That's right—6. Shade in the 6. Continue, counting carefully, and shade in the number you get to when you say "3."

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Write out all of the numbers that are shaded. Do you see a pattern? Describe it.

Name .... Date ....

Patterns in the 100-Table—My Own Number

Activity

Directions: Choose another number and use that number to count off a pattern on this hundreds table.

I chose the number _____. The first number I will shade will be a _____. The next number I will shade in will be a _____.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Using Patterns to Help Us Count

What is the algebra?

Using patterns, exploring patterns to compute totals, translating visual patterns into numerical expressions

What do you need?

- "Using Patterns to Help Us Count" activity sheets (pp. 24–27) for each pair of students
- Overhead transparencies of activity sheets, if available
- 1-inch graph paper (for the extension activity)

Some classroom procedures

Problems that have more than one correct answer or can be solved using more than one method are called *open-ended*. These type of problems help students develop confidence in their problem-solving abilities and discredit the belief that *there is only one right answer and only one right way to solve a problem*.

Each of the two patterns in this activity are open-ended because, while there is a specific number of items in each design, more than one solution strategy can be used find that number.

Place students in pairs and give each pair sheets for both activities. Have them work to find at least two different patterns that allow them to problem solve the number of pictures in the designs without actually counting each item individually. Be sure to give students sufficient time to find as many possibilities as they can and then allow them to share their solutions with the class.

How to extend the lesson

Give students 1-inch graph paper and have them work with a partner to make their own design that can be "counted" by finding patterns instead of counting each item individually.

Activity answers

Student answers may vary.

Name .... Date ....

Using Patterns to Help Us Count . . . Soccer Balls

Directions: We can use many different strategies to help us count how many soccer balls there are in this pattern. Work with your partner to see how many different ways you can find the total number of balls without actually counting each one. Then answer the question on the next page.

Name ______________________ Date ______________

Using Patterns to Help Us Count . . . Soccer Balls (continued)

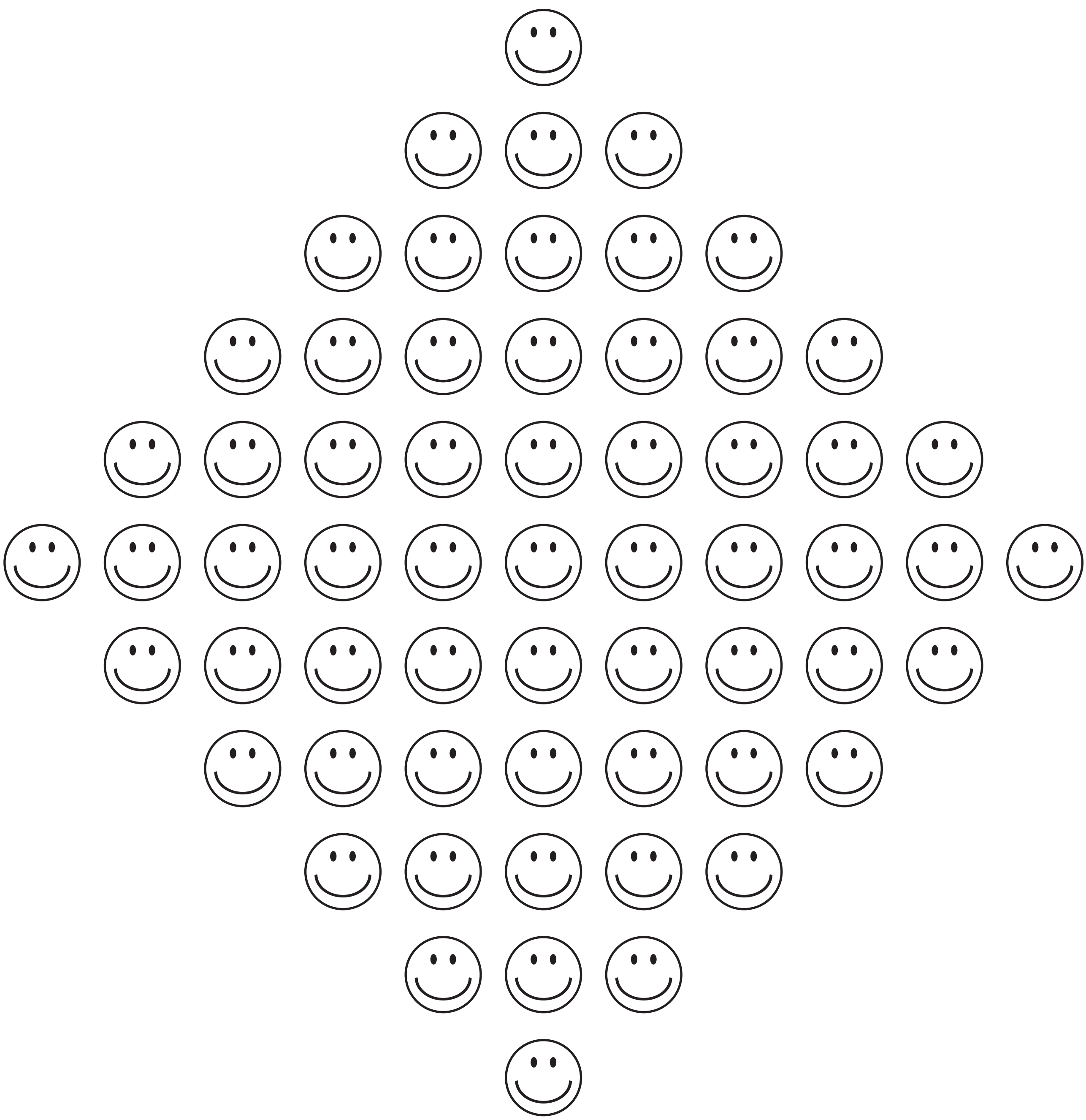
Activity

These are the ways we problem solved how many soccer balls there are in the soccer ball design:

Name .... Date ....

Using Patterns to Help Us Count . . . Smiley Faces

Directions: Work with your partner to see how many different ways you can find the total number of smiley faces without actually counting each one. Then answer the question on the next page.



Name .... Date ....

Using Patterns to Help Us Count . . .
Smiley Faces (continued)

Activity

These are some of the ways we problem solved how many smiley faces there are in the smiley face design:

The Function Machine

What is the algebra?

Functions, expressions, computation

What do you need?

- "Function Machine" activity sheets (pp. 30–31) for each pair of students
- "Design Your Own Function Machine" activity sheets (pp. 32–34) for each student.
- Overhead transparencies of the activity sheets

Some classroom procedures

In mathematics, a function relates each input number to exactly one output number. Function machines give young students a concrete example of a very abstract algebra concept. Sometimes called "What's My Rule?" the function machine consists of three areas: the input numbers, the rule area, and the output area. Students are able to find any of the three areas if the other two are known. For example, if the Input and Rule are known the Output can be found; if the Rule and Output are known, the Input can be found; or if the Input and Output are known, the Rule can be found.

To begin the lesson, show students the overhead of the first activity sheet, "The Function Machine." This activity sheet shows the input and rule; students need to find the output numbers. An example is shown at the top of the page.

On the second activity sheet, "The Function Machine—Find the Rule," the input and output are known and the rule is unknown. Again, work the example at the top of the page with students.

After reviewing the overheads, place students in pairs. Then distribute the two activity sheets students just reviewed. After the groups have finished problem solving, give them a chance to explain their thinking. Ask students to explain their answers as well as *the process they used to find their answers*.

The "Designing Your Own" activity sheet is slightly different. For this activity, each child needs his or her own sheet. Each student is asked to make up his or her own function machines (similar to the ones they just practiced) and then share their problems with partners.

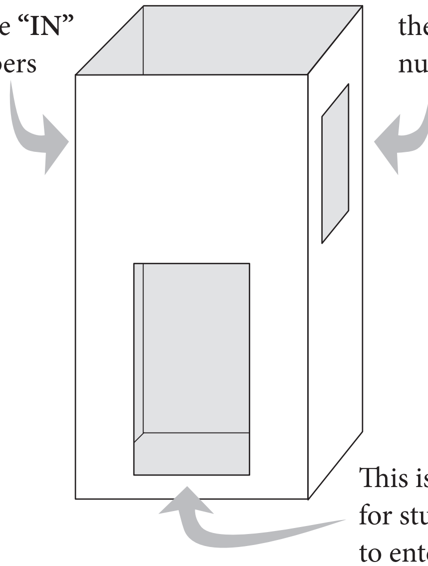
How to extend the lesson

Two sheets of blank function machines are included in the lesson to encourage differentiation. The first sheet has only two input numbers and is less difficult. The second sheet has three input numbers. Both address the same concept and can be used to better meet students' learning needs. You can check and duplicate these problems to make a class "Our Function Machine Book."

Another interesting project for primary students is to set up a "Real-life Function Machine." A very large carton (such as a refrigerator carton) can become a *child-sized function machine* (see the drawing on the next page). Have one student sit or stand inside the carton. He or she is the "Function Machine Computer."

Note that there is a window on either side of the carton; on one side students throw in an input number, and on the other side the student inside the box throws out the correct output number. (That student may need a calculator.) Students can write the input and output numbers in a table on the blackboard. After seeing a number of solutions, students work to find the function. The first child to solve the problem becomes the next "Function Machine Computer."

The Function Machine (continued)



Here is a sample of a table that can be used to record the inputs and outputs.

The Function Rule	
Input	Output

Activity answers

The Function Machine

1. 8 + 6 = 14
   3 + 6 = 9
2. 12 – 4 = 8
   9 – 4 = 5
3. 14 + 7 = 21
   8 + 7 = 15

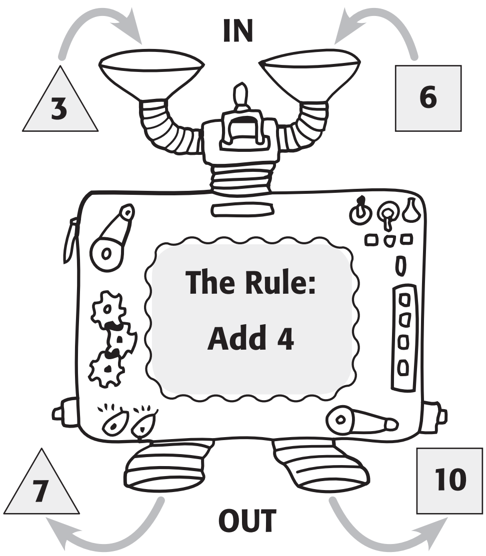
The Function Machine—Find the Rule

1. 6 – 2 = 4
   3 – 2 = 1
   4 – 2 = 2
   Rule: We subtract 2.
2. 10 + 5 = 15
   5 + 5 = 10
   8 + 5 = 13
   Rule: We add 5.
3. 9 + 3 = 12
   10 + 3 = 13
   14 + 3 = 17
   Rule: We add 3.

Name .... Date ....

The Function Machine

There is a wondrous machine called "The Function Machine," and this is how it works. Numbers go into the top. The machine is given a rule to follow for those numbers, and the correct answer comes out the bottom.



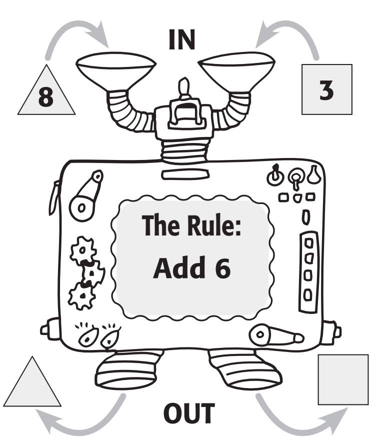
The problems for this Function Machine would look like this:

3 + 4 = 7

6 + 4 = 10

Directions: Now try these. Write out the problems underneath each Function Machine.

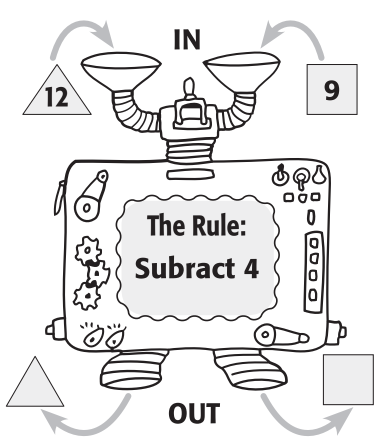
1



_____ +_____ = _____

_____ +_____ = _____

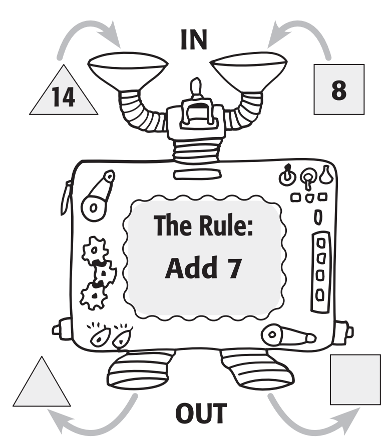
2



_____ +_____ = _____

_____ +_____ = _____

3



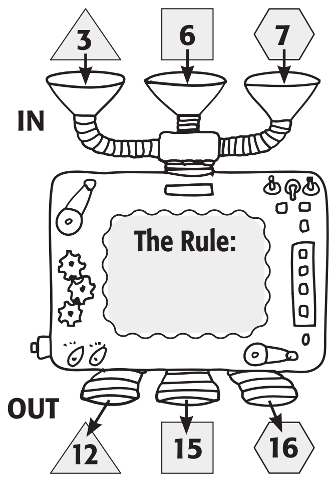
_____ +_____ = _____

_____ +_____ = _____

Name .... Date ....

The Function Machine—Find the Rule

Sometimes we are told the numbers that go into the machine and the numbers that come out of the machine but we don't know the rule. To find the rule, we sometimes need more than two numbers. Here is an example of a "Find the Rule" machine. We have input numbers and answers, or output numbers. Our job is to problem solve what operation to use. Let's look at this example:

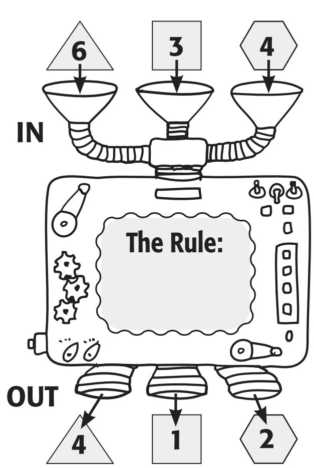


3 ? = 12	6 ? = 15	7 ? = 16
3 + 9 = 12	6 + 9 = 15	7 + 9 = 16

Rule: Add 9.

Directions: Now try these. Write the rule in each of the Function Machines.

1

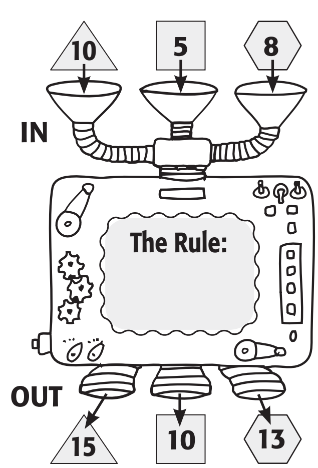


_____ + or – _____ = _____

_____ + or – _____ = _____

_____ + or – _____ = _____

2

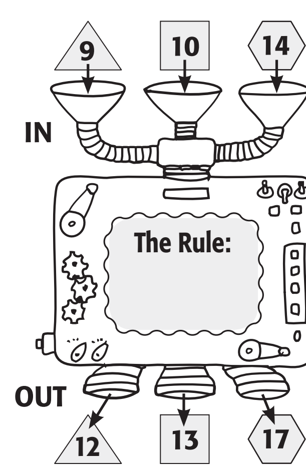


_____ + or – _____ = _____

_____ + or – _____ = _____

_____ + or – _____ = _____

3



_____ + or – _____ = _____

_____ + or – _____ = _____

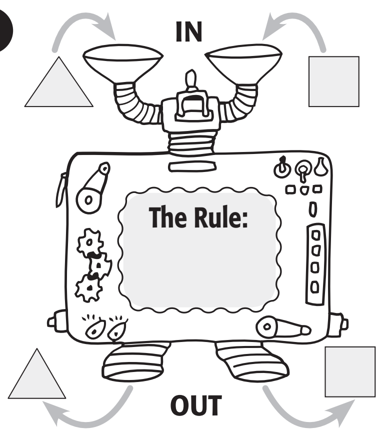
_____ + or – _____ = _____

Name .... Date ....

Design Your Own Function Machine

Directions: For problems 1 and 2, decide what numbers will go in the two "IN" polygons. Then define a "RULE." Write your rule in the space in the middle of the machine. Now let your partner find what the machine will turn out.

1

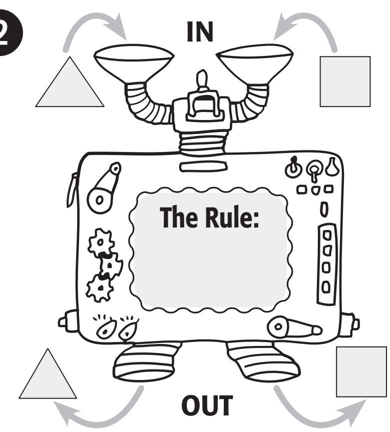


What are the rules?

Problem 1: ____________

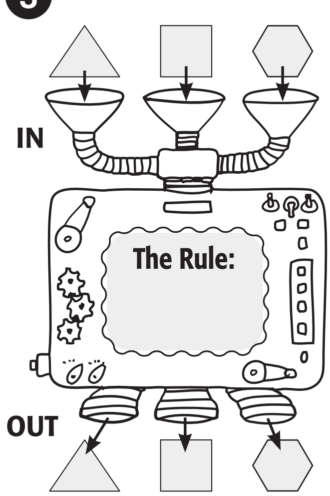
Problem 2: ____________

2



Directions: For problems 3 and 4, write the three numbers you wish to go IN to the machine in the shapes on the top. Now, think up a rule! Use that rule to compute what numbers will go in the correct shapes that come OUT of the machine. Give your function machine problems to your partner and see if he or she can figure out the rules you used for problems 3 and 4.

3

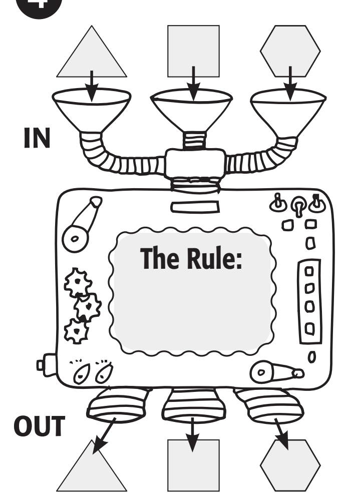


What are the rules?

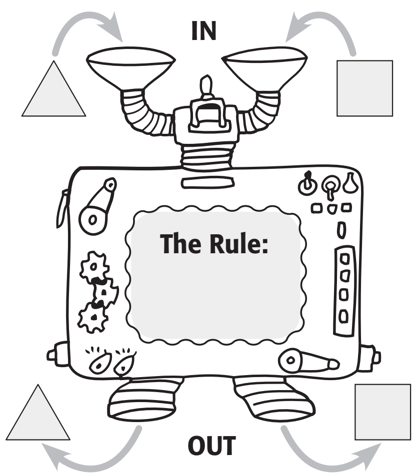
Problem 3: ____________

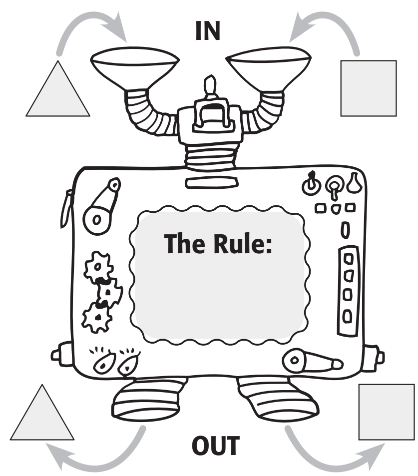
Problem 4: ____________

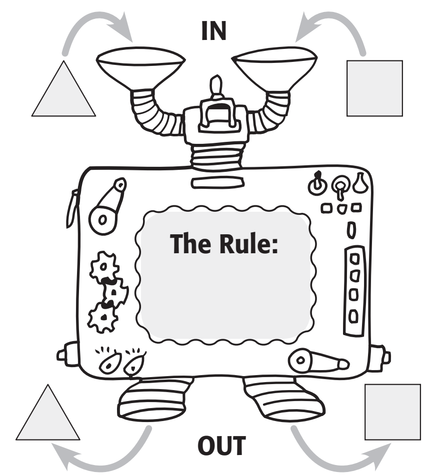
4

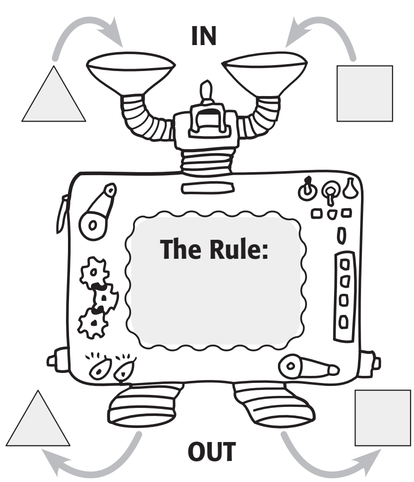


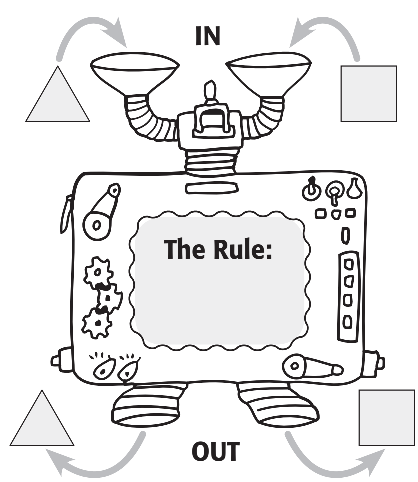
Blank Function Machines

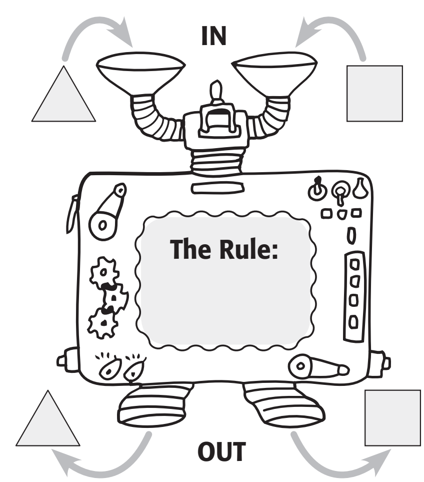


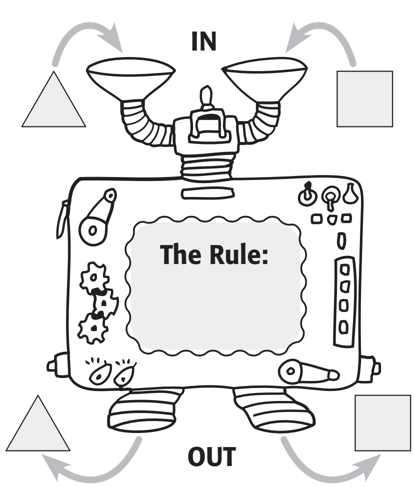


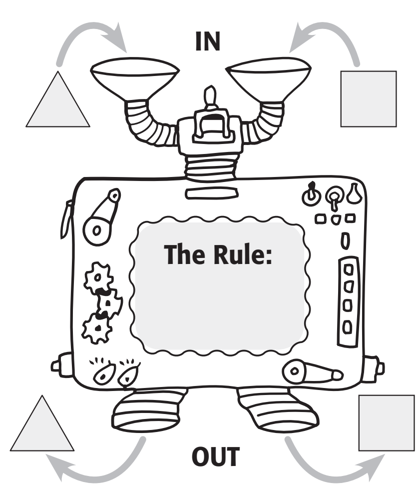


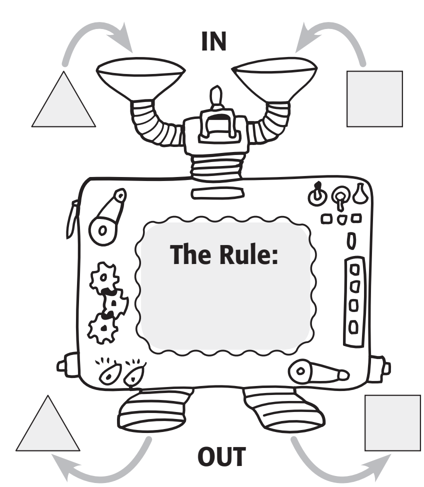




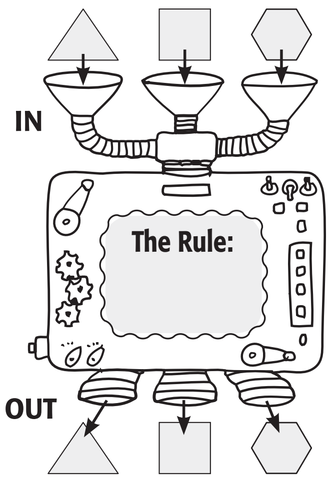


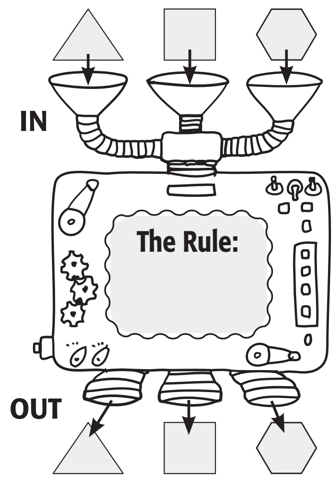


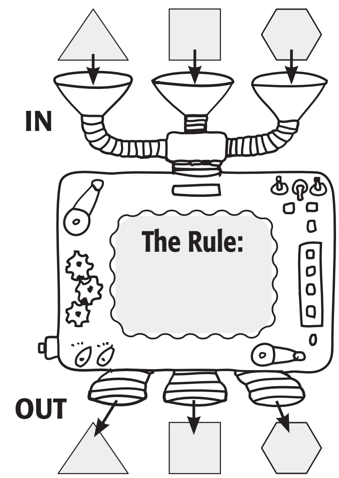


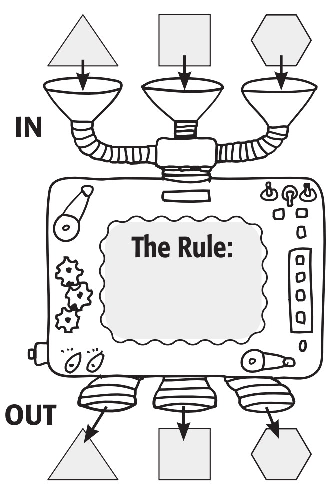


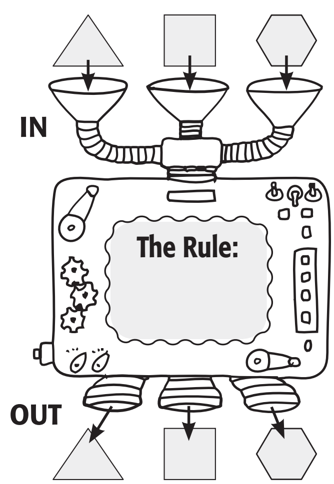
Blank Function Machines

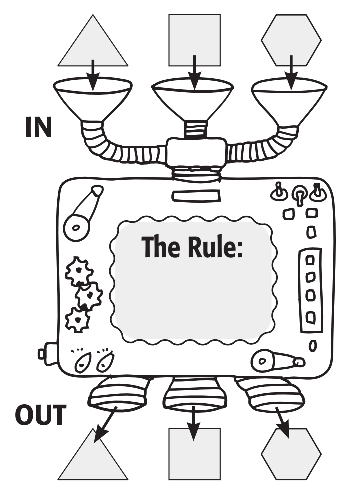


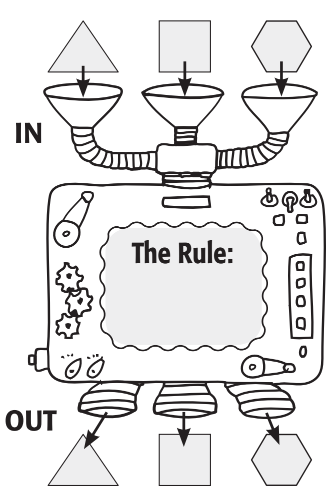


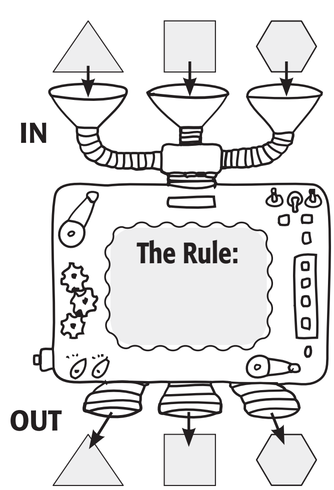


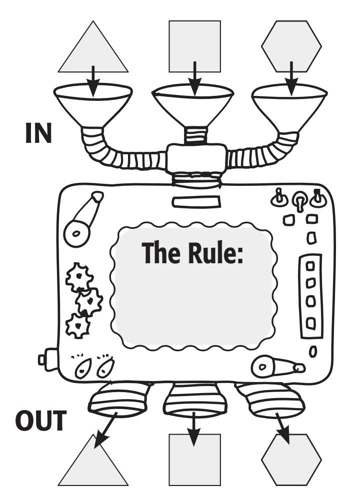












What's My Rule?

What is the algebra?

Understanding the concept of functions, finding patterns in numbers, designing original function tables

What do you need?

- "What's My Rule?" activity sheet (p. 36) for each student
- "Design My Own Rule" activity sheet (p. 37) for each pair of students
- Calculators, if necessary

Some classroom procedures

The activities related to "Function Machines" use geometric shapes to represent the input and output numbers in the functions. The problems in "What's My Rule?" use tables—a more abstract representation of functions. Also, these functions require students to use addition, subtraction, and multiplication to find the rules. These activities can be used to review computation skills.

To help students better understand the meaning of the numbers that appear in the IN and OUT columns and what the arrow signifies, use the following examples:

IN		OUT
1	→	3
3	→	5
4	→	6
6	→	8
10	→	12

When students understand what to do, hand out the "What's My Rule?" activity sheet. Have students complete the sheet and share their solutions with the class. Then place students into pairs and give them the "Design Your Own Rule" worksheet. Have students make up their own functions using addition, subtraction, multiplication, or division (depending on the grade level and math abilities of the students). They can share these original problems with the class.

How to extend the lesson

Some students may be ready for problems that contain more than one operation. The function $2n - 1$ is an example:

IN		OUT
3	→	5
6	→	11
5	→	9
4	→	7
10	→	19

Activity answers

1. IN + 1
2. IN + 7
3. IN × 3
4. IN – 6

Name .... Date ....

What's My Rule?

Activity

Work with a partner to find these rules. Each table has a different rule. Sometimes you will add the same number to the IN number to get the OUT number. Sometimes you will multiply or subtract. Write each rule on the lines provided.

1

IN		OUT
1	→	2
2	→	3
3	→	4
4	→	5
5	→	6
6	→	7
7	→	8

RULE: ______________

2

IN		OUT
1	→	8
2	→	9
3	→	10
4	→	11
5	→	12
6	→	13
7	→	14

RULE: ______________

3

IN		OUT
1	→	3
2	→	6
3	→	9
4	→	12
5	→	15
6	→	18
7	→	21

RULE: ______________

4

IN		OUT
10	→	4
11	→	5
12	→	6
13	→	7
14	→	8
15	→	9
16	→	10

RULE: ______________

Name .... Date ....

Design Your Own Rule

Activity

Work with your partner to design your own "What's My Rule?" tables. You may add, subtract, multiply, or divide, but remember that in each table, the same rule must be used to get every OUT number. For example, in the first table, if you and your partner decide to add 3 to your first IN number, then you must add 3 to all of the other IN numbers. See how creative you can be. Then share your "What's My Rule?" problems with your neighbor.

IN		OUT
	→	
	→	
	→	
	→	
	→	
	→	
	→	

RULE: ____________

IN		OUT
	→	
	→	
	→	
	→	
	→	
	→	
	→	

RULE: ____________

IN		OUT
	→	
	→	
	→	
	→	
	→	
	→	
	→	

RULE: ____________

IN		OUT
	→	
	→	
	→	
	→	
	→	
	→	
	→	

RULE: ____________

Chapter 2

Problem Solving Using Numbers, Pictures, and Symbols

It is the duty of all teachers, and of teachers of mathematics in particular, to expose their students to problems much more than to facts.
—Paul Halmos, 1916–2006

"Allowing the subject to be problematic means allowing students to wonder why things are, to inquire, to search for solutions, and to resolve incongruities. . . . both the curriculum and instruction should begin with problems . . ." (Hiebert et al., 1996, p. 12). Problem solving is the foundation of mathematics and good problems are those that don't have a prescribed or memorized rule or method; solution strategies must be carefully considered—they are not readily known and there is not *one right way to solve the problem*. The open-ended problems, those that have more than one possible solution or allow solutions to be found using different strategies, give students the opportunity to draw upon previous knowledge and become more confident learners of mathematics.

Chapter 2 begins with very unusual mathematical puzzles, "Sudoku Puzzles." Taking advantage of the current interest in Sudoku puzzles, these activities have been designed to give young children experiences using interesting problem-solving strategies. They start out with puzzles using pictures and progress to puzzles using numbers.

Solving problems using "Number Riddles," the chapter's next activity, is not only fun for students but it helps them connect number relationships to language. The riddles are written to encourage students to use algebraic methods to solve addition, subtraction, multiplication, division, and geometry problems.

"Find the Year Puzzles" make important connections for students between historical events and using algebra strategies to find mathematical answers. The birthdays of eight famous Americans are highlighted using clues that ask students to find the missing number in a sequence or a missing number in a computation problem. Students might enjoy making up their own "Find the Year Puzzles" for famous people in their own city or state.

"Math Jokes" will really tickle the mathematical funny bone of students. When students solve arithmetic, money, or geometry problems, they learn the answer to a joke or riddle.

And finally, in "Find the Missing Number 1 and 2," students use number charts to solve word problems. "Find the Missing Number 1" uses a 20-Chart containing the numbers 1 through 20 and asks students to solve problems using addition and subtraction skills. "Find the Missing Number 2" uses a 50-Chart and has problems that relate to all four arithmetic operations.

Using puzzles, riddles, jokes, and patterns in a number table, students have the opportunity to work on motivating activities that will help them develop algebraic thinking.

Sudoku Puzzles

What is the algebra?

Finding patterns, problem solving using pictures and numbers

What do you need?

- A copy of each puzzle page and the accompanying pieces (pp. 42–53) for each student
- Scissors
- Glue

Some classroom procedures

Sudoku puzzles are usually designed in a square of nine 3 × 3 grids. The player must strategically place the numerals 1 through 9 in each 3 × 3 square so that no numeral is used more than once in each individual square, row, or column.

These are more child-friendly Sudoku puzzles that children can do for practice. They consist of puzzles at two different levels of difficulty: four 2 × 2 squares or six 2 × 3 rectangles. Sudoku puzzles for children can be found at http://www.activityvillage.co.uk/_for_kids.htm. This British Web site includes puzzles with 2 × 2, 2 × 3, and 3 × 3 grids at various difficulty levels.

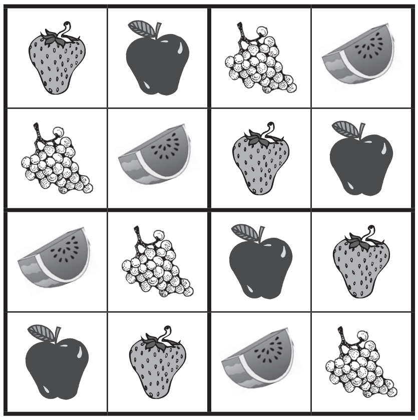
Some of the puzzles in this book use pictures, but as students progress through the levels, there are puzzles with numbers. Each puzzle consists of a grid page and a page of objects that students can cut out and use to fill in the puzzle. Once they are satisfied that they have a solution, they can glue the pieces into place and turn in the puzzle for your assessment.

How to extend the lesson

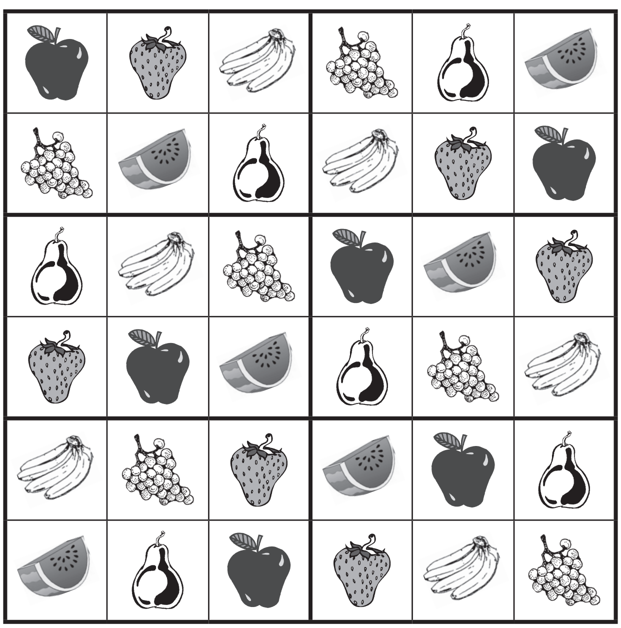
Give students a copy of a blank grid along with pictures of fruit, animals, or numbers, and ask them to design a puzzle to be included in a class puzzle book.

Solutions to Sudoku Puzzles

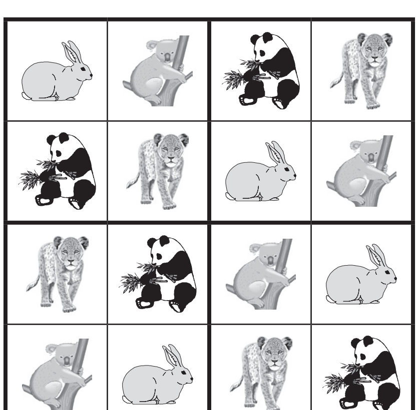
One possible solution to Sudoku Puzzle 1



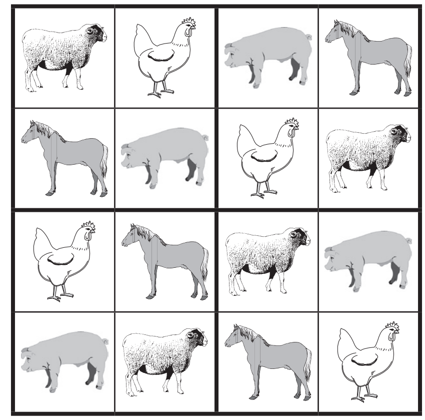
One possible solution to Sudoku Puzzle 2



One possible solution to Sudoku Puzzle 3



One possible solution to Sudoku Puzzle 4



One possible solution to Sudoku Puzzle 5

3	2	4	1
1	4	2	3
2	1	3	4
4	3	1	2

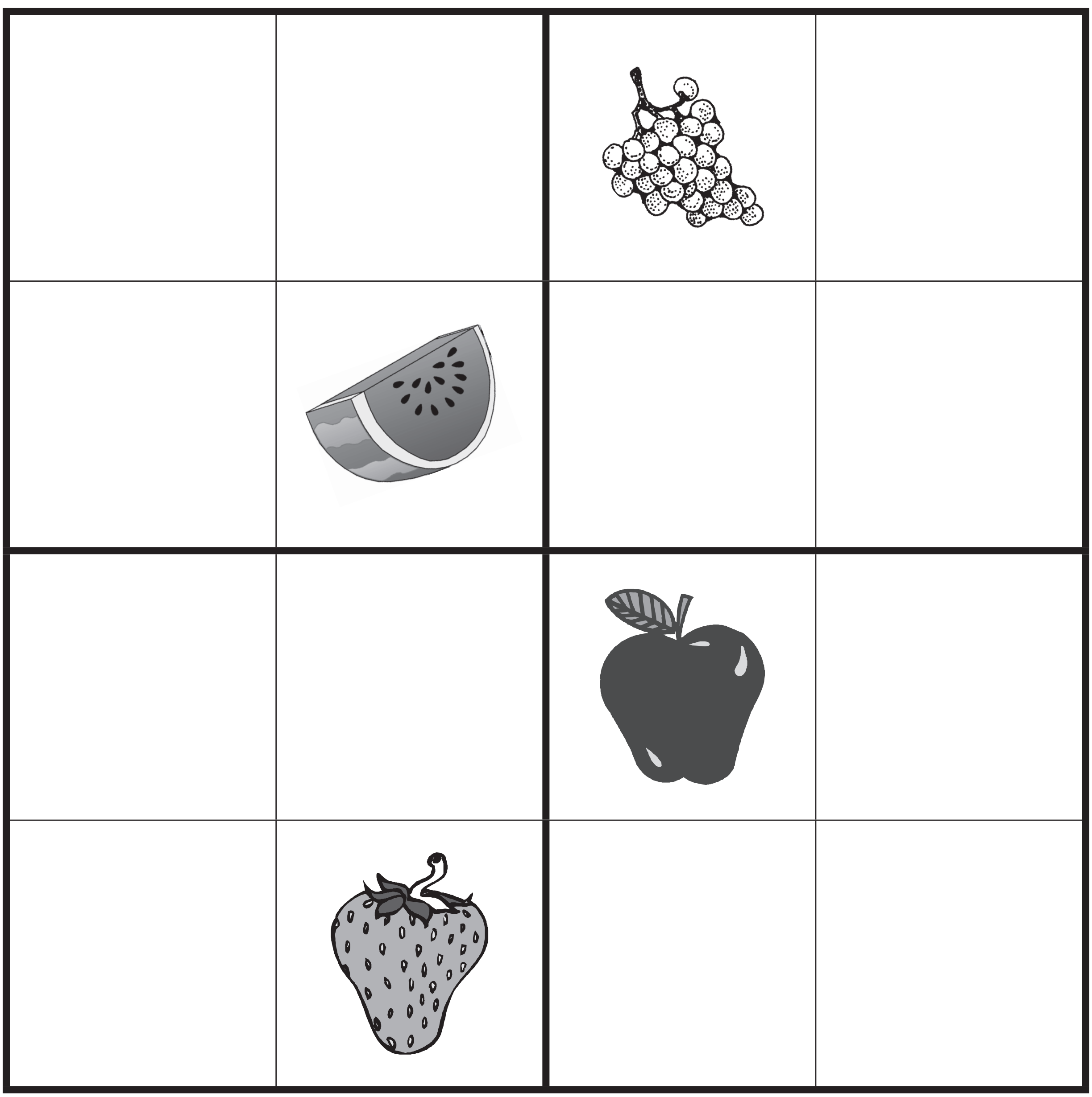
One possible solution to Sudoku Puzzle 6

2	1	5	6	4	3
6	3	4	5	1	2
4	5	6	2	3	1
1	2	3	4	6	5
5	6	1	3	2	4
3	4	2	1	5	6

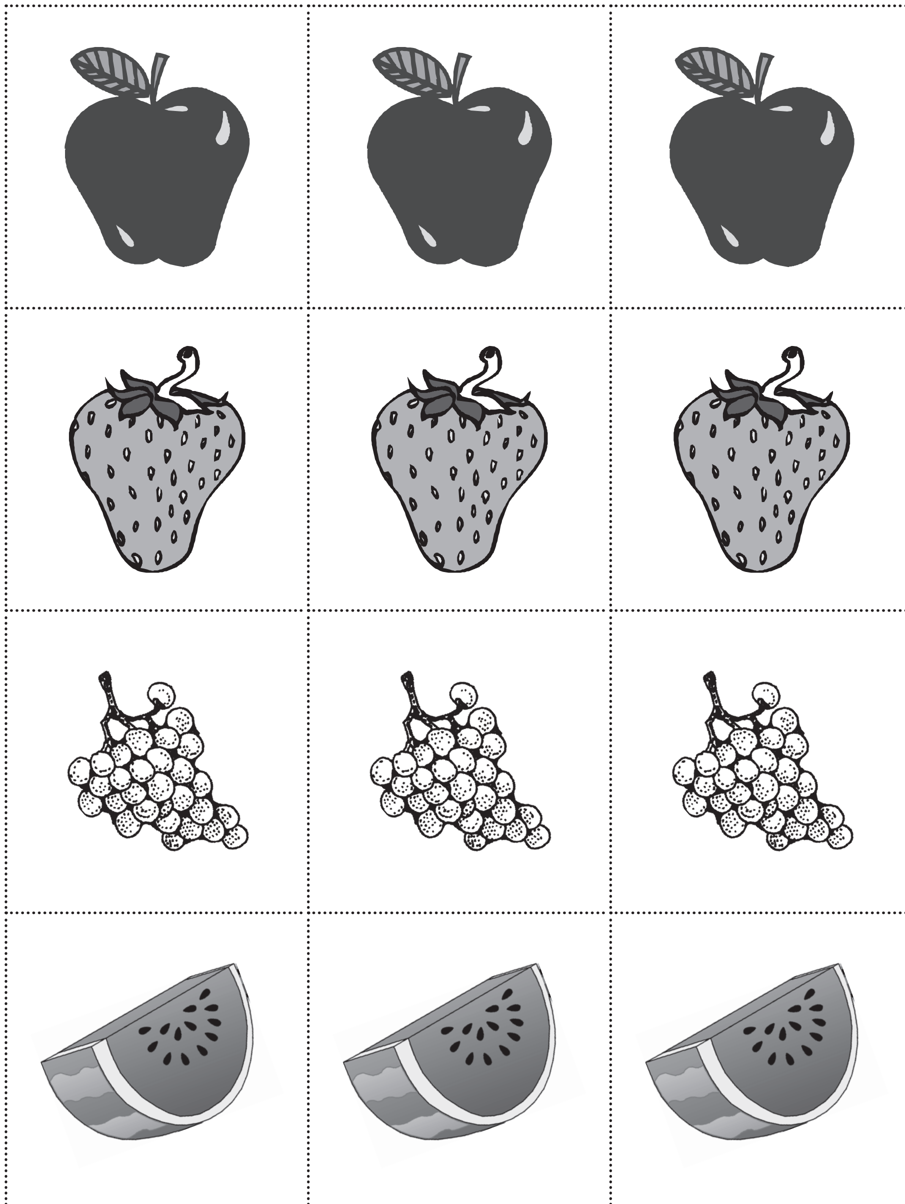
Name .... Date ....

Sudoku Puzzle 1

Directions: Cut out the fruit pictures on the next page and use them to help you solve this puzzle. Each 2 × 2 square must contain one apple, one strawberry, one slice of watermelon, and one bunch of grapes. Each row or column can have only one picture of each fruit.



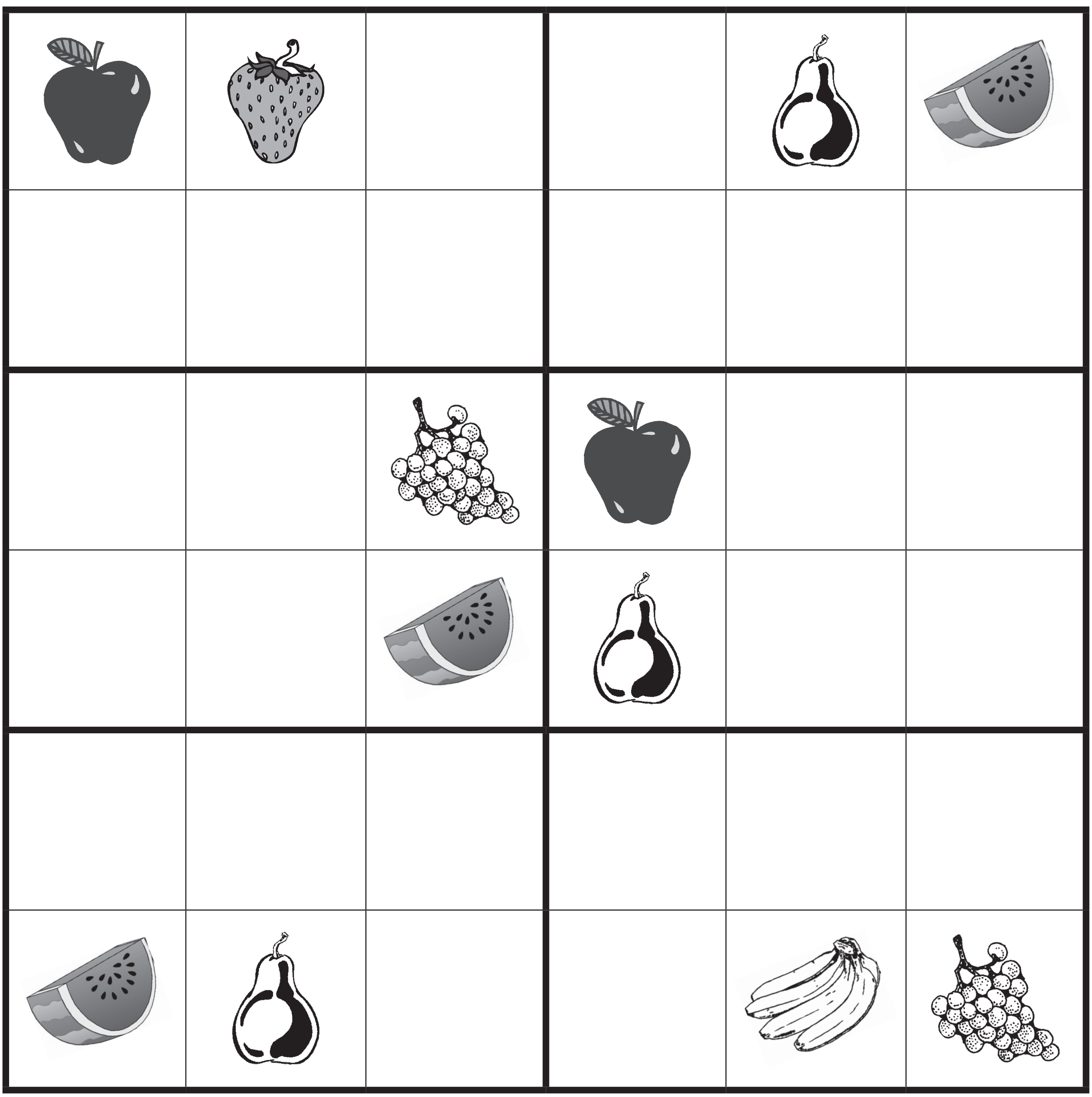
Puzzle Pieces for Sudoku Puzzle 1



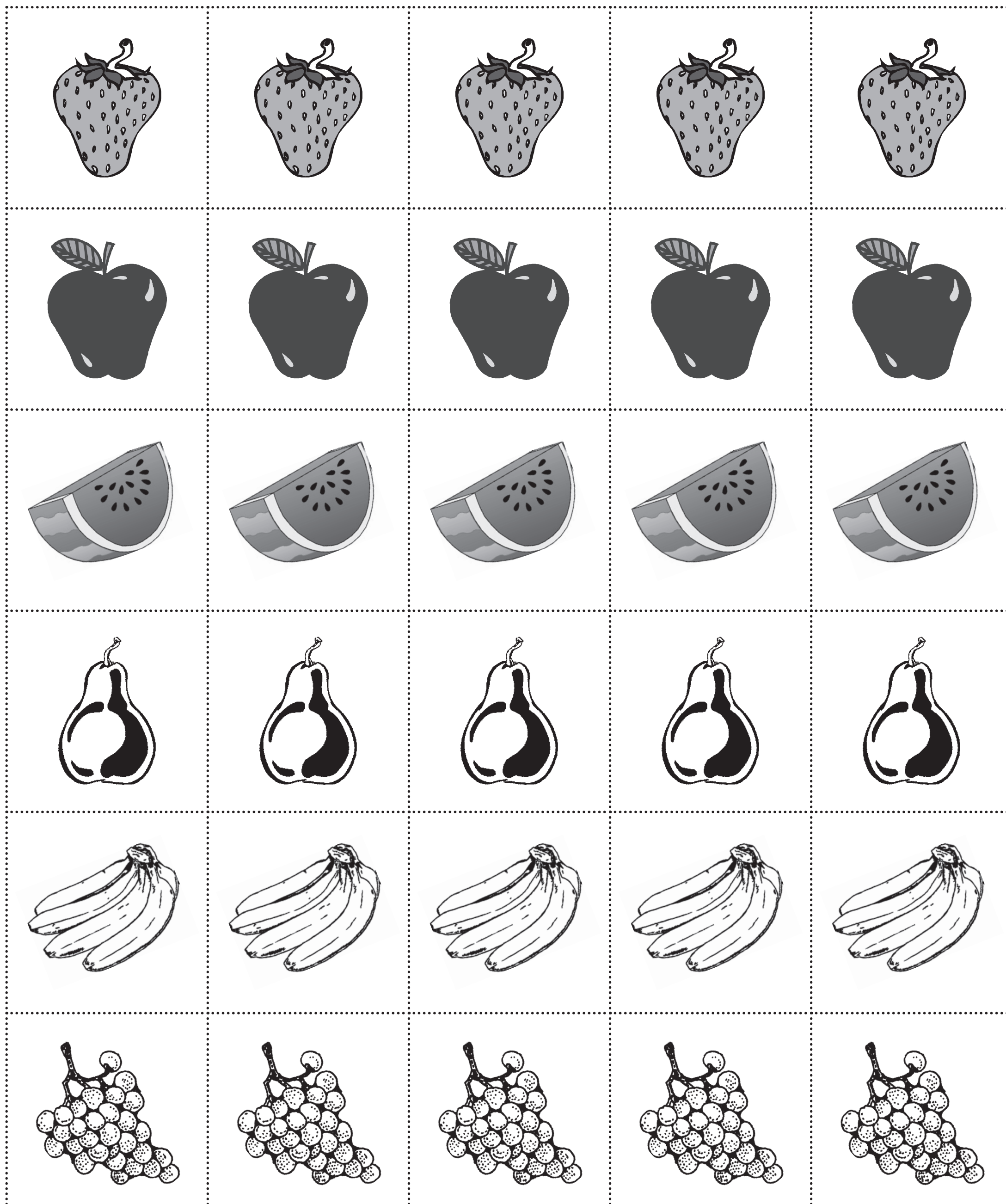
Name .... Date ....

Sudoku Puzzle 2

Directions: This puzzle contains six different varieties of fruit: apples, strawberries, grapes, pears, watermelon, and bananas. Cut out the fruit pictures on the next page and use them to solve this puzzle. Each 2 × 3 rectangle must have each type of fruit in it, and each row or column can have only one picture of each fruit. Happy problem solving!



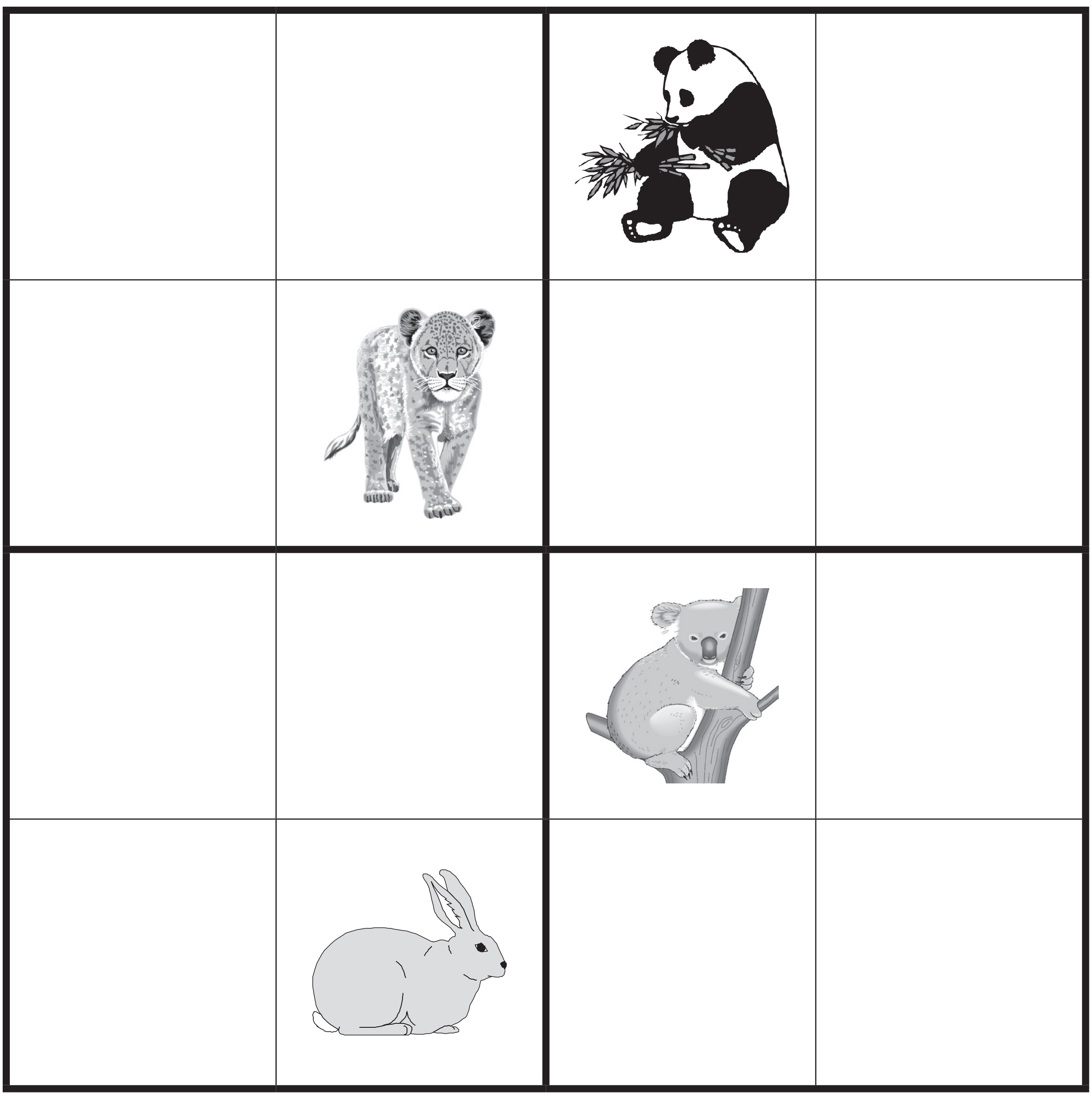
Puzzle Pieces for Sudoku Puzzle 2



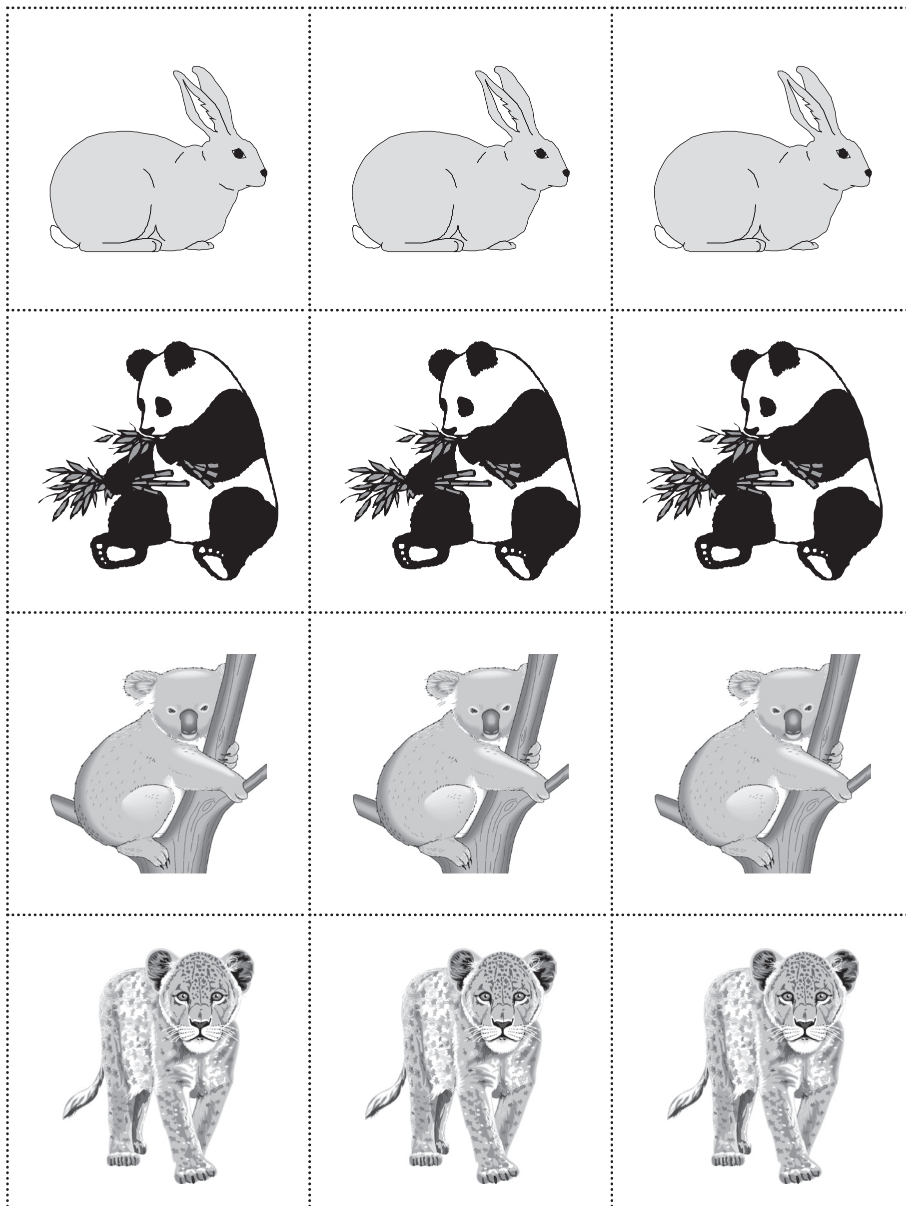
Name .... Date ....

Sudoku Puzzle 3

Directions: Cut out the animal pictures on the next page and use them to help you solve this puzzle. Each 2 × 2 square must contain one panda, one lion cub, one koala, and one rabbit. Each row or column can have only one picture of each animal.



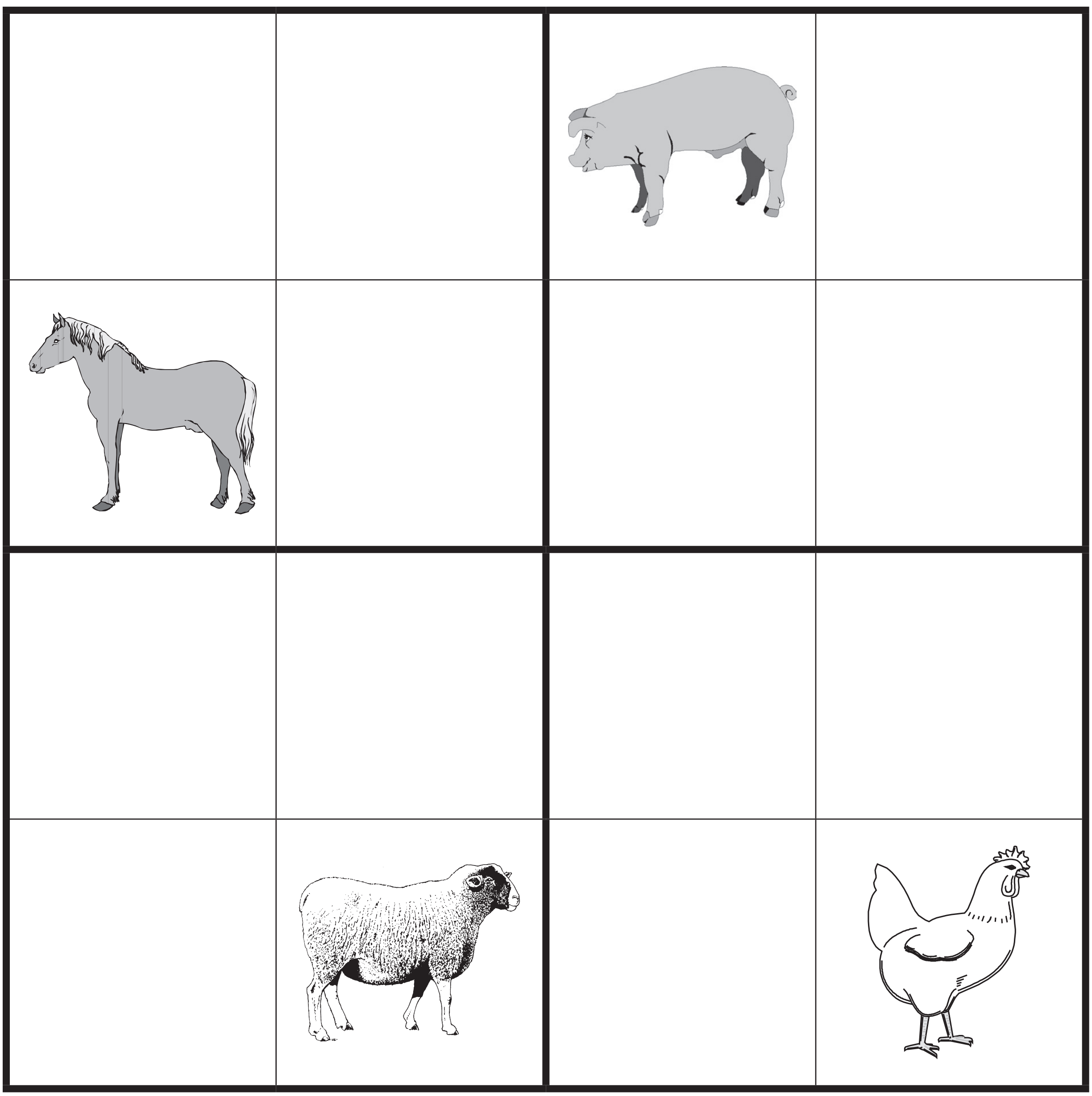
Puzzle Pieces for Sudoku Puzzle 3



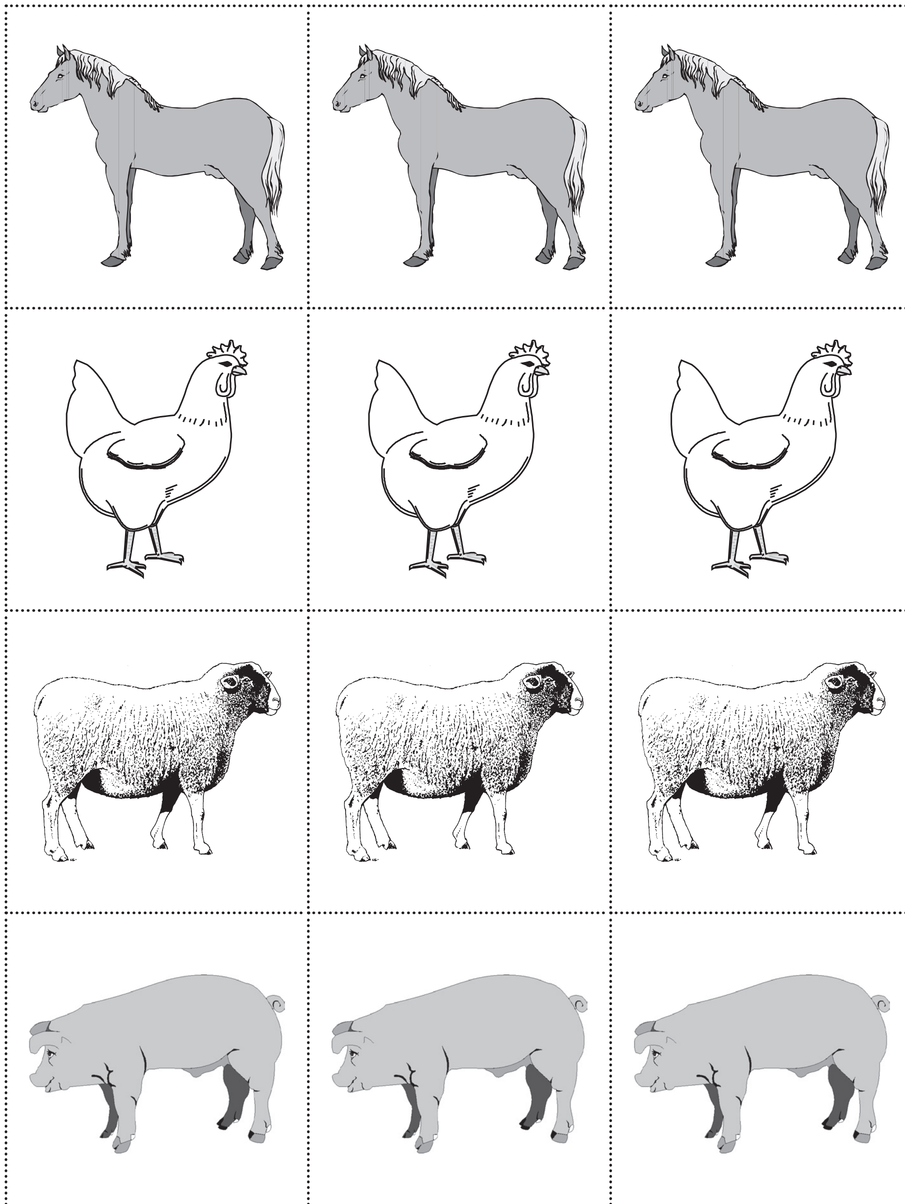
Name .... Date ....

Sudoku Puzzle 4

Directions: Cut out the animal pictures on the next page and use them to help you solve this puzzle. Each 2 × 2 square must contain one pig, one horse, one chicken, and one sheep. Each row or column can have only one picture of each animal.



Puzzle Pieces for Sudoku Puzzle 4



Name .... Date ....

Sudoku Puzzle 5

Activity

Directions: Cut out the numbers on the next page and use them to help you solve this puzzle. Each 2 × 2 square must contain one of each number (1, 2, 3, and 4). Each row or column can have each number only once.

		4	
1			
			4
	3		

Puzzle Pieces for Sudoku Puzzle 5

1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4

Name .... Date ....

Sudoku Puzzle 6

Directions: This puzzle contains six different numbers—1, 2, 3, 4, 5, and 6. Cut out the numbers on the next page to help you solve this puzzle. Each row or column can have each number only once. Each 2 × 3 rectangle must contain each number once. Happy problem solving!

2	1			4	3
		6	2		
		3	4		
3	4			5	6

Puzzle Pieces for Sudoku Puzzle 6

1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6

Number Riddles

What is the algebra?

Converting word clues to symbols, problem solving, using algebraic methods to interpret clues

What do you need?

- A copy of each number riddle page (pp. 55–60) for each student
- An overhead transparency of each number riddle, if available
- An overhead transparency of the sample riddles on this teacher's page

Some classroom procedures

Place a transparency of Sample Riddle 1 on the overhead projector and read it with students. The first column contains the riddle and the second shows the symbolic representation of the riddle. Fill in the second column as you read the riddle with students. Be sure to ask them to tell you how they think the sentence can be translated into math symbols. What is shown is only an example of possible symbolic representations.

Sample Riddle 2 will help students set up problems that require subtraction.

How to extend the lesson

As an enrichment activity have some students work with partners to write their own Number Riddles. They can be poems or just ordinary riddles, such as, "I am equal to 6 + 3 – 1. What number am I?"

Activity answers

Sum-thing Riddle 1: 8

Sum-thing Riddle 2: 17

Reduction Riddle: 7

A Times Rhyme Riddle: 5 and 2

Division Riddle: 3

Rectangular Riddle: 4

Sample Riddle 1	Converted to Symbols
To find my number, add 6—let's go,	□ + 6
But you'll need to know more than you know,	
11 is the sum,	□ + 6 = 11
Isn't this fun?	
The mystery number let's show.	□ + 6 = 11

The mystery number is 5.

Sample Riddle 2	Converted to Symbols
When 8 from this number you take away,	△ – 8
The answer to find a hint I'll say,	
The difference is 4,	△ – 8 = 4
I'll say no more,	
Your knowledge of math will show the way.	△ – 8 = 4

The mystery number is 12.

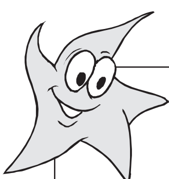
Name .... Date ....

Sum-thing Riddle 1

Activity

When adding my number and 4,
You can find my answer—no chore,
For the sum of these two,
I'll give you a clue,
The answer is 12, now we're through.

Directions: First read the clues then use the space below to help you solve the problem. HINT: Write the word clues as a mathematical sentence.

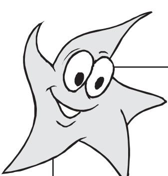


Name .... Date ....

Sum-thing Riddle 2

My number when added to 3,
Calculation is needed—you see,
This problem is true,
I'll give you a clue,
The answer is 20, you're through.

Directions: First read the clues then use the space below to help you solve the problem. HINT: Write the word clues as a mathematical sentence.

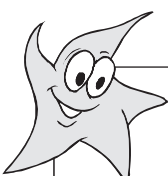


Name .... Date ....

Reduction Riddle

Subtraction can really perplex,
Follow the clues that surely come next,
Take 5 from this number,
The difference is 2,
We're expert in minuses too.

Directions: First read the clues then use the space below to help you solve the problem. HINT: Write the word clues as a mathematical sentence.

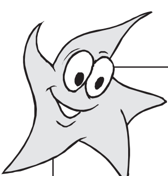


Name .... Date ....

A Times Rhymes Riddle

There are two digits; their product is 10,
Their difference is 3, so then,
One digit is odd,
One's even as well,
Find the numbers; the answer is swell.

Directions: First read the clues then use the space below to help you solve the problem. HINT: Write the word clues as a mathematical sentence.

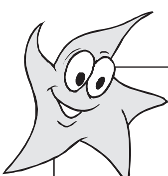


Name .... Date ....

Division Riddle

Let's look at 12 and divide,
By a number we're trying to hide,
Our quotient is 4,
You don't need more,
The mysterious number provide.

Directions: First read the clues then use the space below to help you solve the problem. HINT: Write the word clues as a mathematical sentence.

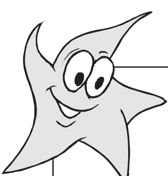


Name .... Date ....

Rectangular Riddle

One side of this rectangle is 6, you know,
The other's unknown—a mystery to show,
The distance around,
Is 20 I've found,
The mystery side no longer confounds.

Directions: First read the clues then use the space below to help you solve the problem. HINT: Write the word clues as a mathematical sentence.



Find the Year Puzzles

What is the algebra?

Solving equations, recognizing number patterns, learning mathematical terms, computing sums, differences, products, and quotients, solving problems

What do you need?

- A copy of each "Find the Year" puzzle sheet (pp. 63–70) for each student
- An overhead transparency of each puzzle sheet
- Calculators, if necessary

Some classroom procedures

These puzzles have been designed to make connections between historical events and beginning algebra concepts. Puzzles are used to help *mathematically celebrate* the historical event described in each puzzle. For example, February 12, Abraham Lincoln's birthday, can be celebrated by solving the puzzle to learn the year he was born.

Depending on the reading level of students, it may be necessary to make an overhead transparency of each of these puzzles and read the clues to them. While the details of the event are short, this is an excellent opportunity to make connections between mathematics and history by spending some time researching each of these famous birthdays. All of these puzzles relate to people who have had a great impact on history—Benjamin Franklin; former Secretary of State Colin Powell; President George Washington; Helen Keller; Dr. Martin Luther King, Jr.; Albert Einstein; Rosa Parks; and Florence Nightingale.

In recent years March 14 (3.14) has been celebrated as π Day (Pi Day) because the first three digits of the irrational number pi are 3.14 (3/14). Because Albert Einstein was born on π Day, his birthday can be included in any school celebration that is being planned!

The clues in each of these puzzles vary but they include solving for a variable, finding a missing number in a sequence of numbers, and using the language of mathematics (sum, difference, product, etc.) to solve a word problem. Give the puzzles to students one at a time around the time that the event or birthday occurred. Have students work with a partner so they can discuss the problem and communicate the mathematical ideas.

How to extend the lesson

You can design similar puzzles to reflect historical events for your state or city. Simply write a clue for each digit in the date (or, for more advanced students, combine digits to form two-digit numbers). Give students the calendar date of the event (such as April 1) and have them find the year of the occurrence.

Here are some great Web sites you can use to enrich these lessons:

- http://www.berwickacademy.org/lincoln/timeline.htm
  Using children's drawings as links, students can explore Lincoln's boyhood years, his years as a young man, his early politics, and his presidency. There are also links to other Web sites and tips for teachers, as well. It is worth a look!
- http://www.kids.gov/k_history.htm
  A government site that has many links to historical events.

Find the Year Puzzles (continued)

Activity answers

Find the Year Puzzle 1: Benjamin Franklin was born on January 17, 1706.

Find the Year Puzzle 2: Colin Powell was born on April 5, 1937.

Find the Year Puzzle 3: George Washington was born on February 22, 1732.

Find the Year Puzzle 4: Helen Keller was born on June 27, 1880.

Find the Year Puzzle 5: Martin Luther King, Jr., was born on January 15, 1929.

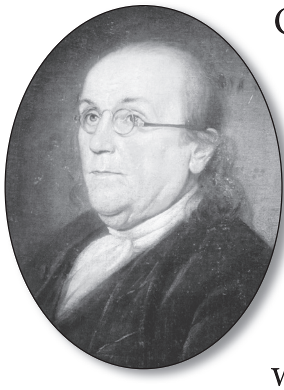
Find the Year Puzzle 6: Albert Einstein was born on March 14, 1879.

Find the Year Puzzle 7: Rosa Parks was born on February 4, 1913.

Find the Year Puzzle 8: Florence Nightingale was born on May 12, 1820.

Name .... Date ....

Find the Year Puzzle 1



On January 17 of this year, **Benjamin Franklin** was born in Boston, Massachusetts. Born the tenth son of a soap maker, he became one of the most famous men of his time.

He published *Poor Richard's Almanack*. Almanacs were books that were printed once a year and contained things like weather reports, recipes, and predictions. His almanac contained many of his famous quotations. One of them was "Be slow in choosing a friend, slower in changing." What do you think this means?

When Ben Franklin retired from business, he became interested in science and is famous for his many inventions, including the "Franklin Stove." His experiment using a kite to find electricity in a thunderstorm made him famous around the world.

Ben Franklin was elected to the Second Continental Congress and worked on a committee that helped Thomas Jefferson write the Declaration of Independence.

Directions: Solve this puzzle to learn Benjamin Franklin's birth year.

- The digit in the tens place is the missing number in this number pattern: ______, 2, 4, 6, 8, 10, 12.
- The digit in the ones place is the same as the number given to the month of June.
- Solve this problem and learn the digit that's in the hundreds place: 2 + ______ = 9.
- Add up all my digits—their sum is 14.

What year was Benjamin Franklin born? ______ ______ ______ ______

Thousands Hundreds Tens Ones

Name .... Date ....

Find the Year Puzzle 2

On April 5 of this year, **Colin Powell**, an American soldier and statesman, was born in New York City. He was raised in the south Bronx by parents who had immigrated from Jamaica. They believed that education was very important.

He went to the City College of New York and George Washington University in Washington, DC.

Colin Powell served in the U.S. Army and during his military career was promoted from second lieutenant to general. He was appointed chairman of the Joint Chiefs of Staff and has received 11 military decorations, including the Purple Heart, the Bronze Star, the Presidential Medal of Freedom and the Legion of Merit. He was also given honorary knighthood from the Queen of England. He became the first African American to serve as Secretary of State.

Directions: Solve this puzzle to learn the year Colin Powell was born.

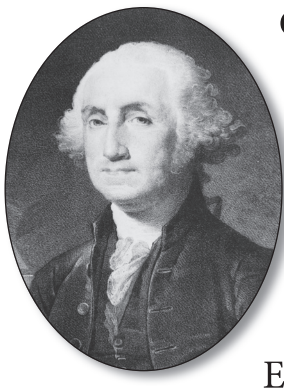
* The digit in the hundreds place is the missing number in this equation: _____ – 7 = 2.
* The digit in the ones place is the same as the number of days in one week.
* The tens digit is the missing number: 10 – 7 = _____.
* If you add up all of the digits, the sum is 20.

What year was Colin Powell born?

_____	_____	_____	_____
Thousands	Hundreds	Tens	Ones

Name ______________________ Date ______________

Find the Year Puzzle 3



George Washington, the first president of the United States, was born on February 22 in Westmoreland County, Virginia, to a family of Virginia planters.

He was the commander-in-chief of the Continental Army during the American Revolution. After the war, when the new Constitution of the United States was ratified, the Electoral College elected George Washington the first president of the United States. He took the oath of office in New York City.

Among the famous Americans he had working with him were Thomas Jefferson and Alexander Hamilton. Thomas Jefferson was his Secretary of State and Alexander Hamilton was his Secretary of the Treasury.

Directions: To learn the year George Washington was born, solve this puzzle.

* The digit in the hundreds place is the missing number in this number pattern: 1, 3, 5, _____, 9, 11.
* Solve this problem and learn the digit that is in the tens place: 6 ÷ _____ = 2.
* Find the missing addend and learn the digit that's in the ones place: _____ + 11 = 13.
* Add up all my digits—their sum is 3 more than 10.

What year was George Washington born?

_____	_____	_____	_____
Thousands	Hundreds	Tens	Ones

Name .... Date ....

Find the Year Puzzle 4

Helen Keller was born on June 27 in Alabama. When she was born, she was able to hear and see. But when she was nineteen months old, she became very ill and lost her sight and hearing.

Helen became very wild, but her mother knew that she was very intelligent. When Helen was seven years old, her mother hired a very special teacher, Anne Sullivan. Anne learned how to communicate with Helen. Because of her love of learning, Helen learned to read, write, and even speak. She became famous around the world for her accomplishments.

Helen Keller showed the world that being deaf and blind did not make a person helpless. She worked hard to protect the rights of people with special needs.

Directions: To learn the year Helen Keller was born, just solve this puzzle.

* The two-digit number formed by the digits in the tens and ones place is the product of 8 and 10.
* The digit in the hundreds place is three more than the number of toes on one foot.
* Find the missing addend and learn the digit that's in the thousands place: _____ + 30 = 31.
* Add up all my digits—their sum is 17.

What year was Helen Keller born?

_____	_____	_____	_____
Thousands	Hundreds	Tens	Ones

Name .... Date ....

Find the Year Puzzle 5

Reverend Martin Luther King, Jr., was born on January 15 in Atlanta, Georgia. His father was the pastor of the Ebenezer Baptist Church in Atlanta.

Dr. King went to Morehouse College in Atlanta. He was awarded a fellowship to attend Boston University and received his doctorate degree from this university.

He was one of the most important civil rights leaders in the United States, giving more than 2,500 speeches to protest the injustices of discrimination. His famous "I Have a Dream" speech, which he delivered at the Lincoln Memorial in Washington, D.C., became famous around the world. One of the lines of this speech was: "I have a dream that my four little children will one day live in a nation where they will not be judged by the color of their skin but by the content of their character." What do you think Dr. Martin Luther King, Jr., meant by this?

At 35 years old, he was the youngest man to win the Nobel Peace Prize. He donated his prize money (more than $54,000) to the civil rights movement.

Directions: To learn the year Dr. King was born, solve this puzzle.

* The digit in the hundreds place is the missing number in this number pattern: 0, 3, 6, ______, 12, 15.
* Find the missing divisor and learn the digit that is in the tens place: 6 ÷ ______ = 3.
* Solve this problem and learn the digit that's in the ones place: 20 –_____ = 11.
* Add up all my digits—their sum is 1 more than 20.

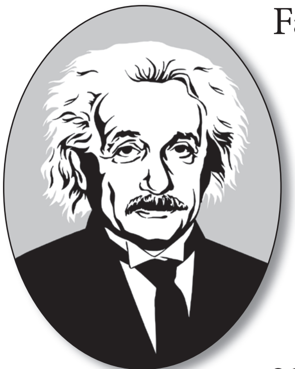
What year was Martin Luther King, Jr., born?

______ ______ ______ ______

Thousands Hundreds Tens Ones

Name .... Date ....

Find the Year Puzzle 6



Famous physicist **Albert Einstein** was born on March 14 in Germany. Later, his family moved to Italy, and Einstein was trained as a teacher in physics and mathematics.

Albert Einstein became a U.S. citizen and taught physics at Princeton University, becoming one of the most important scientists of the twentieth century.

As a child, Albert received a compass from his father and was fascinated that the needle in the compass would always point to the north. Albert's father explained that there were magnetic forces that drew the needle to the north pole of the Earth. This started young Einstein on his search for scientific knowledge. He later won the Nobel Prize in physics.

Directions: Solve this puzzle to learn the year this scientist was born.

* The digit in the tens place is the same as the number of days in one week.
* The digit in the ones place is 2 more than the number in the tens place.
* The digit in the hundreds place is the same as the number of sides of an octagon.
* Add up all my digits—their sum is equal to five groups of 5.

What year was Albert Einstein born?

______	______	______	______
Thousands	Hundreds	Tens	Ones

Name .... Date ....

Find the Year Puzzle 7

Activity

February 4 is the birthday of **Rosa Parks**, "the mother of the civil rights movement." She was born in Alabama. On her 77th birthday she said, "I would like to be known as a person who is concerned about freedom and equality and justice and prosperity for all people."

In 1955 Rosa Parks refused to give up her seat on a Montgomery, Alabama, bus to a white man. After she was arrested, she helped organize the Montgomery bus boycott. She said, "For more than a year, we stayed off those buses. We did not return to using public transportation until the Supreme Court said there shouldn't be racial segregation."

Read more about Rosa Parks and her role in the civil rights movement in the United States.

Directions: To learn the year Rosa Parks was born, solve this puzzle.

* The digit in the hundreds place is the missing quotient: $18 \div 2 =$ _____.
* The number in both the tens and thousands place is the same as the number given to the month of January.
* Find the missing addend and learn the digit that is in the ones place: $8 +$ _____ $= 11$.
* Add up all my digits—their sum is 1 less than 15.

What year was Rosa Parks born?

_____	_____	_____	_____
Thousands	Hundreds	Tens	Ones

Name .... Date ....

Find the Year Puzzle 8

Florence Nightingale was born on May 12 in Florence, Italy, but worked mostly in England. She is best remembered for her work as a nurse, but very few people know that she used math to help her figure out how to help soldiers after they had been wounded in battle.

By using graphs that looked very much like pie charts, she showed the world how important it was to have sanitary conditions in hospitals. She saved many lives. She also worked very hard to improve conditions in hospitals that helped very poor patients.

Directions: To learn the year Florence Nightingale was born, solve this problem.

* The two-digit number in the tens and ones place is the missing product in this equation: $4 \times 5 =$ _____.
* The digit in the hundreds place is the missing number in this sequence: 4, _____, 12, 16, 20, 24.
* The digit in the thousands place is the same as the missing factor in this equation: _____ $\times 235 = 235$.
* Add up all my digits—their sum is 11.

What year was Florence Nightingale born?

_____	_____	_____	_____
Thousands	Hundreds	Tens	Ones

Math Jokes

What is the algebra?

Using a variable, finding missing addends, subtrahends or minuends, factors or quotients, solving money problems, finding perimeter, identifying geometric shapes

What do you need?

- A "Math Jokes" puzzle sheet (pp. 72–77) for each student
- Computation tables or calculators, if needed

Some classroom procedures

These puzzles can be used with students in grades 1, 2, 3, or 4. Because of the different math skills required in each puzzle, they will work well in classrooms that differentiate the curriculum based upon student needs. Some of the problems are addition and subtraction; some are simple multiplication and division. One problem asks students to find the sum of a group of coins and one requires students to find the perimeter of simple geometric shapes. Choose which puzzles would best suit your students. All of the "joke puzzles" follow the same blueprint:

1. Each problem is assigned a letter (instead of a number).
2. Students solve each problem and look for the answer below the puzzle.
3. They place the letter of the problem at the bottom of the page above the answer to the problem.
4. When all of the problems are solved, the answer will appear at the bottom of the page.

How to extend the lesson

Puzzles are appreciated by older as well as primary students. This is a great project to work across grade levels! Students in grades 5 and 6 can design puzzles that can be used with primary-age students. If there is time, they can even mentor younger students to solve their puzzles.

Activity answers

Math Joke 1:
"Where were the first French fries made?"
In grease

Math Joke 21:
"Why do we measure snakes in inches?"
They have no feet.

Math Joke 3:
"What do telephones do on January 1?"
Ring in the new year

Math Joke 4:
"If people keep canaries and parakeets as pets, what do ghosts keep?"
Boo jays

Math Joke 5:
"What follows a cat wherever she goes?"
The tail

Math Joke 6:
"What has 4 legs but can't walk?"
A table

Name ______________________ Date ______________

Math Joke 1

Where were the first French fries made?

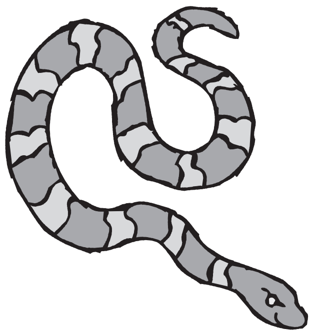
Directions: Solve each problem. Write the letter of the problem above the answer shown at the bottom of the page.

R 2 + 5 = ___	**G** ___ −1 = 4	**E** 5 – ___ = 2
I ___ + 3 = 4	**N** ___ – 2 = 2	**S** 8 – ___ = 6
A 4 + 2 = ___		

1	4		5	7	3	6	2	3

Name .... Date ....

Math Joke 2



Why do we measure snakes in inches?

Directions: Solve each problem. Write the letter of the problem above the answer shown at the bottom of the page.

T ___ + 3 = 10	**E** 6 + ___ = 10	**F** 11 – ___ = 10
H 2 + 3 = ___	**A** ___ –2 = 6	**O** 6 – ___ = 3
V ___ + 8 = 10	**Y** ___ –2 = 7	**N** 2 + ___ = 8

7	5	4	9

5	8	2	4

6	3

1	4	4	7

Name .... Date ....

Math Joke 3

What do telephones do on January 1?

Directions: Solve each problem. Write the letter of the problem above the answer shown at the bottom of the page.

G ____ × 2 = 18	**H** 20 ÷ ____ = 5	**E** 7 × ____ = 21
I 20 ÷ ____ = 10	**T** 35 ÷ 35 = ____	**R** ____ × 7 = 35
W ____ ÷ 2 = 3	**N** ____ × 4 = 32	**Y** ____ × 4 = 28
A 10 × ____ = 0		

__ __ __ __   __ __   __ __ __   __ __ __   __ __ __ __

5 2 8 9   **2 8**   **1 4 3**   **8 3 6**   **7 3 0 5**

Name .... Date ....

Math Joke 4

If people keep canaries and parakeets as pets, what do ghosts keep?

Directions: Solve each problem. Write the letter of the problem above the answer shown at the bottom of the page.

J ___ × 4 = 16	**O** ___ ÷ 3 = 4	**S** 5 × ___ = 15
A 24 ÷ ___ = 4	**B** ___ × 3 = 27	**Y** 24 ÷ ___ = 3

___	___	___
9	12	12

___	___	___	___
4	6	8	3

Name .... Date ....

Math Joke 5

What follows a cat wherever she goes?

Directions: Count up the coins. Write the letter of the problem above the answer at the bottom of the page.

A

Total Equals ________

H

Total Equals ________

E

Total Equals ________

T

Total Equals ________

L

Total Equals ________

I

Total Equals ________

55¢	41¢	60¢

55¢	27¢	57¢	28¢

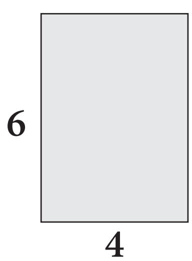
Name .... Date ....

Math Joke 6

What has 4 legs but can't walk?

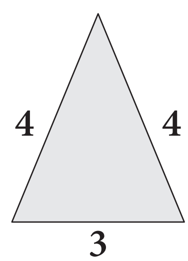
Directions: Find the perimeter of each shape. Write the letter of the problem above the answer at the bottom of the page.

L



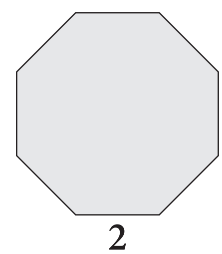
Rectangle

T



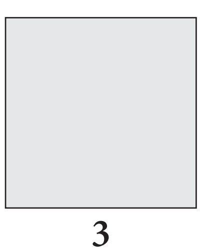
Isosceles triangle

E



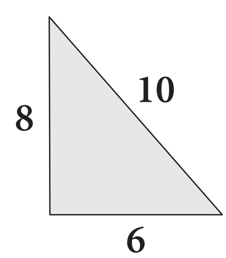
Regular octagon

B



Square

A



Right triangle

24

11	24	12	20	16

Find the Missing Number

What is the algebra?

Understanding a variable, solving open-ended problems, using mathematical terms

What do you need?

- "Find the Missing Number 1" or "Find the Missing Number 2" activity sheet (pp. 80–81) for each student
- Overhead transparencies of activity sheets, if available
- Overhead transparency of 20-Chart or 50-Chart, if available

Some classroom procedures

As an introduction to these activities, review the important math vocabulary used in the problems. Vocabulary words in "Finding the Missing Number 1" are *sum* and *difference*. Vocabulary words in "Finding the Missing Number 2" are *sum*, *difference*, *product*.

In both activities, students will use number charts—either a 20-Chart or a 50-Chart—to help with computation. To prepare them to use these charts, place a transparency of the number chart being used in each lesson on the overhead and go over sample problems. A possible example might be: "If you were to add 6 to this number, you would get a sum of 16. What is the missing number?" Have students discuss possible strategies to solving this problem using the number chart. It is important that students use prior knowledge to help them find a strategy that works for them.

In some instances students might want to count backward from the answer; others may find it easier to use inverse operations. Give students the opportunity to share their solution strategies with the rest of the class.

Now hand out the activity sheets. Note that the problems found on "Find the Missing Number 1" are for students in grades 1 and 2. These problems ask students to solve either addition or subtraction problems in which one of the numbers is missing. While the majority of the problems have only one possible answer, two of them have multiple solutions and students are asked to supply three of these solutions.

"Find the Missing Number 2" is for students in grades 3 and 4. These problems require students to find sums, differences, and products when one of the numbers is unknown. One of the problems involves simple division. Two of these problems are also open-ended and have multiple solutions. They require students to find five different solutions.

The last problem on both activity sheets asks students to write their own problem. Students can work with a partner on their original problems and then share their problems with the rest of the class.

How to extend the lesson

Ask students to convert the "word problem" to an "equation." For example, "When you add 5 to this number, the sum is 18" can be written as:
_____ + 5 = 18.

Find the Missing Number (continued)

Activity answers

Find the Missing Number 1

1. 5
2. 13
3. 14
4. 8
5. 11
6. 5
7. Answers will vary.
   Possible solutions: 1 + 9, 2 + 8, 3+ 7, 4 + 6, 5 + 5.
8. Answers will vary.
   Possible solutions: 20–16, 19–15; 18–14, etc.

Find the Missing Number 2

1. 14
2. 40
3. 9
4. 15
5. 28
6. Answers will vary.
   Possible solutions: 1 + 24, 2 + 23, 3 + 22, etc.
7. Answers will vary.
   Possible solutions: 50–30, 49–29, 48–28, etc.

Name .... Date ....

Find the Missing Number 1

Directions: Use the 20-Chart to help you find the missing numbers.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

1. When you add these two numbers, the sum is 12. One of the numbers is 7. What is the other number? _____

2. When you add 5 to this number, the sum is 18. What is this number? _____

3. When you subtract 12 from this number, the difference is 2. What is this number? _____

4. The sum of two numbers is 11. One of the numbers is 3. What is the other number? _____

5. When you subtract 8 from this number the difference is 3. What is this number? _____

6. Start at 20 and subtract this number. You end up at 15. What number did you subtract? _____

7. The sum of two numbers is 10. Can you find three different solutions to this problem? ___________ ___________ ___________

8. The difference between these two numbers is 4. Can you find three different solutions to this problem? ___________ ___________ ___________

9. Write your own problem: ______________________________________

____

Name .... Date ....

Find the Missing Number 2

Directions: Use the 50-Chart to help you find the missing numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

1. The sum of two numbers is 27. One of the numbers is 13.
   What is the other number? _____
2. When you subtract 12 from this number, the difference is 28.
   What is this number? _____
3. The product of two numbers is 45. One of the factors is 5.
   What is the other factor? _____
4. When you divide this number by 3, you get 5 as an answer.
   What is this number? _____
5. This number plus 12 is 40. What is this number? _____
6. The sum of two numbers is 25. Can you find five different solutions to this problem? ________ ________ ________ ________ ________
7. The difference between two numbers is 20. Can you find five different solutions to this problem? ________ ________ ________ ________ ________
8. Write your own problem: ____

____

Chapter 3

Understanding Equivalence and Equations

Equations are more important to me, because politics is for the present, but an equation is something for eternity.
—Albert Einstein, 1879–1955

"The equal sign is one of the most important symbols in elementary arithmetic, in algebra, and in all mathematics . . . " (Van de Walle, 2007, p. 260). While students may be able to tell you that "what appears on both sides of the equal sign in an equation are equal" only about 10% of the students in any grade from 1 to 6 answered this problem correctly: 8 + 4 = ___ + 5. While the correct answer is 7, the most common responses were 12 and 17! (Falkner, Levi, & Carpenter, 1999). It has been found that using open sentences or equations where one of the numbers is missing from the problem is a good way to introduce young children to the meaning of the equal sign and the concept of equivalency.

The first activity, "Domino Equations," is a hands-on, concrete way to help students understand the meaning of the equal sign. Two dominoes are pictured as equal but one has a question mark where the dots should be. Students are asked to find the value of the unknown (the variable) on the incomplete domino. Then, working with partners, students design their own domino equations, thereby reinforcing the meaning of an equation. For those students in need of a hands-on manipulative, look in the Appendix for paper sets of dominoes.

Learning about the value of money is an important primary mathematics concept. "Equations with Money" combines finding the value of a collection of coins and dollar bills with an unknown quantity—one of the coins is a mystery. Paper versions of coins and illustrated dollar bills are supplied in the Appendix to give students the opportunity to "cut and paste" their own money equations.

Five different toys are pictured and priced in "Find the Missing Toy." But this *find the total cost* activity has a bit of a twist—two toys are purchased but only one of the toys is known—the other is a mystery. By using their algebra skills, scissors, and glue, students can make a picture of their equation.

Equations have often been described using a model of a balance scale where the value of both sides of the equation must be balanced. In "Balance the Scales with Shapes" students are told the weight of both sides of the scale and one of the shapes and asked to balance the scales by finding the weight of the other shape. In "Balance the Scales with Cards" students are given a visual model that allows them to choose the numbers or pictures on the cards to help them solve the problem.

The activity titled "The Unbalanced Scale" has a slightly different twist—these scales are unbalanced because one side of the scale weighs more than the other. An *equation* can be formed using the information provided for each side of the scale. When students problem solve the weight of one of the shapes, they can find the weight of the missing shape.

"Spinner Math" is a game played by two students and they score points when they form correct missing number sentences. Students find two numbers using a spinner and then add or subtract to find the missing number.

Dominoes are used by many primary teachers to help students learn addition and subtraction concepts, but the "Domino Algebra Game" uses a very unusual set of dominoes—one side of the domino shows an open number sentence and the other side the answer to one of the problems on another domino. Students play with a set of 24 algebra dominoes, trying to match problems with answers.

And finally, "Frog Jumps" uses a number line and a jumping frog to encourage students to find the number sentence that is formed when a frog jumps from 0 to 25 in two different jumps. After students find the missing addend, they must write out the number sentence that illustrates the model shown on the number line.

By using a variety of different types of open number sentences and making their solutions fun, you can help students gain an understanding of the meaning of the equal sign in an equation and the important concept of equivalency.

Domino Equations

What is the algebra?

Solving equations, finding missing addends

What do you need?

- "Domino Equations" activity sheets (pp. 86–87) for each student
- "Design Your Own Domino Equations" activity sheet (p. 88) for each pair of students
- Addition or subtraction tables, if needed
- Overhead transparency of the first page of the activity, if available
- Set of dominoes (available in the Appendix)

Some classroom procedures

Since dominoes have dots (called *pips*), they are a concrete representation of numerals in these simple equations. The variable (question mark) has been placed in various spots in each equation so that students become familiar with solving a variety of different versions of open sentences or equations.

Show students the overhead transparency of the first activity sheet, and ask students how they might solve the first problem. Encourage them to verbalize their solution by discussing the strategies they used to find the missing addend. For example, a student might say, "I knew that the answer (sum) on each side had to be 7, so I said, '3 plus what number equals 7?' and I found the answer of 4." Those students who believe that the one side of an equation represents the problem and the other side the answer might respond incorrectly to this question by stating, "The answer is 7 because 2 + 5 is 7." To help these students understand that the equal sign means that both sides of the equation have the same value, ask, "What is the sum of the dots on the side of the domino that does not have the question mark?" When students have calculated the value of that domino, explain that the sum of the number and the question mark must be the same as it was on the other side. Ask students again, "What was the value of the first side?" "What must we add to get the same sum on the second side?" Be sure to work examples where the variables (or question mark) appear on different sides of the equal sign.

When you think students understand the concepts, hand out the two-page worksheet. When students have completed the worksheet, place them in pairs. Then give each pair of students a copy of the "Design Your Own Domino Equations" sheet and ask them to design their own domino equations to share with the rest of the class.

How to extend the lesson

Domino Equations can be made using Double Nine dominoes. This gives students experiences using sums greater than 12 and a greater variety of addends.

Activity answers

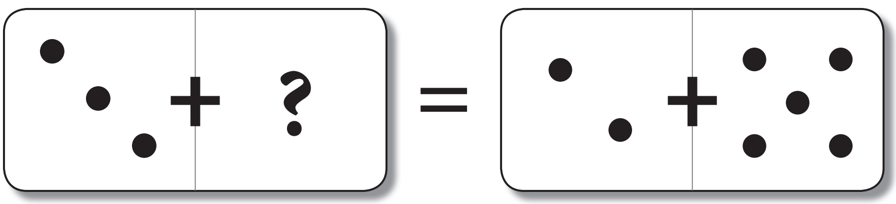
1. 4
2. 3
3. 3
4. 5
5. 4
6. 2
7. 4
8. 5
9. 0
10. 3

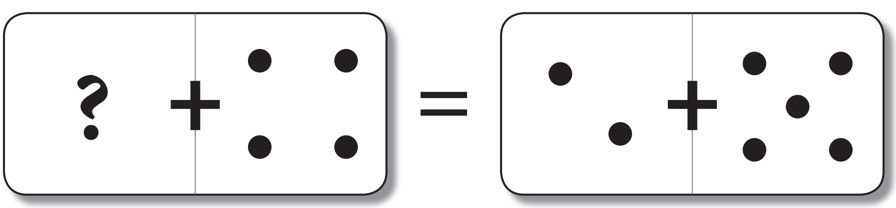
Name .... Date ....

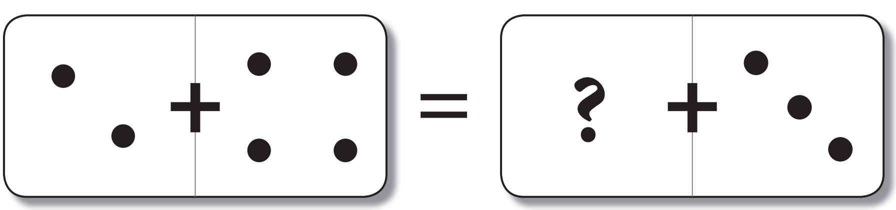
Domino Equations

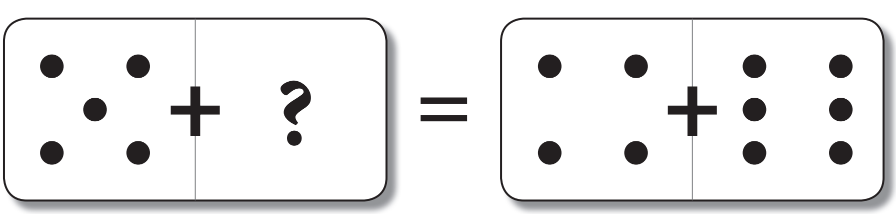
Activity

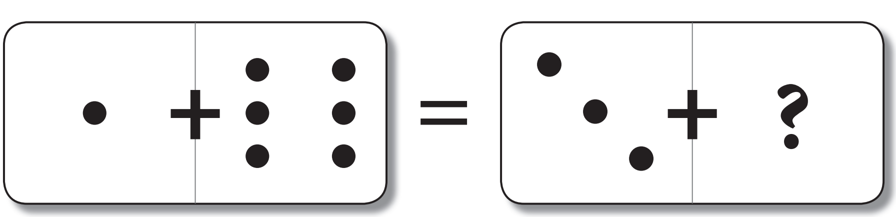
Directions: The dominoes in each problem form an equation. The sum of the dots on each domino is equal but one of the addends is missing. Find each missing addend.

1 
? = ____

2 
? = ____

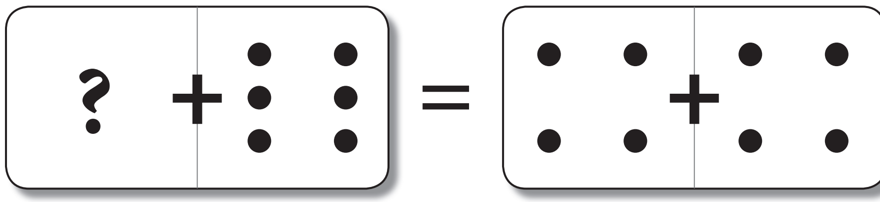
3 
? = ____

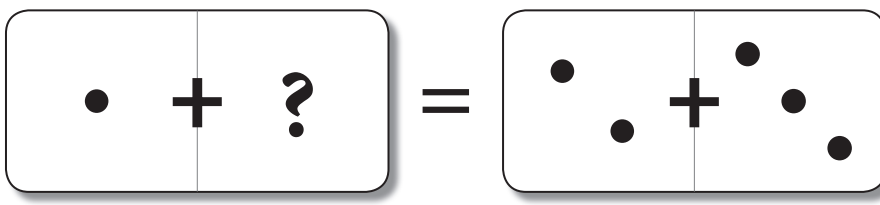
4 
? = ____

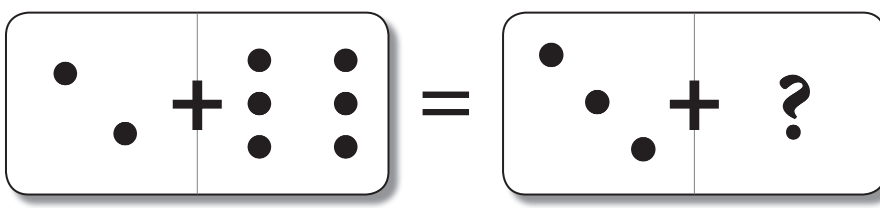
5 
? = ____

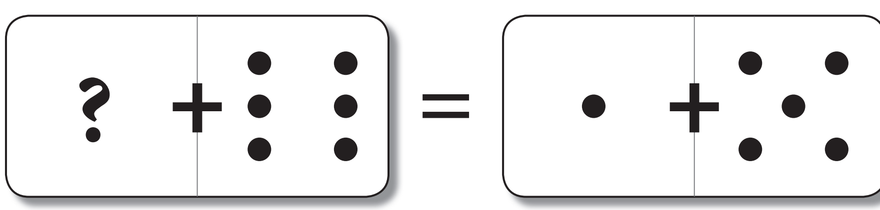
Name .... Date ....

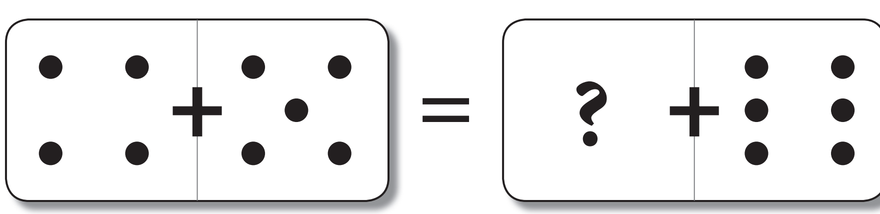
Domino Equations (continued)

6  ? = ______

7  ? = ______

8  ? = ______

9  ? = ______

10  ? = ______

Name .... Date ....

Design Your Own Domino Equations

Activity

Directions: Use these blank dominoes to design your own "Domino Equations." Then share them with another group. Be sure to place a question mark (?) in one space to stand for the unknown number of dots.

1. [+] = [+]   ? = ____

2. [+] = [+]   ? = ____

3. [+] = [+]   ? = ____

4. [+] = [+]   ? = ____

5. [+] = [+]   ? = ____

Equations with Money

What is the algebra?

Understanding how variables are used, solving equations, solving measurement problems using algebra strategies

What do you need?

- "Equations with Money" activity sheets (pp. 90–91) for each student
- "Design Your Own Money Equations" activity sheet (p. 92) for each pair of students
- Overhead transparencies of the activity sheets, if available
- Play coins or coin manipulatives (available in the Appendix)
- Glue
- Scissors

Some classroom procedures

Learning about money is an important math skill that is stressed at the primary level. This activity combines the understanding of the value of each coin with the concept of variables (the missing coin becomes the variable).

In this activity, students are shown a collection of coins and a sum but one of the coins is unknown. Their job is to calculate the value of the missing coin. (Coins may not be shown to scale.) This is an example of one of the problems:

 +  + ________ = 25¢

If plastic coins are available, some students may find it easier to solve the problems when they can use hands-on manipulatives. If these are not available, a sheet of images of each coin can be found in the Appendix.

Show students a transparency of the "Equations with Money 1" sheet. Explain that only one coin is missing from each equation. Demonstrate solving the first equation. Then hand out the activity sheet. Page 1 is addition problems and page 2 is subtraction problems.

When students have finished the two activity sheets, have them pair up. Hand out "Design Your Own Money Equations," along with the sheets of coin images from this book's Appendix. Have students cut out coins and paste them right into the problems on the activity sheet. Give each group of students the opportunity to share their problems with the rest of the class.

How to extend the lesson

Some students may be ready to work with a larger amount of money, such as $5, $10, and $20 amounts. Additional problems can be designed to help students solve equations using bills with larger denominations and coins with larger sums.

Looking at the real world, students can work out problems such as these:

"You have saved $1.45 from your allowance to buy a book about sports. The book costs $2.00. Use your collection of coins to help you problem solve how much more money you need to buy the book."

Newspaper advertisements can also be useful in helping students find missing sums or differences.

Activity answers

Equations with Money 1

1. 10¢ **2.** 1¢ **3.** 25¢ **4.** 50¢

Equations with Money 2

1. 25¢ **2.** 25¢ **3.** 5¢ **4.** 1¢

Name .... Date ....

Equations with Money 1

Directions: These money problems show a sum but there is one coin missing in each problem. Put on your problem-solving caps to find that missing coin.

❶  + ______ +  = **$0.16**

❷  +  + ______ = **36¢**

❸ ______ +  +  = **85¢**

❹ ______ +  +  = **$1.00**

Name .... Date ....

Equations with Money 2

Directions: These money problems show a difference but there is one coin missing in each problem. Put on your problem-solving cap to find that missing coin.

1.  — ______ = $0.75

2. ______ —  = 24¢

3.  — ______ = 45¢

4.  — ______ = 9 ¢

Name .... Date ....

Design Your Own Money Equations

Activity

Directions: Now it's time to make up your own equations. Design two equations that add and two that subtract. Write your problems in the spaces—be sure to leave one box blank for the unknown coin.

+ + =

+ + =

− =

− =

Find the Missing Toy

What is the algebra?

Using a variable, finding missing addends

What do you need?

- "Find the Missing Toy" Activity sheet (p. 94) for each student
- Addition tables or calculators, if needed
- Glue or tape
- Scissors

Some classroom procedures

In this activity, students are shown five different toys and their prices. The prices and names of each of the toys are labeled. Hand out the activity sheet. Then go over each of the toys and their names with students so that they understand what each of the pictures represents.

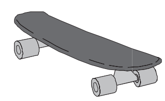
Give students the opportunity to solve each of the problems by cutting out the toys at the bottom of the page and gluing them in the correct place. When the students have completed the activity, have them share their solutions with other class members.

How to extend the lesson

Pair students and ask them to use the toys on the page to make up other addition and/or subtraction problems. Use the additional pictures supplied on the activity sheet for this extension activity. An example might be:

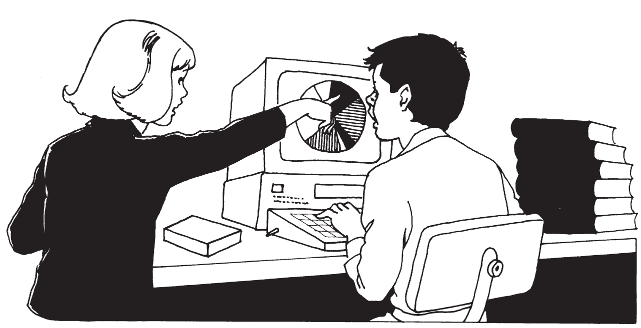
 +  = □

or

 +  = □

Activity answers

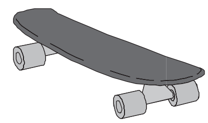
1. jack-in-the-box
2. soccer ball
3. teddy bear
4. skateboard
5. train

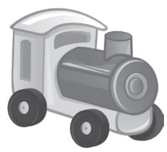


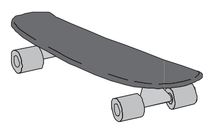
Name .... Date ....

Find the Missing Toy

Directions: Use the toys and prices to help you find the missing toy in each problem.

$1	$5	$3	$8	$4
				
train	soccer ball	jack-in-the-box	skateboard	teddy bear

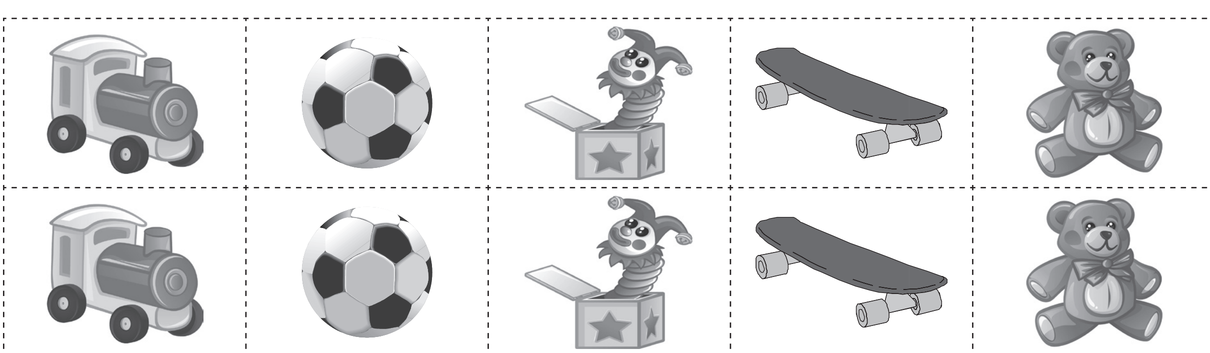
1.  + = $4

2. +  = $13

3.  + = $9

4. +  = $11

5.  + = $5



Balance the Scales with Shapes

What is the algebra?

Understanding equivalency, introducing the concept of a variable, solving problems

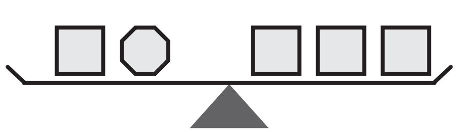
What do you need?

- "Balance the Scales with Shapes" activity sheets (p. 97) for each student
- "Balance the Scales with Shapes—Design Your Own" activity sheet (p. 98) for each pair of students
- Overhead transparencies of activity sheets, if available
- Scissors
- Glue

Some classroom procedures

Some students find it difficult to understand the concept of equivalency. In the activity "Balancing the Equation with Shapes," students are asked to calculate the weight of a shape (representing a variable) on one side of the scale (representing an equation) when they know the entire weight in the scale. Because the scales are balanced, by finding the sum of the value of two shapes, students find the weight of the unknown shape.

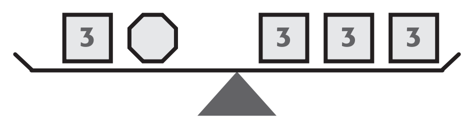
In each problem, one side of the scale contains two or more of the same shape; therefore, each of these shapes weigh the same amount. Students are asked to problem solve the unknown weight of a shape when the weight of one of the shapes and the total weight of the scale are known. Use the following problem as an example to begin the lesson. Explain that the three squares all weigh the same amount. Then give them this information:



The total weight of the scale is 18 pounds and each square weighs 3 pounds.

Ask: (1) What is the total weight of the squares on the scale?
(2) How much must be added to the weight of the squares to get the total weight (18 pounds)?

To make connections between the diagram and the arithmetic problem, write this problem on the board:



$$3 + 3 + 3 + 3 + \bigcirc = 18$$

Place students in pairs and hand out the activity sheet. Have pairs solve the problems. When they have completed these problems, go over the solutions in class.

Give the groups a chance to make up their own problems. Hand out the "Design Your Own" activity sheet. Then ask students to supply the following information in their original problems:

Both sides of the scale weigh ____________.
The ____________ weighs ____________.
How much does the ____________ weigh?

The blanks in the problem are explained below:

1. Both sides of the scale weigh (<u>the total weight including the mystery weight</u>)?
2. The (<u>geometric shape whose weight is given</u>) weighs (<u>the weight of that shape</u>).
3. How much does the (<u>shape with the unknown weight</u>) weigh?

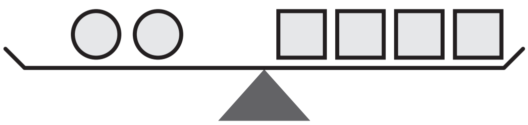
Have each group work another group's problem to see if they arrive at the same answer.

All of the original problems can be published in a student book and used at another time.

Balance the Scales with Shapes (continued)

How to extend the lesson

Extend the lesson by having students set up scales like the one shown here::



Tell the students: Because both sides of the scale have the same weight, the scale is balanced but we don't know the weight of any of the shapes. There are many possible solutions. See how many you can find. Do you see a pattern? What is it?

Solutions: Because the ratio of the number of squares to the number of circles is 2:1, each circle will be twice the weight of each square. For example, if each square weighs 1 pound then each circle will weigh 2 pounds.

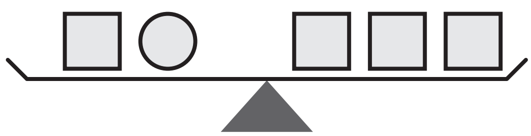
Activity answers

1. 10 pounds
2. 12 pounds
3. 8 pounds
4. 5 pounds

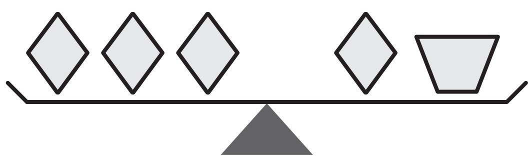
Name .... Date ....

Balance the Scales with Shapes

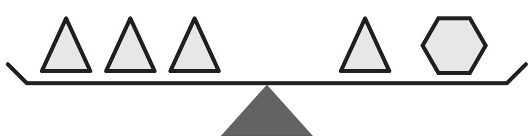
Directions: These scales are balanced because they weigh the same amounts on each side. On each scale in a particular problem, the same shapes have the same weight. Use the information you are given to find the weight of the unknown shape.

1 

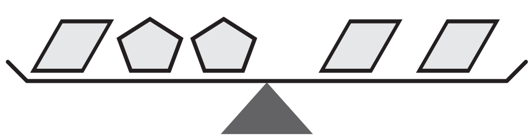
The total weight of the scale is 30 pounds and each square weighs 5 pounds.
How much does the circle weigh? _____

2 

The total weight of the scale is 36 pounds and each rhombus weighs 6 pounds.
How much does the trapezoid weigh? _____

3 

The total weight of the scale is 24 pounds and each triangle weighs 4 pounds.
How much does the hexagon weigh? _____

4 

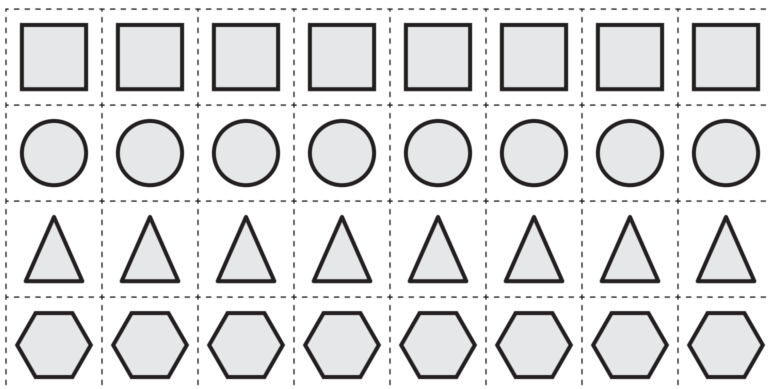
The total weight of the scale is 40 pounds. Each rhombus weighs 10 pounds.
How much does each pentagon weigh? ____

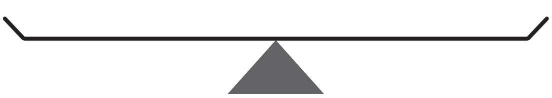
Explain how you solved these problems:

Name .... Date ....

Balance the Scales with Shapes—Design Your Own

Directions: Work with a partner. Cut out the shapes you will need for your problem and place them on a balanced scale. Then write your problem in the space next to that problem. Remember that the value of each shape can change when it appears in a different problem.

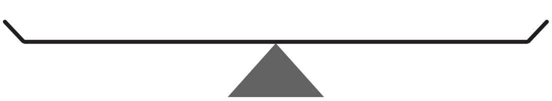




The total weight of the scale is ____________.

The ____________ weighs ____________.

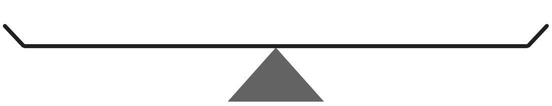
How much does the ____________ weigh?



The total weight of the scale is ____________.

The ____________ weighs ____________.

How much does the ____________ weigh?



The total weight of the scale is ____________.

The ____________ weighs ____________.

How much does the ____________ weigh?

Balance the Scales with Cards

What is the algebra?

Finding equivalent values, working with variables, writing equations, solving problems

What do you need?

- "Balance the Scales with Cards" activity sheets (pp. 100–101) for each student
- "Balance the Scales with Cards—Design Your Own Equations" activity sheet (p. 102) for each pair of students
- Addition tables or calculators, if needed
- Decks of playing cards, if available
- Scissors
- Glue

Some classroom procedures

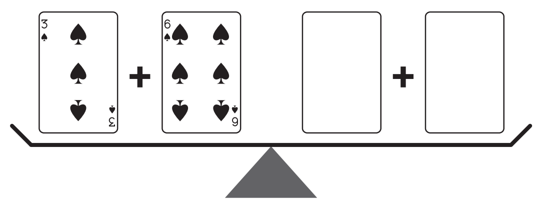
This activity gives students additional practice with the concept of equivalency. Because it uses a standard deck of playing cards, it allows visual learners to employ those types of strategies. Students can either use the number shown on the card or they can count the number of pictures to help them solve the number sentence.

Hand out the "Balance the Scales with Cards" activity sheet. Students are asked to find the unknown number (the variable) and then write the equation (using numbers) that applies to the visual model. Discuss the strategies students used to solve the problems and their answers.

After students have worked on the problems individually, pair them and hand out the "Design Your Own" activity sheet. Have students design their own balances using the cards provided in this activity. Have students cut out the cards and paste them on scales making an equation with an unknown quantity. They can share these original problems with the rest of the class.

How to extend the lesson

Try open-ended problems with students such as the one shown. In this problem, multiple answers are possible.



Give students the opportunity to discuss how they solved their problem, asking them if they saw a pattern that helped them solve it more quickly.

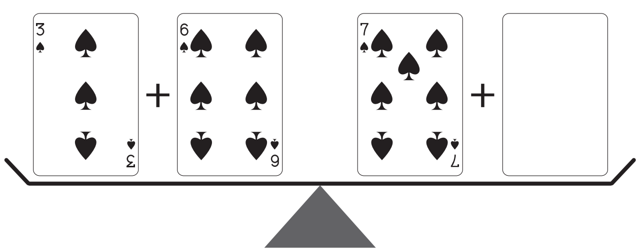
Activity answers

1. 2
2. 5
3. 4
4. 2
5. 5
6. 3
7. 8
8. 7

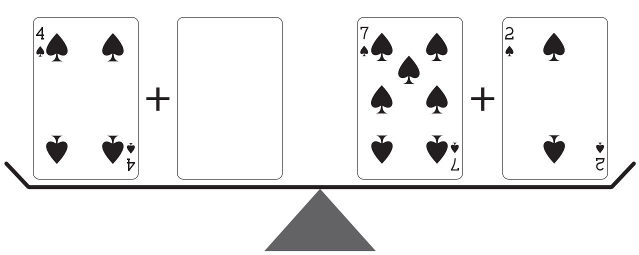
Name .... Date ....

Balance the Scales with Cards

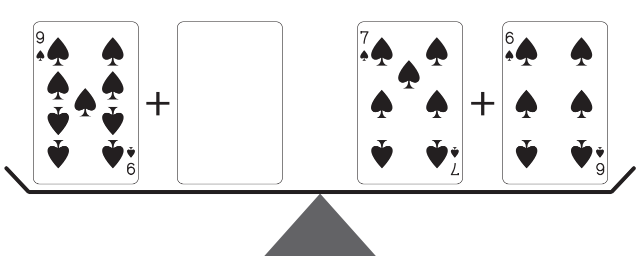
Directions: Each of these problems is a *picture* of an equation. The blank card represents an unknown number. For example, the equation for problem 1 is $3 + 6 = 7 + \square$. Write this equation on the line to the right of problem 1. Now use your problem-solving skills to work out what the missing number must be in order that both sides of the equation will have the same value. ***Clue:*** Because the left side of the equation has a value of 9, the right side also must have a value of 9. Write the value of the unknown number in the space provided. Now find the value of the missing numbers in the rest of the problems.

1 

____ + ____ = ____ + $\square$

2 

____ + $\square$ = ____ + ____

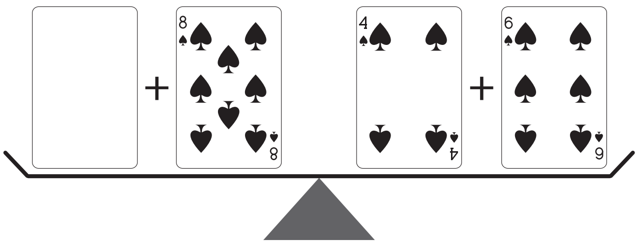
3 

____ + $\square$ = ____ + ____

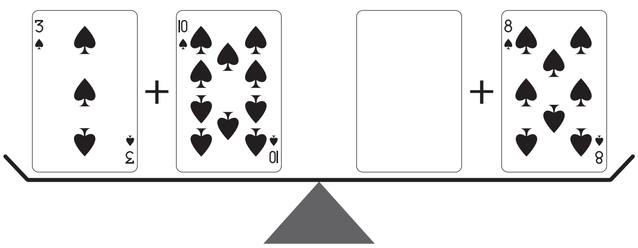
Name .... Date ....

Balance the Scales with Cards (continued)

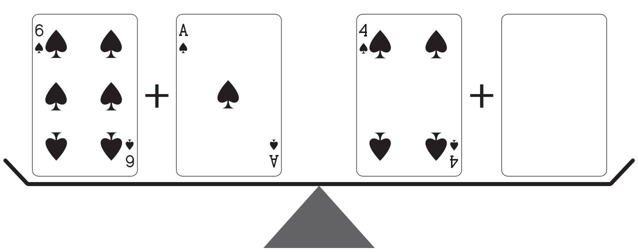
Activity

4 

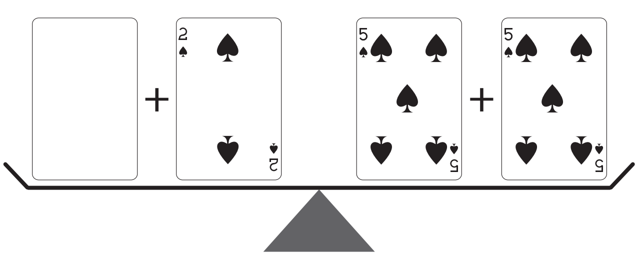
☐ + ____ = ____ + ____

5 

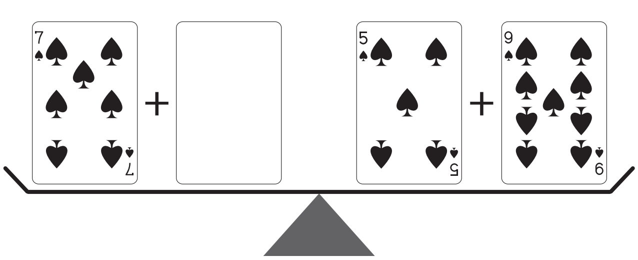
____ + ____ = ☐ + ____

6 

____ + ____ = ____ + ☐

7 

☐ + ____ = ____ + ____

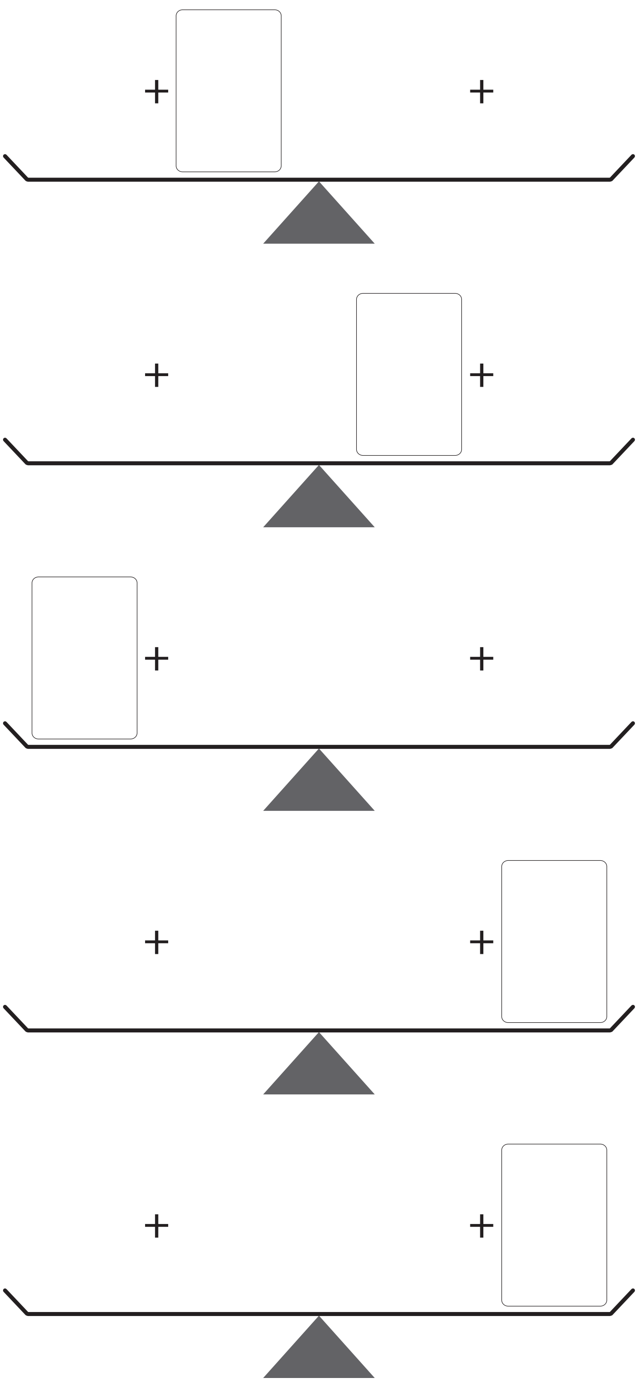
8 

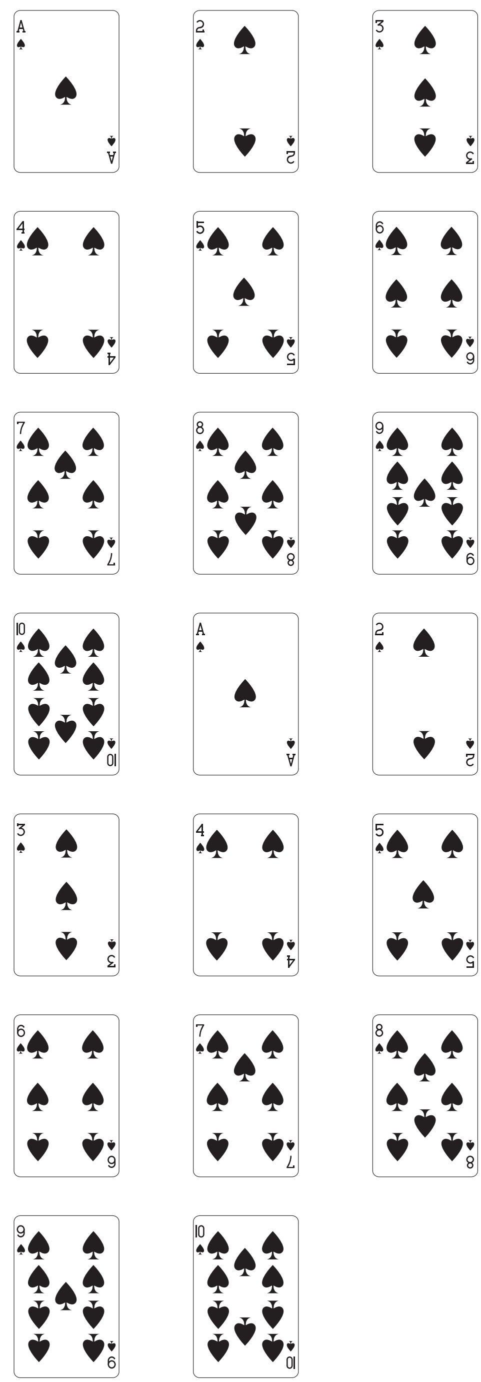
____ + ☐ = ____ + ____

Name .... Date ....

Balance the Scales with Cards—Design Your Own Equations

Directions: Use these cards to make your own equations. Cut out the cards you need and paste them on the scales. When you have finished, share your equations with the rest of the class.





The Unbalanced Scale

What is the algebra?

Using variables, solving open sentences, solving problems using visual clues

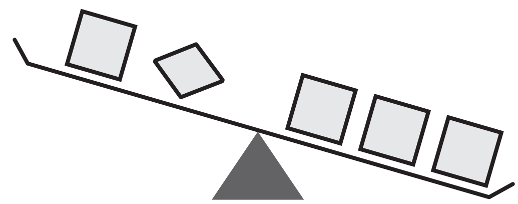
What do you need?

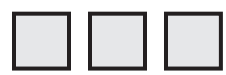
- "The Unbalanced Scale" activity sheet (p. 105) for each student
- "The Unbalanced Scale—Design Your Own" activity sheets (pp. 106–107) for each pair of students
- Overhead transparencies of activity sheets, if available
- Scissors
- Glue

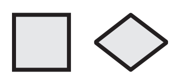
Some classroom procedures

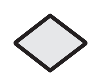
In "The Unbalanced Scale," students are asked to calculate the weight of a shape (representing a variable) when they know the entire weight of each side of the scale. The scales are not balanced because one of the sides weighs more than the other.

In each problem, one side of the scale contains two or more of the same shape. Explain to students that in the same problem the same shapes must weigh the same amount. However, their weight may vary from problem to problem. Students are asked to problem solve the unknown weight of a shape when the weight of one of the shapes is known, as is the total weight of the side. Use problem 1 on the sheet as an example to begin the lesson. Note that the algebra that corresponds to the problem is indicated. *It is not recommended that this information be shared with students because this type of solution is very abstract*. The purpose of the lesson is to make a concrete connection between scales and the weight of the objects.



 The right side weighs 15 pounds.

 The left side weighs 12 pounds.

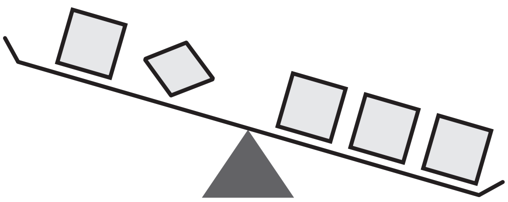
How much does  **weigh?** __________

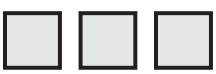
When you think students have the concepts, hand out the activity sheet. Have students work the problems. If pattern blocks are available, students can use them to develop additional problems on the "Design Your Own" activity sheet. You can make a pattern of the balance scale on an overhead transparency or card stock to simulate the unbalanced scale used in this problem.

The Unbalanced Scale (continued)

How to extend the lesson

Have students set up equations to reflect what shapes and values there are on each side of the balance. These types of problems willl require students to use variables. Following is an example of this type of advanced problem:

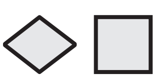



The right side weighs 15 pounds.

Written as an equation: $3x = 15$

$x = 5$

Students know the square weighs 5 pounds.


The left side weighs 12 pounds.

Written as an equation: $5 + y = 12$

$y = 7$

So the diamond weighs 7 pounds.

Activity answers

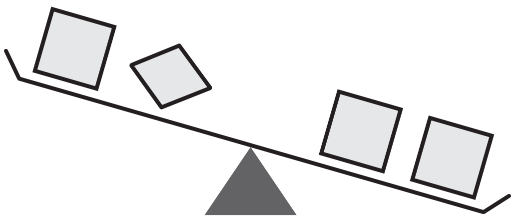
1. 4 pounds
2. 18 pounds
3. 6 pounds

Name .... Date ....

The Unbalanced Scale

Directions: These scales are not balanced because each side weighs a different amount. Use the information you are given to find the weight of the unknown shape on each scale below. On each scale in a particular problem, the same shapes weigh the same amount.

1

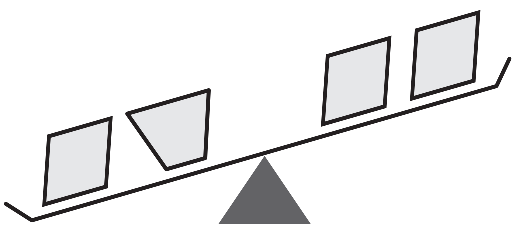


The right side weighs 16 pounds.

The left side weighs 12 pounds.

How much does ◇ weigh? __________

2

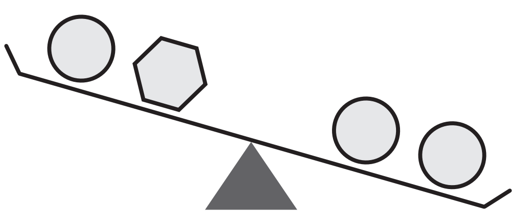


The right side weighs 20 pounds.

The left side weighs 28 pounds.

How much does weigh? __________

3



The right side weighs 18 pounds.

The left side weighs 15 pounds.

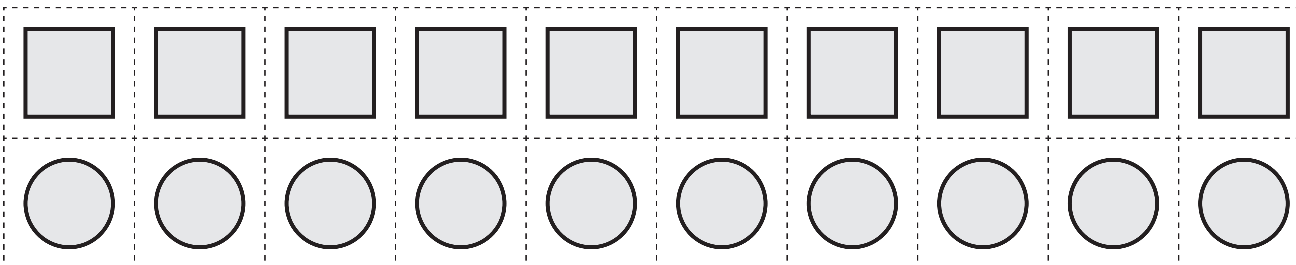
How much does ⬡ weigh? __________

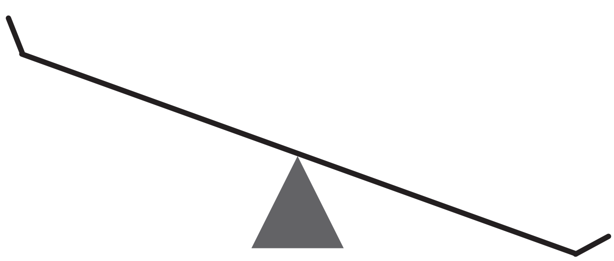
Explain how you solved these problems:

Name .... Date ....

The Unbalanced Scale—Design Your Own

Directions: Work with a partner. Cut out these shapes and place them on an unbalanced scale. Complete the problem that is next to each scale. Share your problems with another group. Remember that the value of each shape can change when it appears in a different problem.



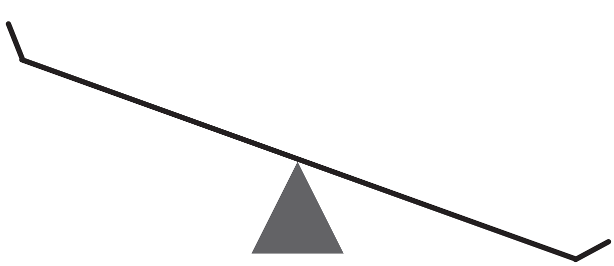


Our Problem

The value of the right hand side of the scale is _______________.

The value of the left hand side of the scale is _______________.

Find the value of the _______________.



Our Problem

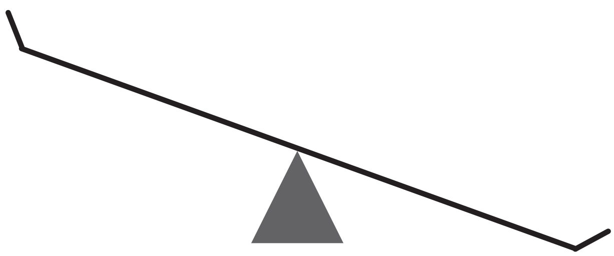
The value of the right hand side of the scale is _______________.

The value of the left hand side of the scale is _______________.

Find the value of the _______________.

Name .... Date ....

The Unbalanced Scale—Design Your Own (continued)

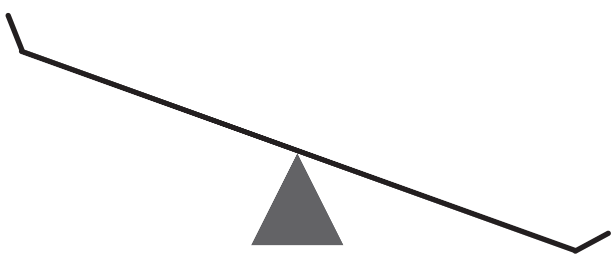


Our Problem

The value of the right hand side of the scale is _______________.

The value of the left hand side of the scale is _______________.

Find the value of the _______________.

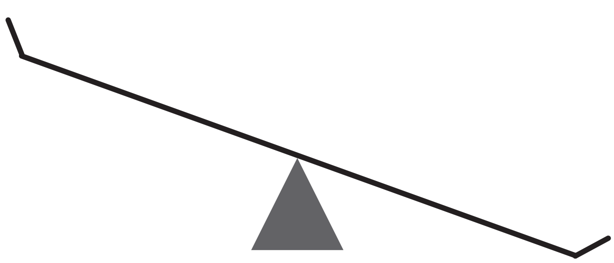


Our Problem

The value of the right hand side of the scale is _______________.

The value of the left hand side of the scale is _______________.

Find the value of the _______________.

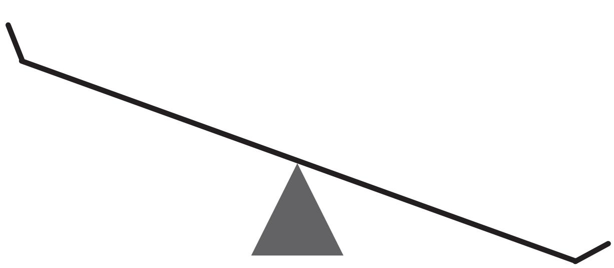


Our Problem

The value of the right hand side of the scale is _______________.

The value of the left hand side of the scale is _______________.

Find the value of the _______________.



Our Problem

The value of the right hand side of the scale is _______________.

The value of the left hand side of the scale is _______________.

Find the value of the _______________.

Spinner Math

What is the algebra?

Solving equations with a missing variable, solving addition and subtraction problems

What do you need?

- One "Spinner Math Game" activity sheet (p. 111) for each pair of students
- One "Spinner Math Game Sheet" (p. 112) for each student
- One spinner for each pair of students (Choose either a single large spinner on p. 109, which also appears in the student edition, or smaller spinners on p. 110.)
- Pencils and paper clips to use with spinners
- Overhead transparency of a spinner, if available
- Overhead transparency of a Game Sheet, if available

Some classroom procedures

Use transparencies of the "Spinner Math Game Sheet" and a spinner to demonstrate how to play this game. Show how a paper clip and pencil can be used as a spinner by placing a paper clip in the center of the spinner and using a pencil to anchor the paper clip. Spin and write the number in the blank below labeled "Spin #1." Spin a second time to obtain the number for the space labeled "Spin #2." Use one line of the game sheet to record these numbers. Ask students, "Will we need to add or subtract a number from (point to and name the first number) to get an answer of (point to and name the second number)?" Write the equation on the game sheet.

Now give students the game sheet and read the directions to them. Show them the sample spins at the top of the page and how they are recorded below. Say, "We need to subtract because we are starting with a 7 and ending with a 5. What number must we subtract from 7 to get an answer of 5?"

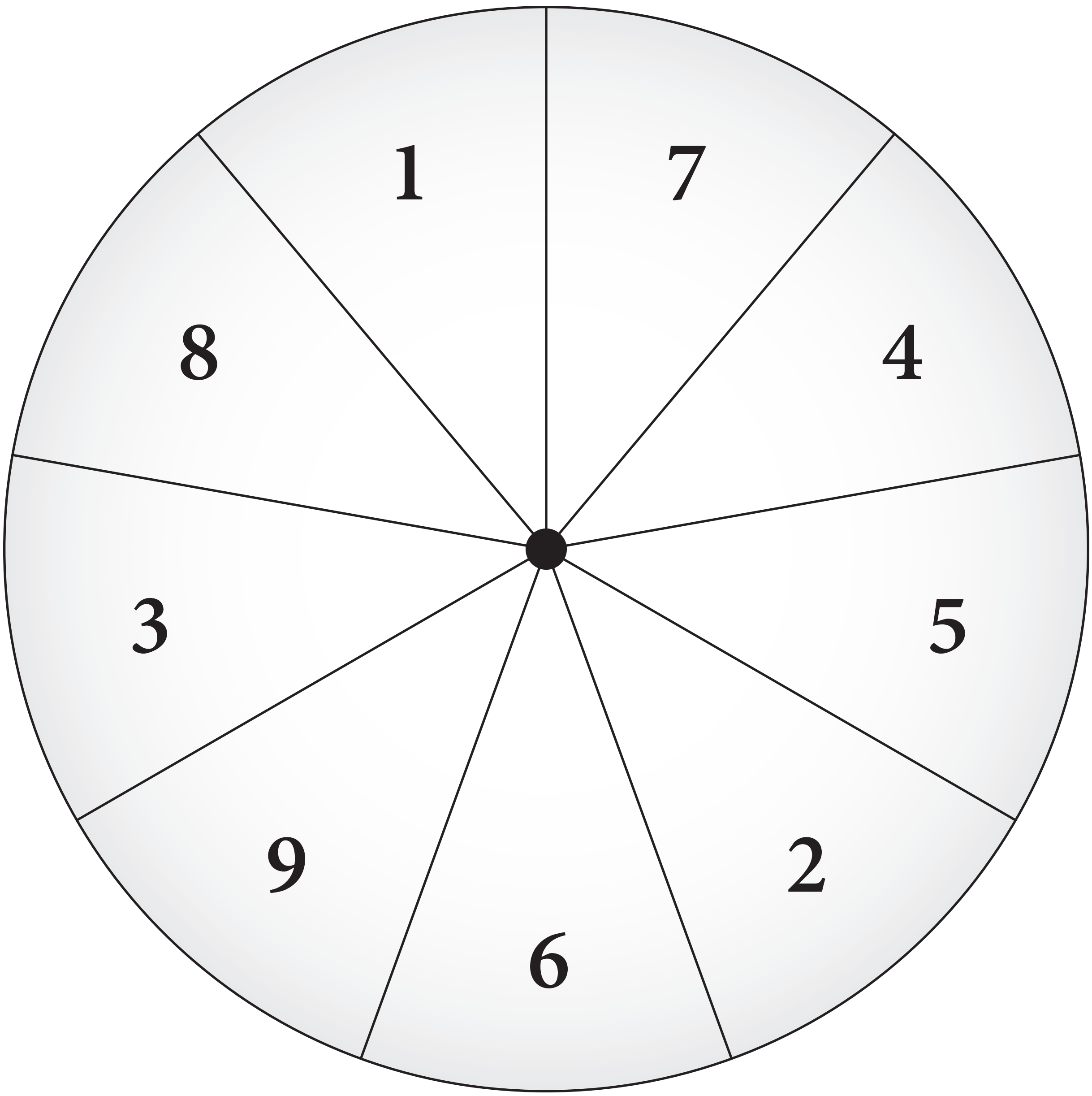
Read the second example on the page to students. Emphasize that in this problem the first spin produced the number "3" and the second spin produced the number "8." Ask, "Should we put an addition sign or a subtraction sign in this space? (Point to the space below the + or – sign.) Continue to solve this problem with the class.

Give each pair of students 1 spinner (which you can copy on card stock and laminate), 1 pencil, 1 paper clip, and 2 "Game Sheets." Have each student write his or her name on the top of the game sheet. Remind students that if their partner agrees that their equation is correct, they can put a check in the box and that will count for 1 point. If their partner does not agree that the equation is correct, then you, the teacher, will determine if the player gets a point. The first player to get 10 points is the winner of the game.

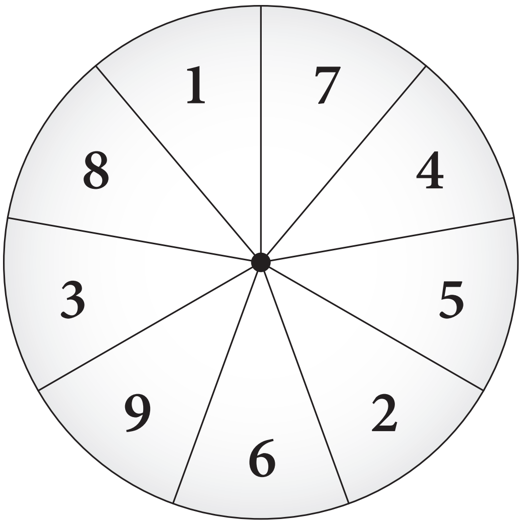
How to extend the lesson

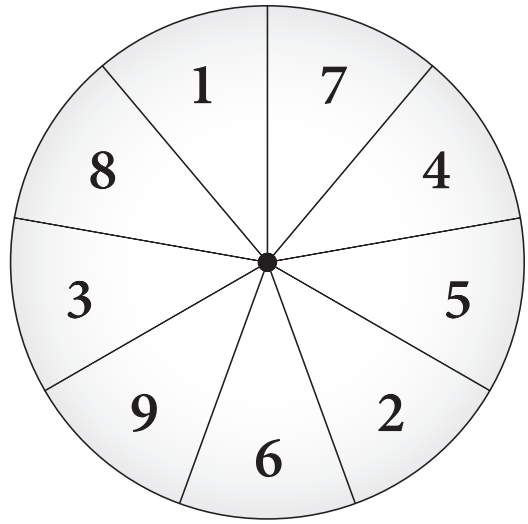
Extend the lesson by using spinners with larger numbers (such as 9, 10, 11, 12, 13, 14, 15, 16, 17 or any combination of larger numbers). See the Appendix for blank spinners.

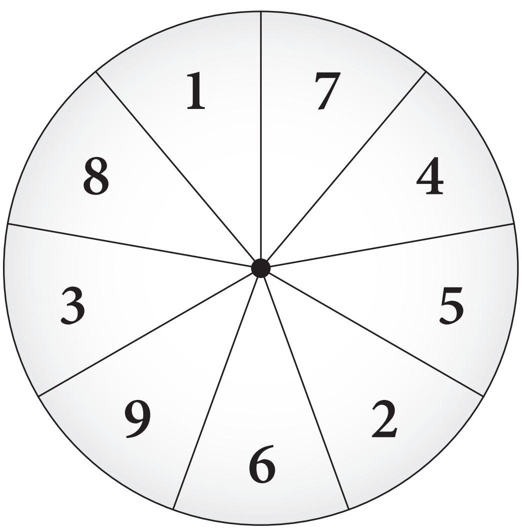
Large Spinner for Spinner Math

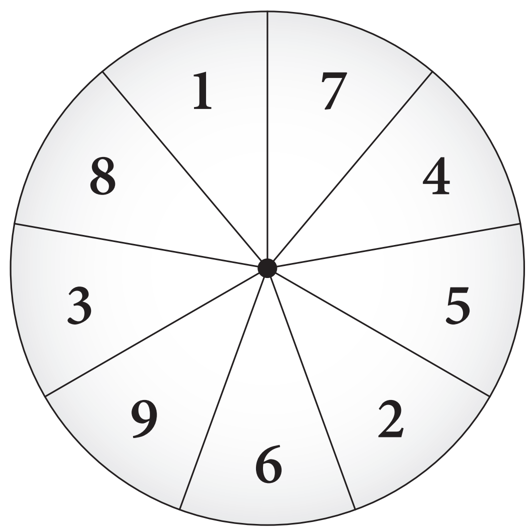


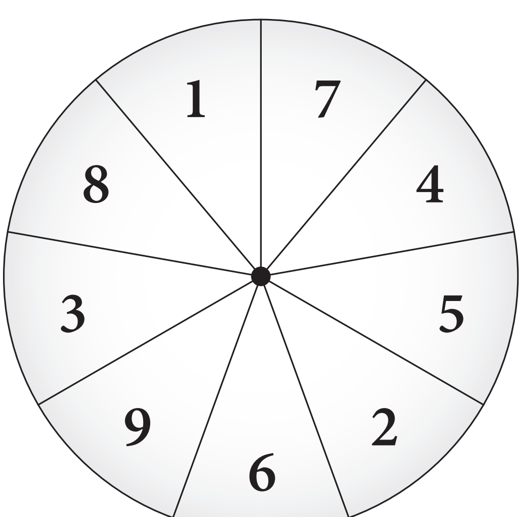
Small Spinners for Spinner Math

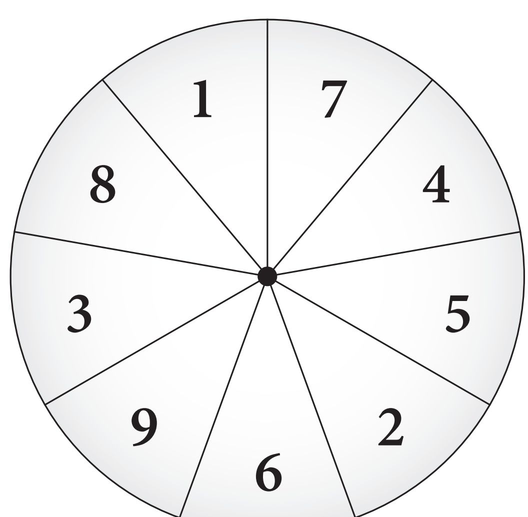








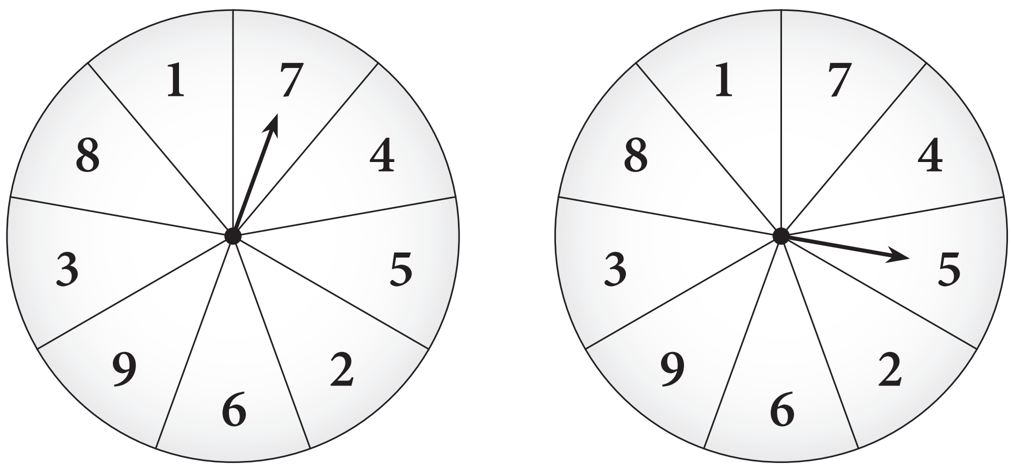




Name .... Date ....

Spinner Math Game

Directions: Play with a partner. Spin the spinner once and write down that number on your game sheet under “Spin #1.” Then spin again. Write that number on your game sheet under “Spin #2.” Decide if you will need to add or subtract the mystery number from the Spin #1 number to get the Spin #2 number. Look at this example: Spin #1 was the number 7; Spin #2 was the number 5. We subtract to get from 7 to 5.



Spin #1	+ or —	Mystery Number		Spin #2
7			=	5

What mystery number goes in the rectangle to make this equation true? ________

Now try this one. Suppose the two spins show these numbers:

Spin #1	+ or —	Mystery Number		Spin #2
3			=	8

Will you add or subtract the mystery to get an answer of 8? ________

What is the mystery number? ________

Now get a game sheet and a spinner. Write your name on top of your game sheet and play the game.

Name .... Date ....

Spinner Math Game Sheet

Spin #1	+ or —	Mystery Number		Spin #2	Point (✓)
			=		
			=		
			=		
			=		
			=		
			=		
			=		
			=		
			=		
			=		

Domino Algebra Game

What is the algebra?

Solving equations, using variables, solving problems using computation skills

What do you need?

- One set of "Domino Algebra Playing Pieces" (p. 114) for each pair of students
- "Domino Algebra Game Score Card" (p. 115) for each pair of students
- One die for each group (to determine which child goes first)

Some classroom procedures

Copy a set of Algebra Dominoes on cardstock for each pair of students. Laminate them, if possible. Students then cut apart the dominoes before playing the game. Because this is not a traditional set of dominoes, there are only 24 instead of the usual 28. Thus, it is best if only two students play. The rules are slightly different than the traditional game:

1. Turn the dominoes facedown and count them to make sure there are 24.
2. Move them around to "shuffle" the deck.
3. Roll one die. The person who rolls the largest number goes first.
4. Choose 7 dominoes from the pile that is facedown in the middle of the table.
5. Turn them over but be sure that only you can see the dominoes in your hand.
6. Neatly stack the remaining dominoes on the table, out of play.
7. The first person places one of his or her dominoes, faceup, on the table.
8. The other player must place a domino with an equivalent value next to the first domino in his or her hand—it doesn't matter which end. Make sure that like values are always touching. If the player is not able to match either end, he or she must draw a domino from the stack on the table.
9. Now it's the first player's turn to match either end of a domino on the table.
10. A player receives 1 point each time he or she is able to match a domino and 2 points if the domino has a star in the corner. These problems are multiplication or division problems and so are worth more points.
11. If a player is not able to match any of the dominoes and there are no more in the stack, then that player loses his or her turn.
12. The first player to get 15 points is the winner.

How to extend the lesson

See the Appendix for a set of blank dominoes you can use to develop multiplication, division, or beginning fraction concepts. A fraction domino game might have equivalent fractions on different dominoes for students to match. A multiplication game might have multiplication facts on one side of the dominoes and the products on the other. You can design dominoes for measurement (e.g., 12 inches on one side of one of the dominoes and 1 foot on another domino). The variations are endless!

Domino Algebra Playing Pieces

$8 \div \square = 2$ *	12	$8 + \square = 18$	12	13	$28 + \square = 48$
3	$10 \div \square = 5$ *	5	$8 + \square = 14$	$\square - 6 = 13$	15
$\square - 8 = 4$	10	6	$12 - \square = 4$	$3 \times \square = 21$ *	8
4	$\square - 9 = 7$	$\square + 8 = 25$	2	1	$\square + 7 = 18$
17	$15 - \square = 6$	11	$\square + 3 = 8$	$\square + 11 = 24$	14
$5 \times \square = 15$ *	12	$\square - 7 = 7$	9	50	$20 + \square = 35$
75	$100 \div \square = 2$ *	$25 \div 3 = \square$ *	12	30	$25 \times 4 = \square$ *
$\square - 7 = 11$	100	20	$18 + 12 = \square$	19	$235 \div 235 = \square$ *

Domino Algebra Game Score Card

Directions: Follow the directions your teacher has given you to play the Domino Algebra Game. You get 1 point each time you are able to match a regular domino but you get 2 points if the domino has a star in the corner. These problems are multiplication or division problems and so are worth more points. Each time you score a point, use a tally mark: (|) It is easier to keep track of your points if you group them in fives like this: 

Your Name:	**Your Name:**

Your Name:	**Your Name:**

Your Name:	**Your Name:**

Your Name:	**Your Name:**

Frop Jumps

What is the algebra?

Solving equations using inverse operations, using arithmetic compensation

What do you need?

- "Frog Jumps" activity sheets (pp. 117–118) for each student
- An overhead transparency of number line activity pages, if available

Some classroom procedures

An algebra equation is balanced; one side of the equal sign has the same value as the other side. To solve equations, we use arithmetic compensation. If we decrease one number by a certain amount, we must increase another number by the same amount for the answer to stay the same. For example:

−5

$$30 + 15 = 45 \quad \rightarrow \quad 25 + 20 = 45$$

+5

If we decrease the 30 by 5, we must increase the 15 by 5 for our answer to remain.

Place the first "Frog Jumps" transparency on the overhead and give each student a copy of the sheet. Read each problem to students and work out these problems as a class. The frog jumps from 0 to 10 and then 15 more units (these are called "frog units" to maintain the flavor of the lesson). In the next jump however, the first jump only gets the frog to 6—this is 4 units less than the first. Students should be encouraged to use arithmetic compensation. Because we subtracted 4 from the 10, to keep the answer the same (25) we must add 4 to the 15.

After students understand the process, hand out the second page of the activity. In this next series of problems, students write out addition equations using a variable. Notice that sometimes the variable is the first addend and sometimes it is the second. When students have solved the problems on this sheet, give them the opportunity to explain their solution strategies.

How to extend the lesson

Put students into pairs and give them a ruler, a calculator, and perhaps some frog stickers. Have them make up three problems to share with the class. Be sure students understand that all three problems must start at 0 and end with the same number. Encourage them to keep the total less than 30.

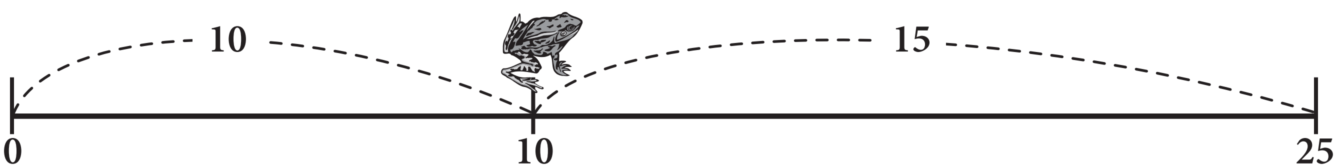
Activity answers

1. 19
2. 7
3. 4
4. 15; 15 + 10 = 25
5. 12; 13 + 12 = 25
6. 22; 22 + 3 = 25
7. 2; 2 + 23 = 25

Name .... Date ....

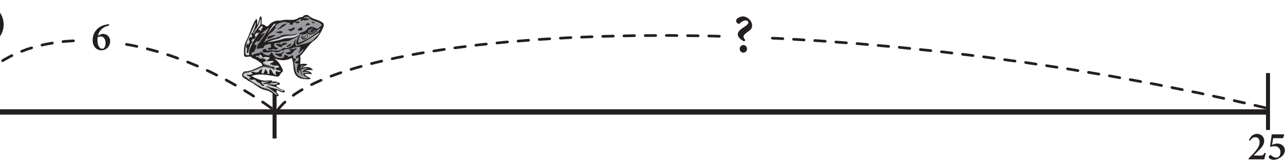
Frog Jumps

Watch the frog jump. Her first jump is 10 frog-units from the beginning. She then jumps 15 more frog-units and ends up at 25 frog-units. This can be written as an addition problem: 10 + 15 = 25.



Frog seems to be getting tired. Look at her next jump.

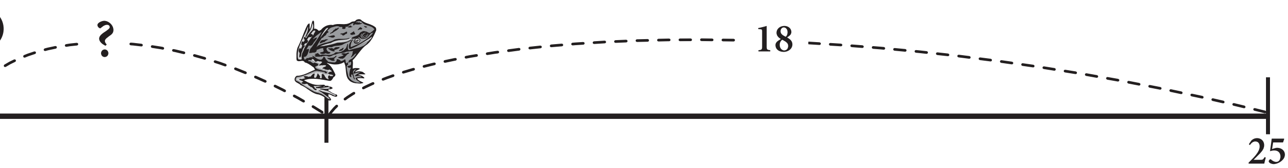
1



Her first jump lands her at only 6 frog-units. This is 4 less than the last jump. But she tries very hard to get to 25 and makes it on the second jump—so she must jump 4 more frog units than she did the first time. This addition problem look like this: 6 + ? = 25. **?** = ______

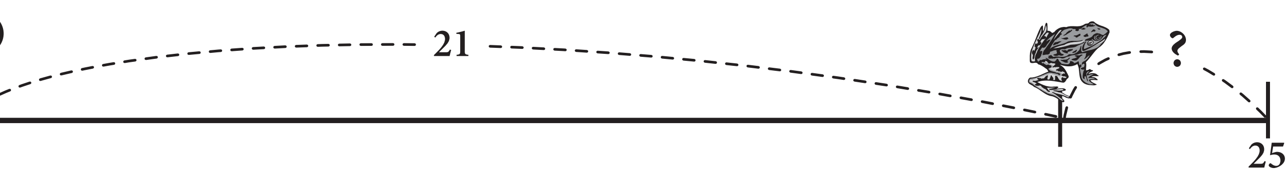
See if you can solve these:

2



The number sentence is **?** + 18 = 25. **?** = ______

3

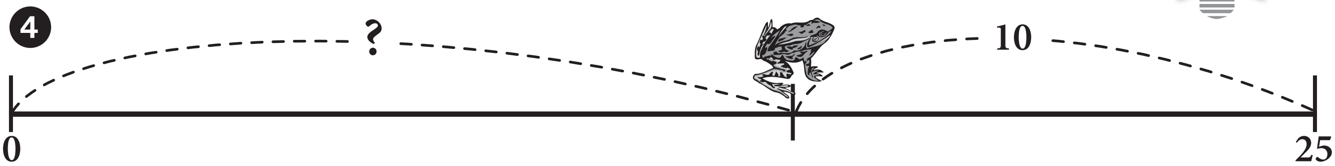


The number sentence is 21 + **?** = 25. **?** = ______

Name .... Date ....

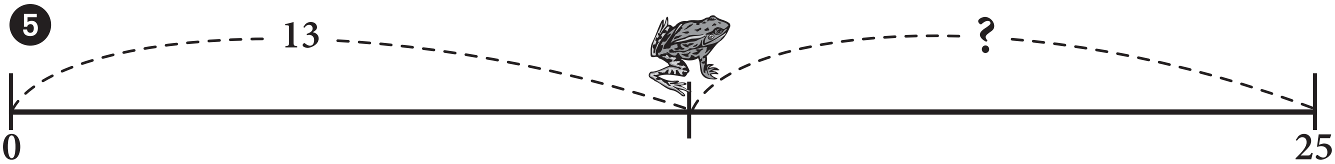
Frog Jumps (continued)

4



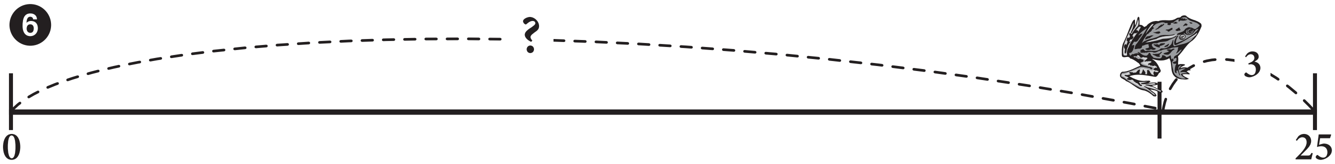
Find out how far Frog jumped and then write out the correct number sentence.

5



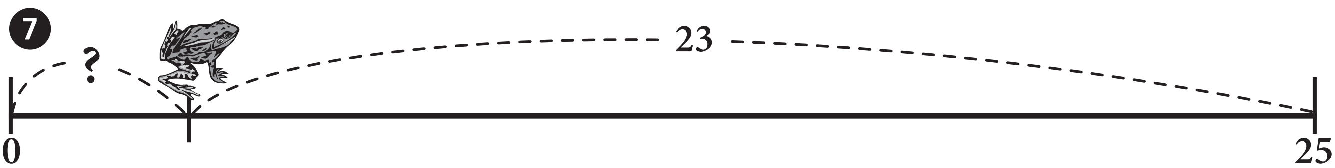
Find out how far Frog jumped and then write out the correct number sentence.

6



Find out how far Frog jumped and then write out the correct number sentence.

7



Find out how far Frog jumped and then write out the correct number sentence.

Making Connections to Children's Literature

Aber, L. M. (2002). *Grandma's Button Box*. Minneapolis, MN: Kane Press.

When she spills her grandmother's button box, Kelly decides to sort them by size, color, and shape. Classifying helps students develop important mathematical reasoning skills. This book was written for students ages 4–8.

Anno, M. (1999). *Anno's Magic Seeds*. New York: Penguin Group.

Jack is given two golden seeds by a wizard and told to eat one and bury the other. The one seed that he ate will satisfy his hunger for a year and the magic seed that was planted will give him two more seeds the following year. Jack continues this same pattern for a number of years but then decides to plant both seeds. The following year his two seeds produce four seeds. He eats one and plants three—Students develop mathematical patterns using Jack's planting strategies. Although written for children ages 5–8, this book will appeal to children of all ages!

Driscoll, L. (2005). *Super Specs*. Minneapolis, MS: Kane Press.

Does Molly really have X-ray vision or is she solving these problems by using number patterns? Written for children ages 4–9, this book cleverly combines an interesting story with an important mathematics concept.

Kassirer, S. (2001) *What's Next, Nina?* Minneapolis, MN: Kane Press.

The author skillfully integrates the concept of patterning into a wonderful story about a little girl who has broken a necklace and has to put it back together in the right order. This book was written for students in grades 1–3.

Kroll, V. L. (2005). *Equal Shmequal*. Boston, MA: Charlesbridge Publishing.

Mouse and a group of friends decide it would be fun to have a tug of war—until they realize that they have to make the two sides equal! The way the animals decide on how to make the sides equal makes this a fun math adventure that will appeal to children ages 5–8.

Murphy, S. J. (1996). *Ready, Set, Hop*. New York: HarperCollins Publishers.

Knowing how equations are built is part of understanding how to interpret and write number sentences. When two frogs have a jumping contest, students are given the opportunity to set up equations to determine which frog is the champion hopper.

Murphy, S. J. (1999). *Rabbit's Pajama Party*. New York: HarperCollins Publishers.

Rabbit has some friends over for a sleepover. After the story, the reader is asked to recreate the events, putting them in the proper sequence. This book is for children ages 5–8.

Murphy, S. J. (2000). *Beep Beep, Vroom, Vroom*. New York: HarperCollins Publishers.

Little Molly decides to play with her brother's red, yellow, and blue cars—but there's going to be trouble because her brother told her to leave them alone. Now she has to return them to their right order before he comes back! This book focuses on recognizing and extending patterns and was written for children in grades K–2.

Murphy, S. J. (2001). *Dinosaur Deals*. New York: HarperCollins Publishers.

This is a clever story to help children understand the concept of equivalent values. Mike and his little brother attend a dinosaur card trading fair hoping to find a Tyrannosaurus Rex card. The two brothers need to make a series of equivalent trades to obtain the desired trading card. This book is appropriate for use with children ages 6–9.

Slater, T., & Burns, M. (1999). *—98, 99, 100, You're It*. New York: Scholastic.

Three little girls are playing hide-and-seek and decide to have some fun by varying the way they count to 100—by ones, fives, tens, and twenties. A cute story that will help children recognize number patterns and count to 100. This book was written for children in grades preK–2.

Tang, G. (2003). *Math-Terpieces: The Art of Problem-solving*. New York: Scholastic.

Tang uses poetry and fine art to show students the importance of using combinations and looking for patterns to find the number of objects on a page. Some of the artists that are featured are Renoir, Mondrian, Jackson Pollock, and van Gogh. This book is for students in grades 2 to 5.

Tang, G. (2005). *Math Potatoes: Mind-stretching Brain Food*. New York: Scholastic.

In another book for grades 2 to 5, Tang uses poetry and a wide variety of illustrations to encourage students to solve problems more efficiently by using patterns and design. Tang's books will interest students of all ages.

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Kamii, C. K. (1982). *Number in Preschool & Kindergarten: Educational Implications of Piaget's Theory*. Washington, DC: National Association for the Education of Young Children.

Kamii, C. K. (1985). *Young Children Invent Arithmetic: Implications of Piaget's Theory*. NY: Teachers College Press.

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National Council of Teachers of Mathematics (2001). Carole Greenes, ed. *Navigating through Algebra: In Pre-kindergarten—Grade 2*. Reston, VA: NCTM.

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Van de Walle, J. A. (2007). *Elementary and Middle School Mathematics: Teaching Developmentally*, 6th ed. Boston, MA: Pearson Education.

Interesting Web Sites

http://whyslopes.com/ParentCenter/Primary_School_Math.html

This Web site identifies the skills and concepts that are important for children to know from primary school to college. The ages are grouped as follows: 5–6, 6–7, 7–8, 8–9, and so on. Many of the concepts listed have links to algebra.

http://nlvm.usu.edu/en/nav/vlibrary.html

A site provided by Utah State University called the National Library of Virtual Manipulatives. The URL takes you to the Virtual Library, where you can choose a variety of different strands of mathematics, including number and operation, algebra, geometry, measurement, and data analysis and probability. The algebra (Grades PreK–2) site contains activities that are interactive and use pattern blocks, pentominoes, color patterns, and more. The algebra (Grades 3–5) contains base blocks, a tool for graphing, space blocks, and other puzzles.

http://arcytech.org/java/

This interactive Web site contains Java appelets to help students learn "All About Money," "It's About Time," and problem solving with pattern blocks.

http://illuminations.nctm.org/ActivitySearch.aspx

The National Council of Teachers of Mathematics Web site contains activities for students in grades PreK–12. While not all of the activities on this Web site relate to algebra, many require the type of reasoning that is very important to develop reasoning skills in young children.

http://www.mathsyear2000.org/magnet/minus3/alllook/index.html

This site asks students to flip monsters to all face left or right—a good activity for kindergarteners and preschoolers.

http://www.oswego.org/ocsd-web/games/spookyseq/spookycf5.html

Ghost shapes help students find the missing numbers in a sequence when skip-counting by 5s. Good for first and second graders.

http://www.oswego.org/ocsd-web/games/spookyseq/spookytrino.html

Ghost shapes help students find the missing number when the number sequences add triangular numbers. Good for second and third graders.

http://www.oswego.org/ocsd-web/games/spookyseq/spookysqno.html

Ghost shapes help students find the missing number when the number sequences add square numbers. Good for third and fourth graders.

Appendix

Many of the activities in *Primary Algebra* require students to use manipulatives or game pieces that may not be available to the user. These are supplied in the Appendix and can be duplicated on card stock and laminated for permanency.

The six pattern block shapes can be run off on the colors in which they are manufactured because cardstock is available in the necessary colors. Squares are orange, equilateral triangles are green, hexagons are yellow, trapezoids are red, rhombus Is are blue, and rhombus IIs are white. In addition to the algebra activities in *Primary Algebra*, these manipulatives can be used in many different ways: helping students learn to recognize geometric shapes, bringing a visual model to beginning fraction concepts, and helping students learn about symmetry and design.

The 100-Chart can be used with "Patterns in the 100-Chart," but it can be used in many other activities as well. Seeing the patterns in skip counting and addition and subtraction patterns are just two other ways the chart can be used with primary students.

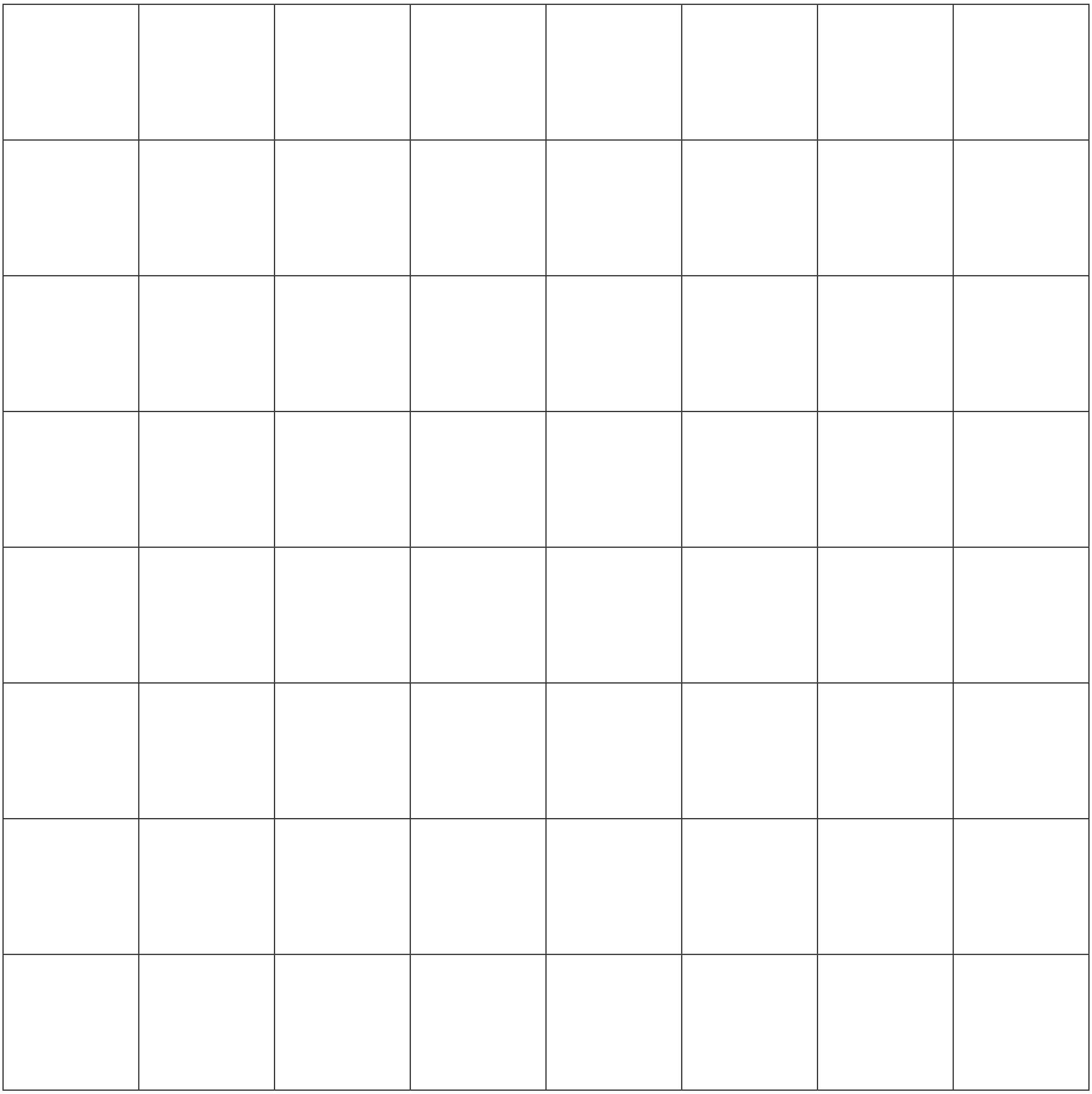
"Equations with Money" asks students to cut out the coin that will make their number sentence or equation true. Give sheets of these to groups of students to share and, with scissors and glue, the fun begins!

The algebra dominoes needed for "Domino Algebra Game" are supplied in the lesson, but a 28-piece set of regular double-six dominoes is supplied as an extra bonus. Laminate these so that you can use them over and over again. A set of blank dominoes is supplied as a tool to make the games suggested in the lesson extension in the "Domino Algebra Game." These games are wonderful ways to help students practice and review skills and concepts in a fun way.

And finally, should your students enjoy playing a game with a spinner, a collection of blank spinners is supplied to help you design your own games. There are four spinners: thirds, fourths, sixths, and eighths.

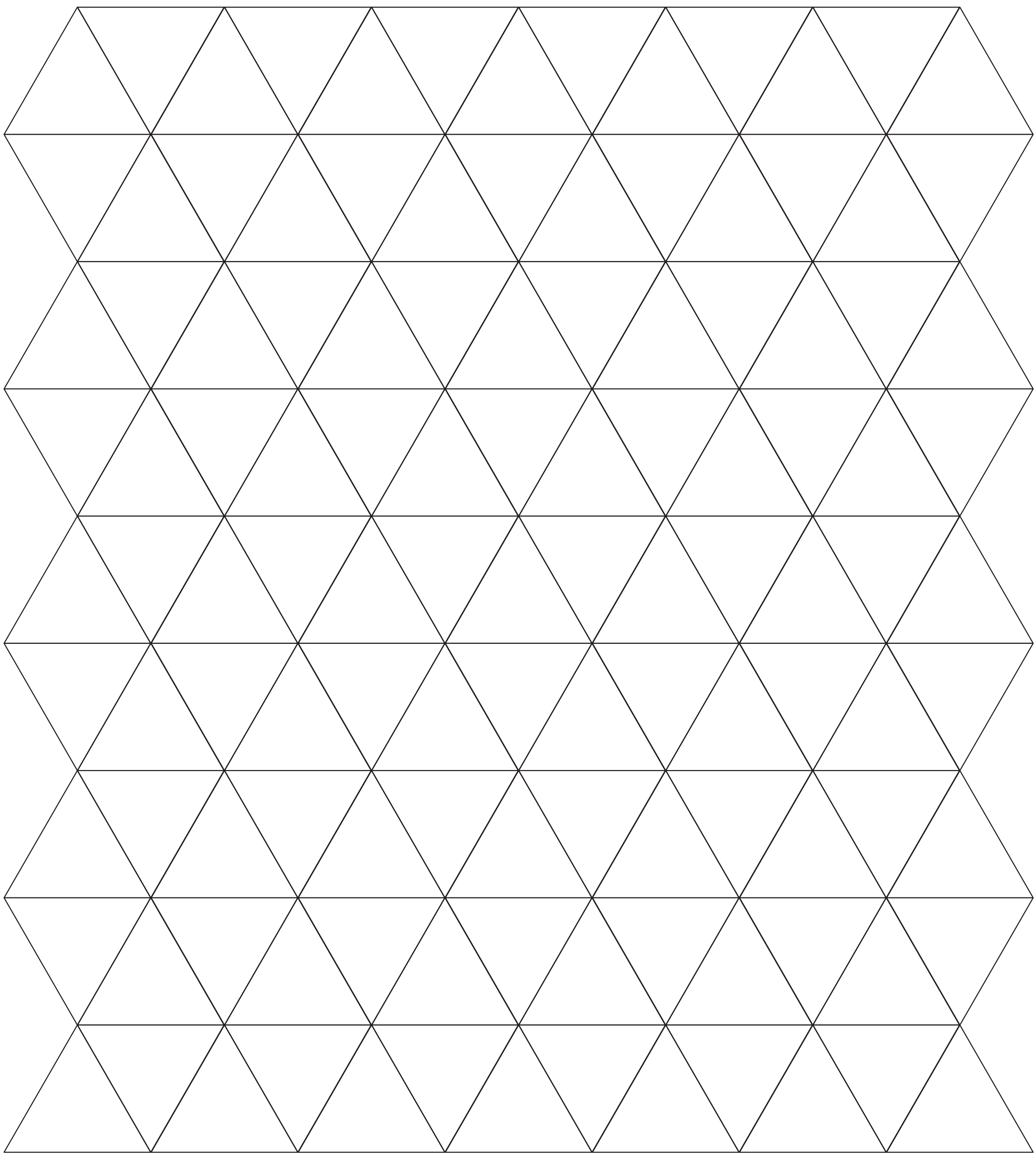
Pattern Block Pieces

Squares



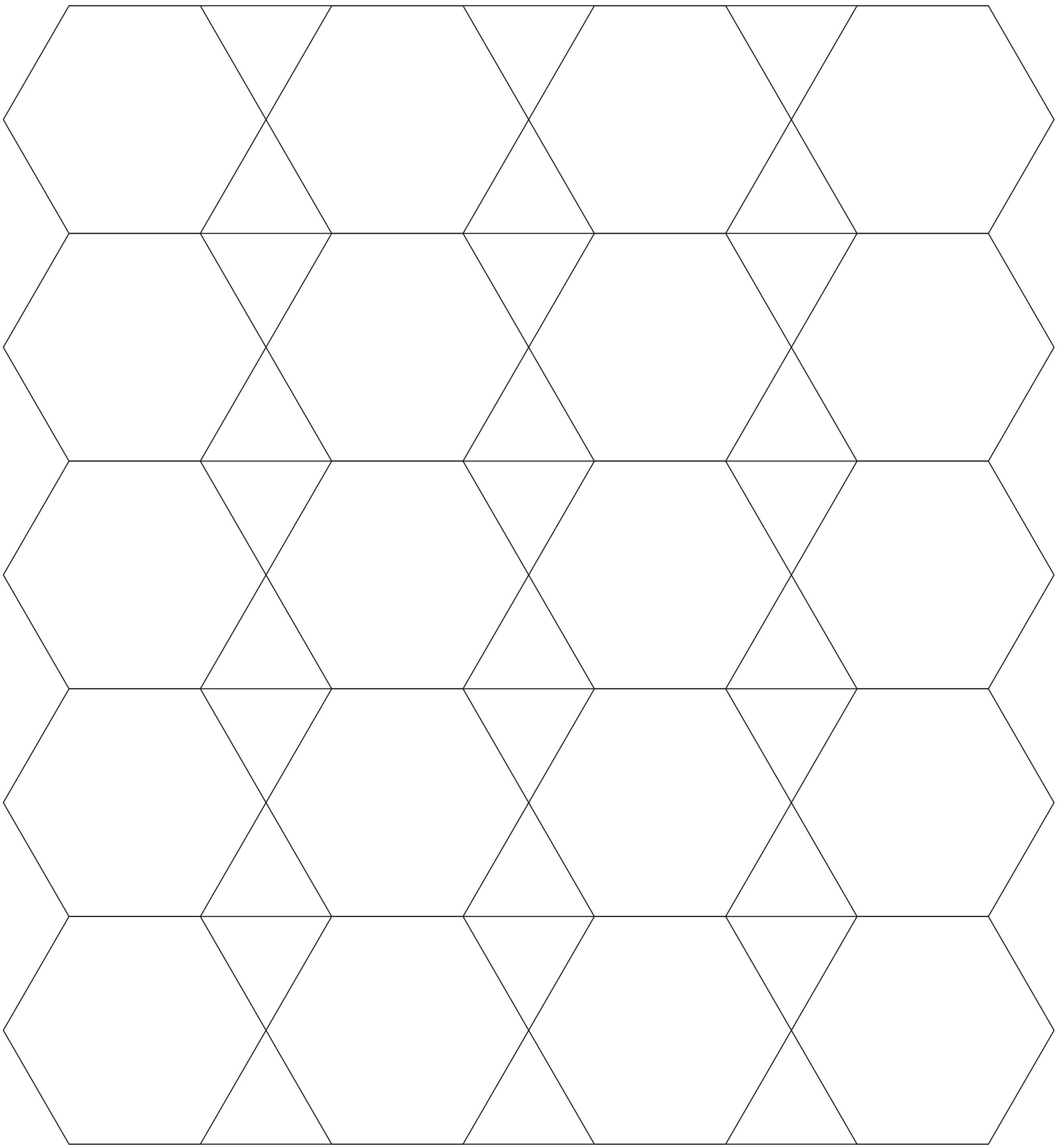
Pattern Block Pieces

Equilateral Triangles



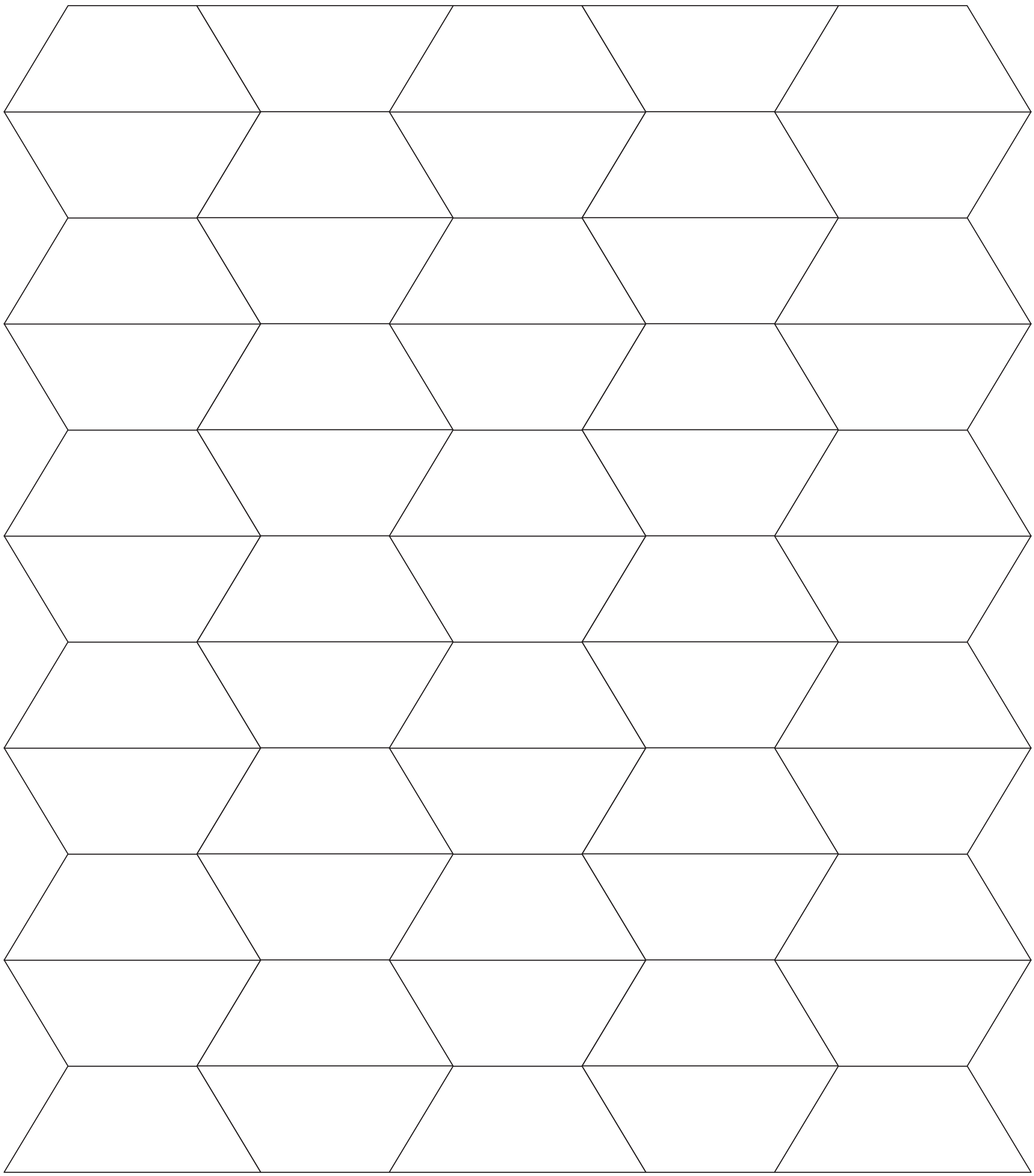
Pattern Block Pieces

Hexagons



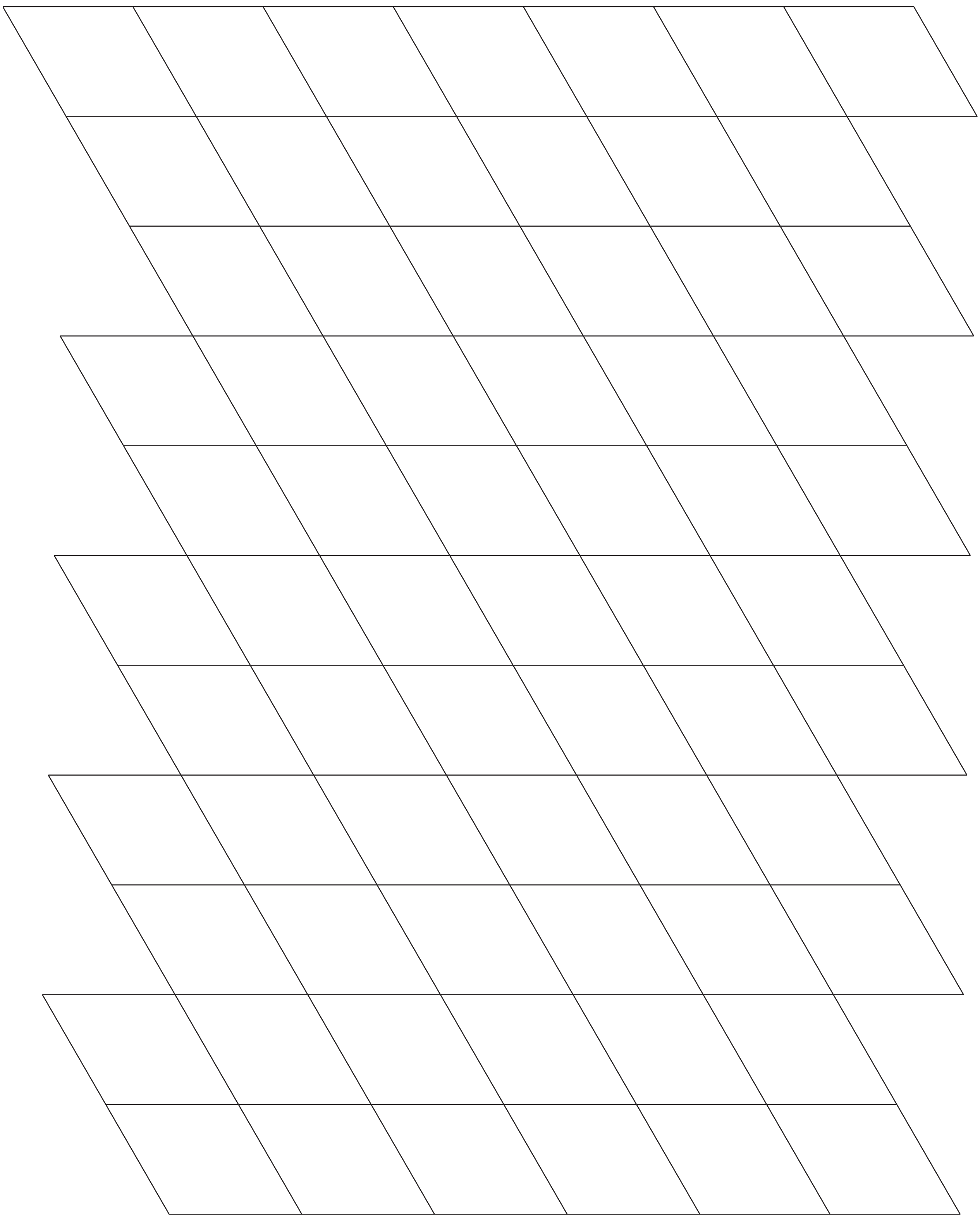
Pattern Block Pieces

Trapezoids



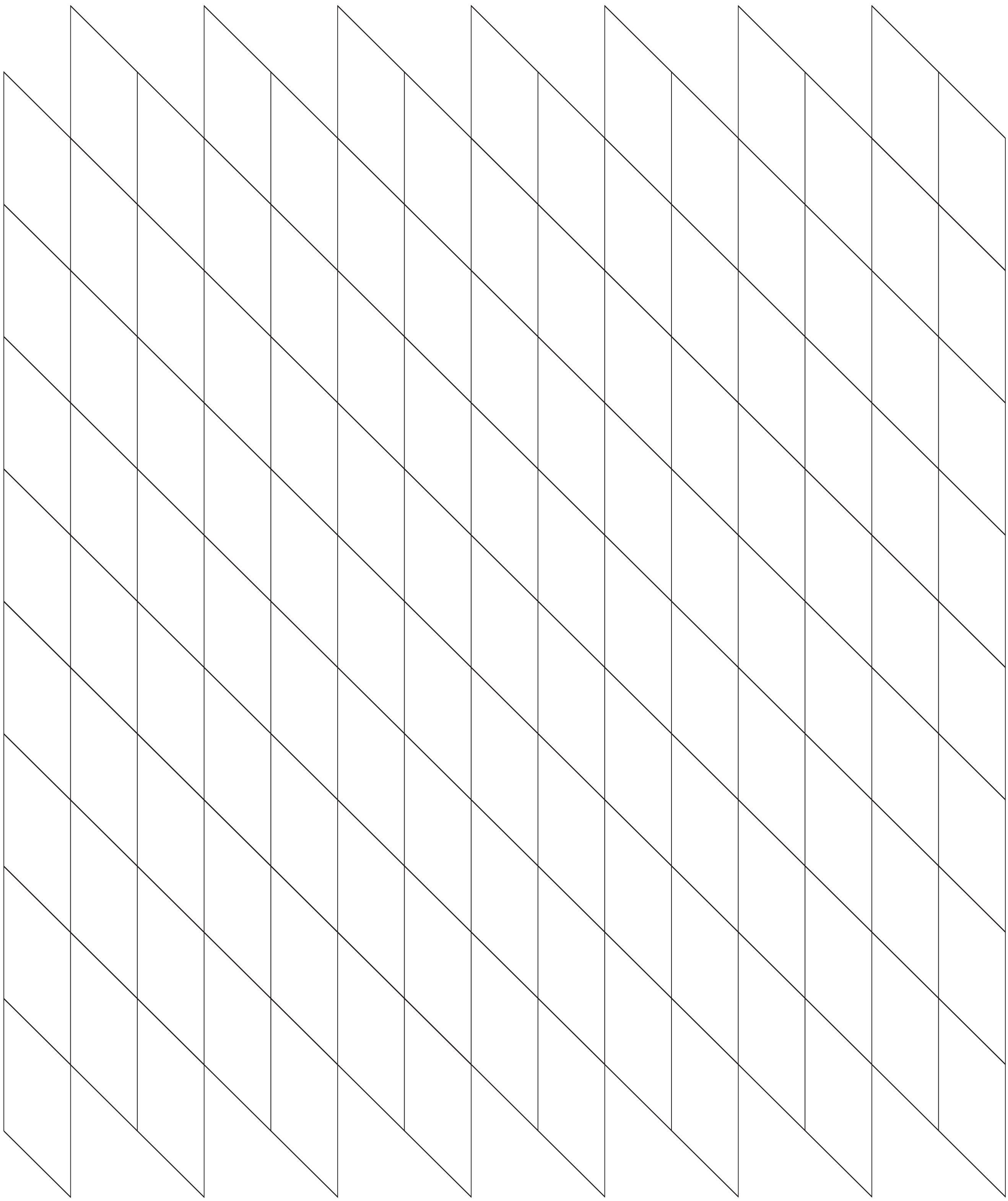
Pattern Block Pieces

Rhombus I



Pattern Block Pieces

Rhombus II



100-Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Table of Pennies

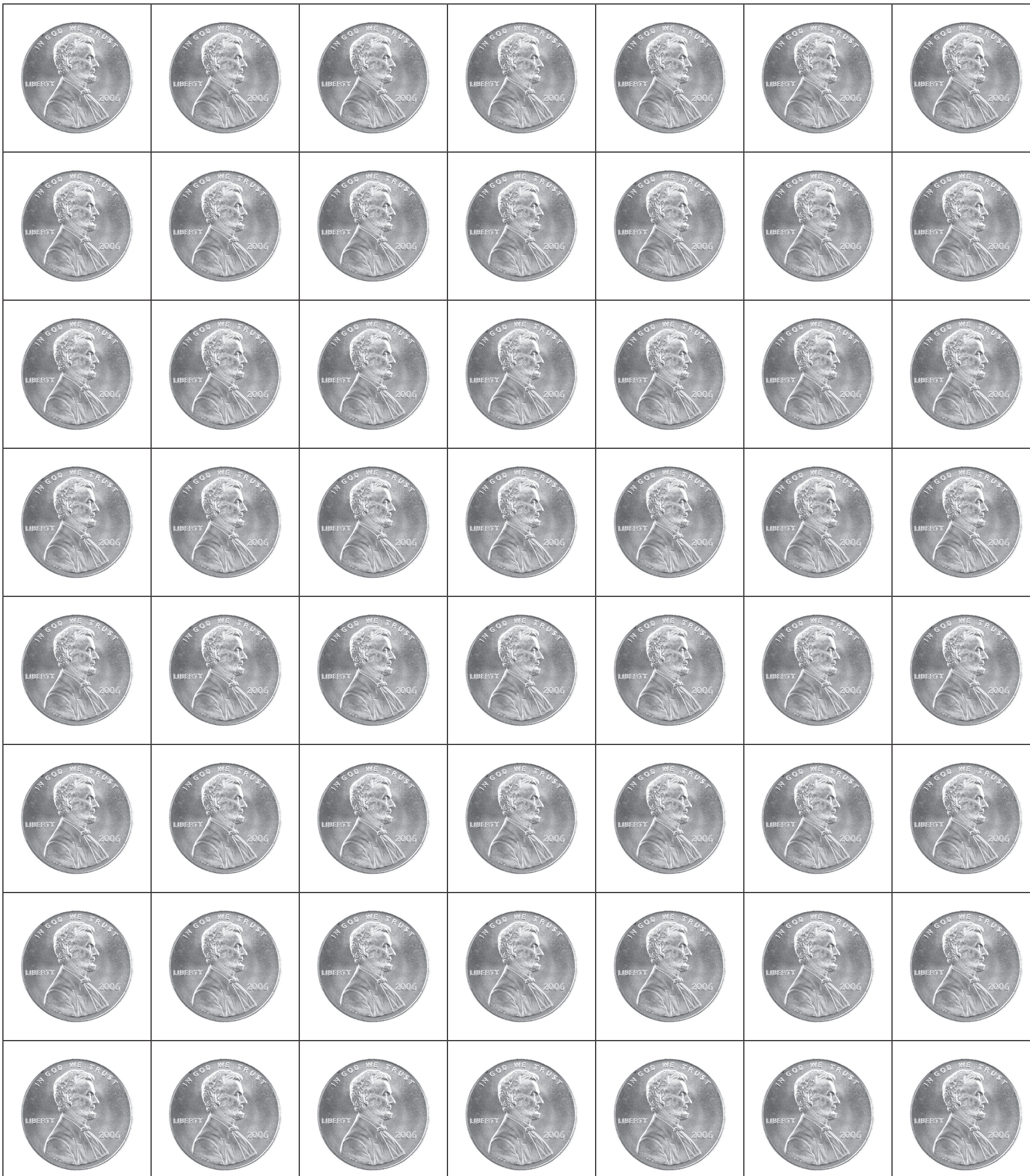


Table of Nickels

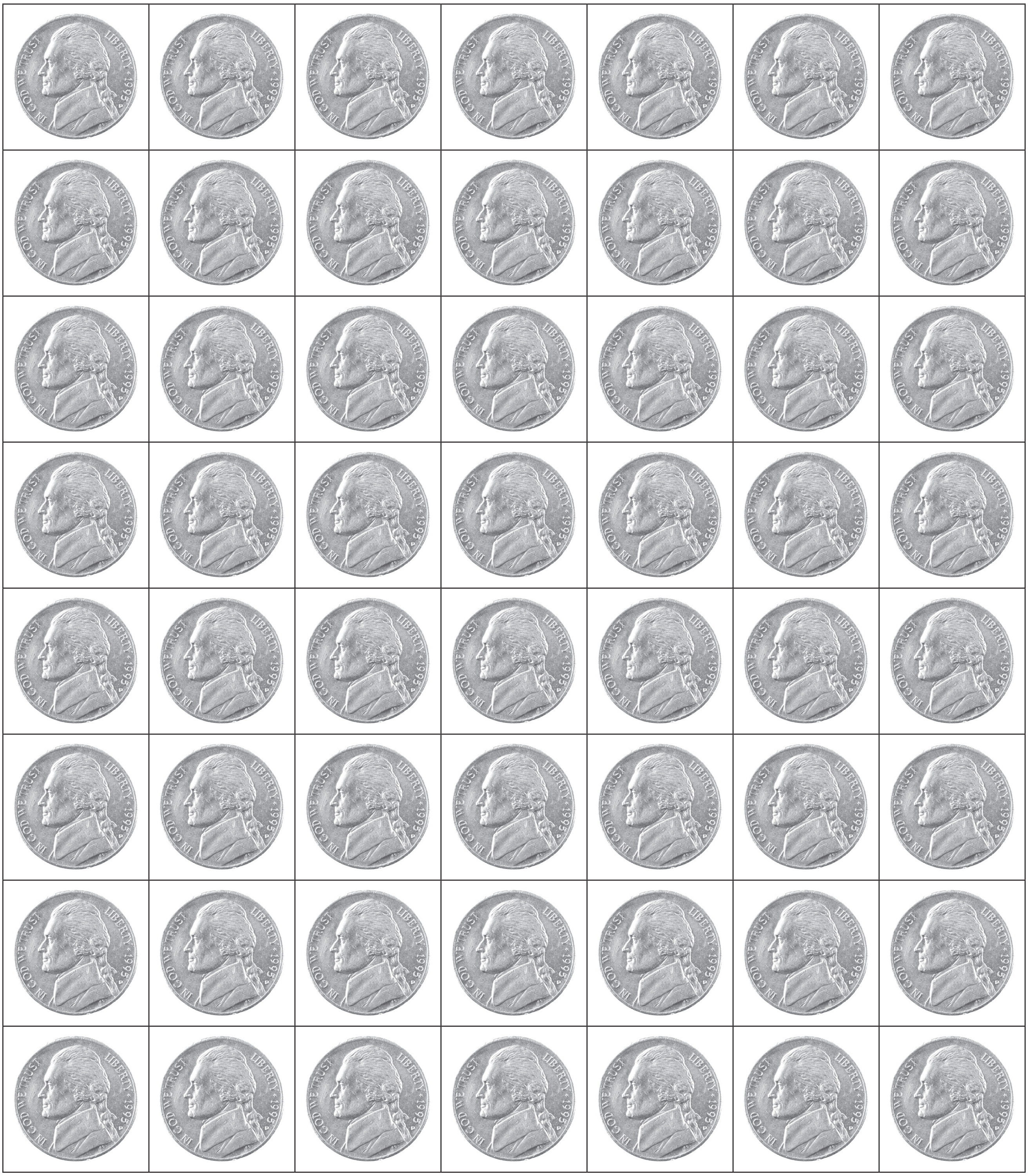


Table of Dimes

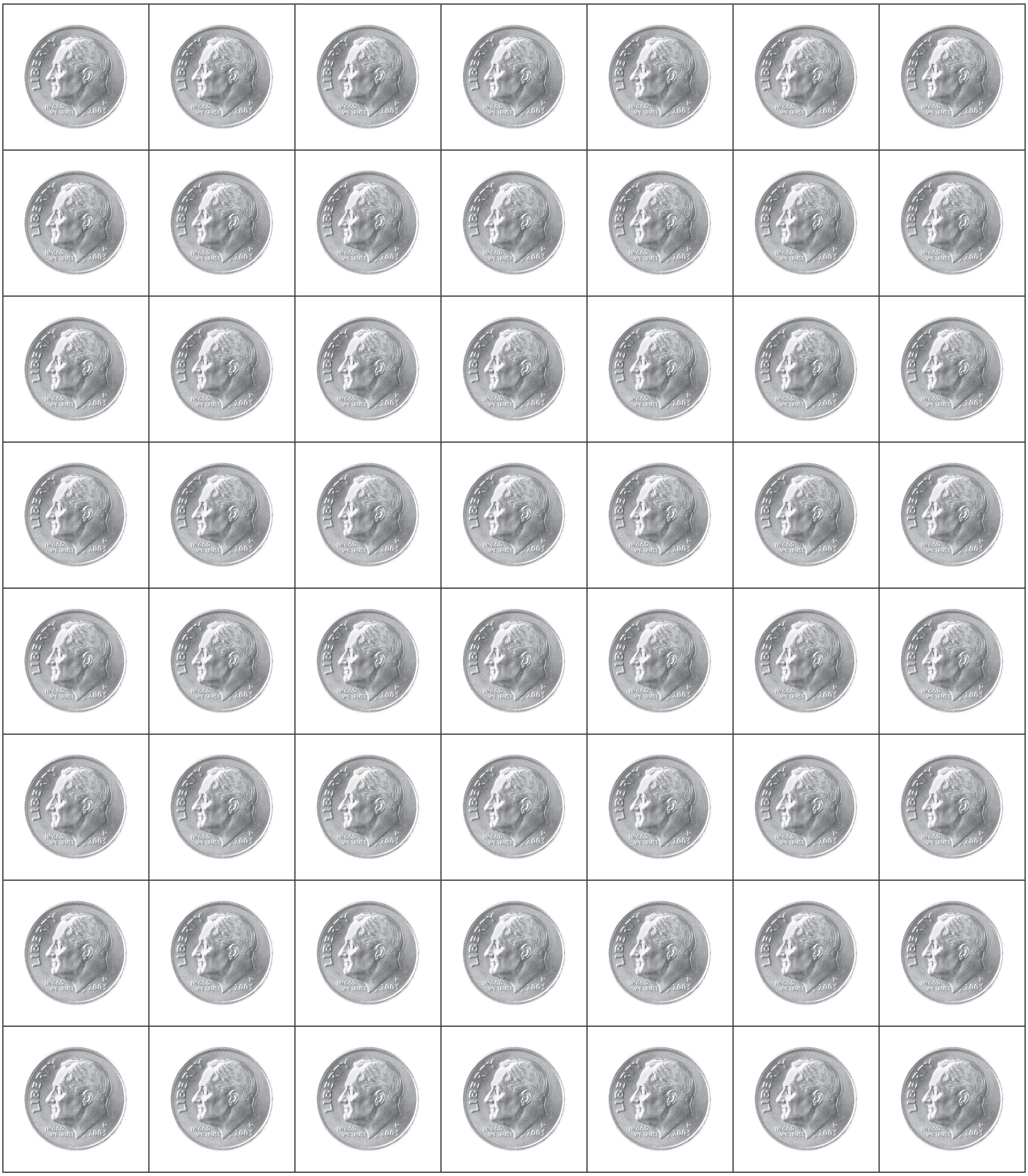
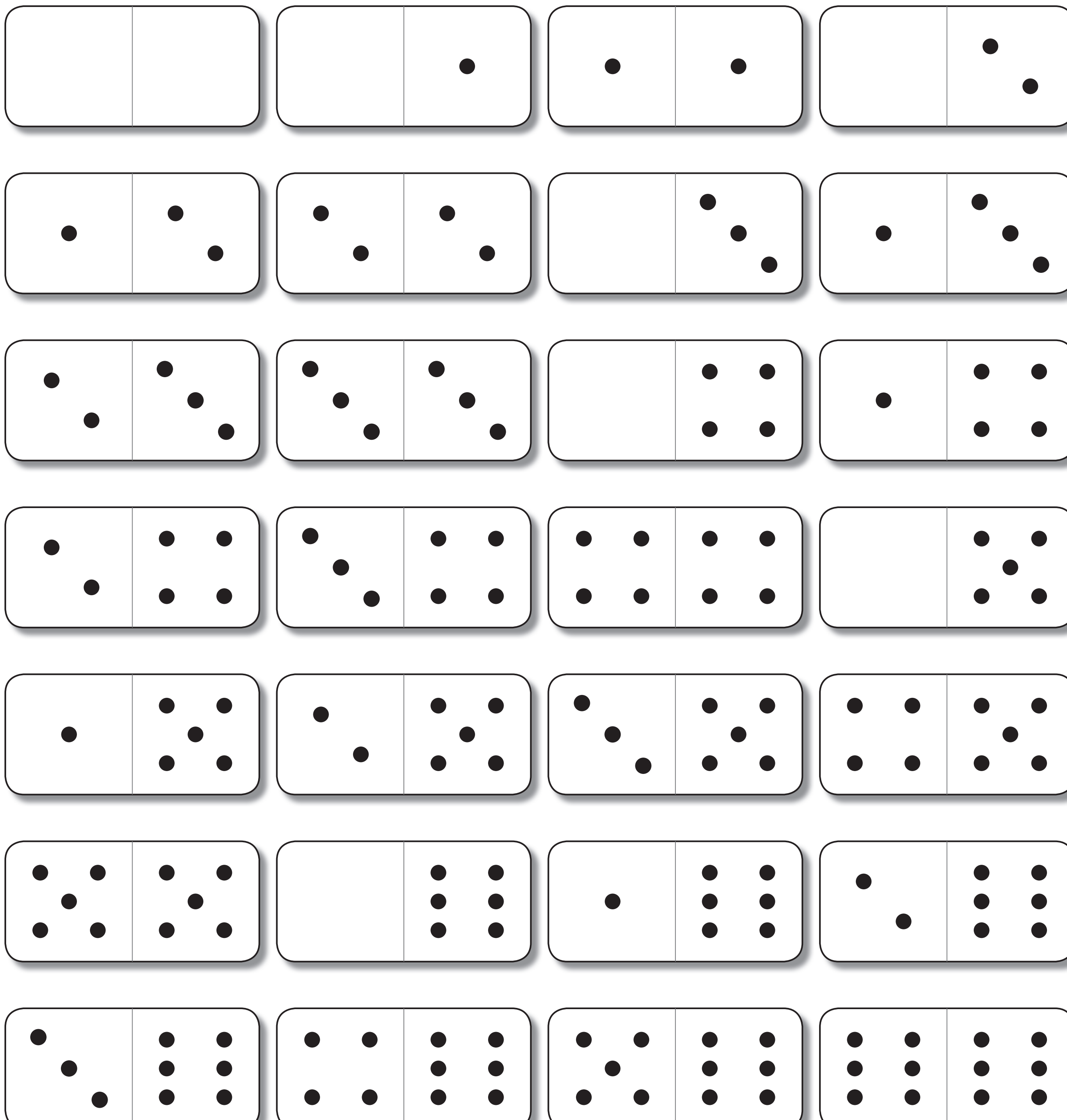


Table of Quarters

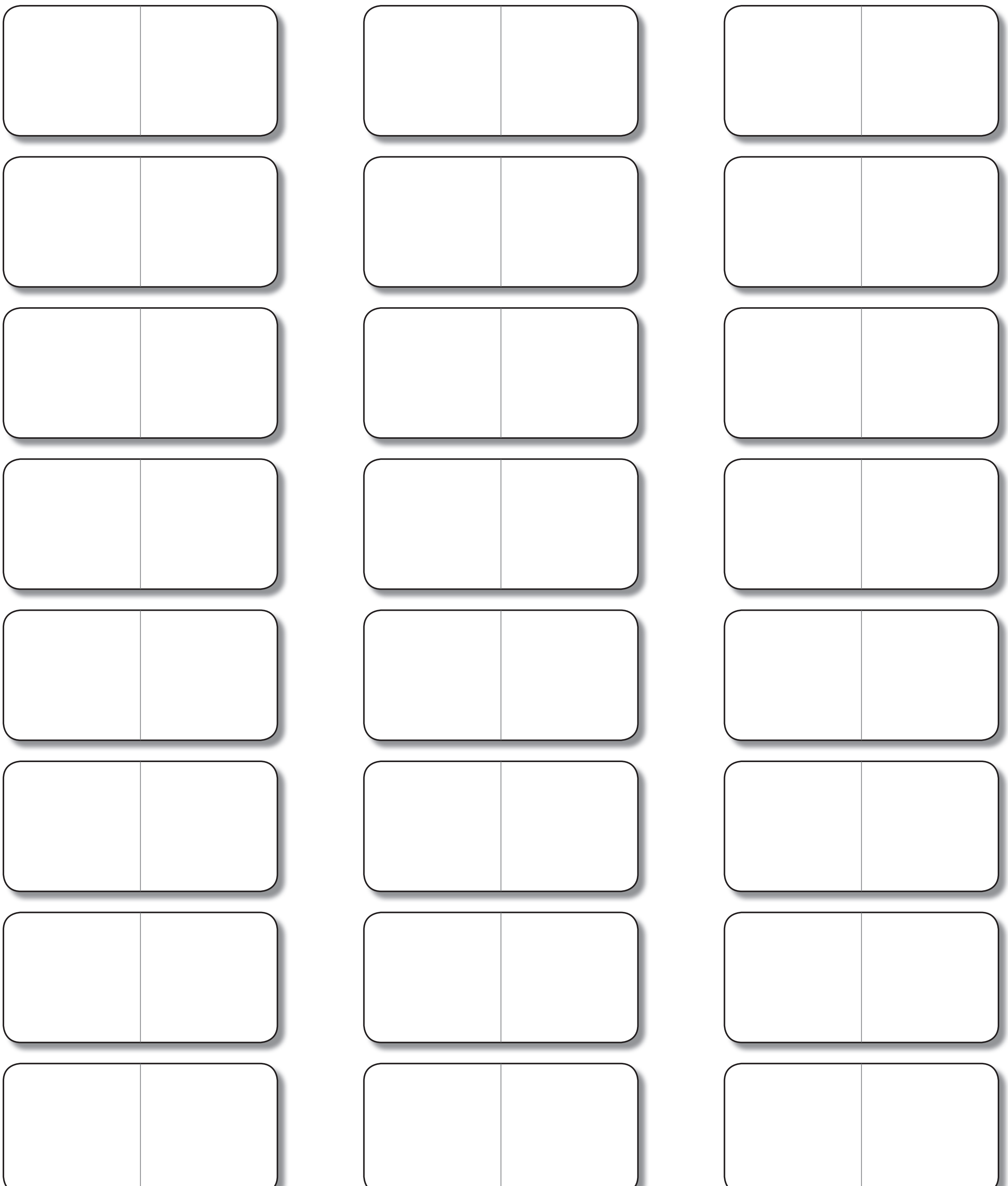
Table of Half-Dollars

Table of $1 Bills

28-Piece Set of Dominoes



Blank Domino Playing Pieces



Blank Spinners