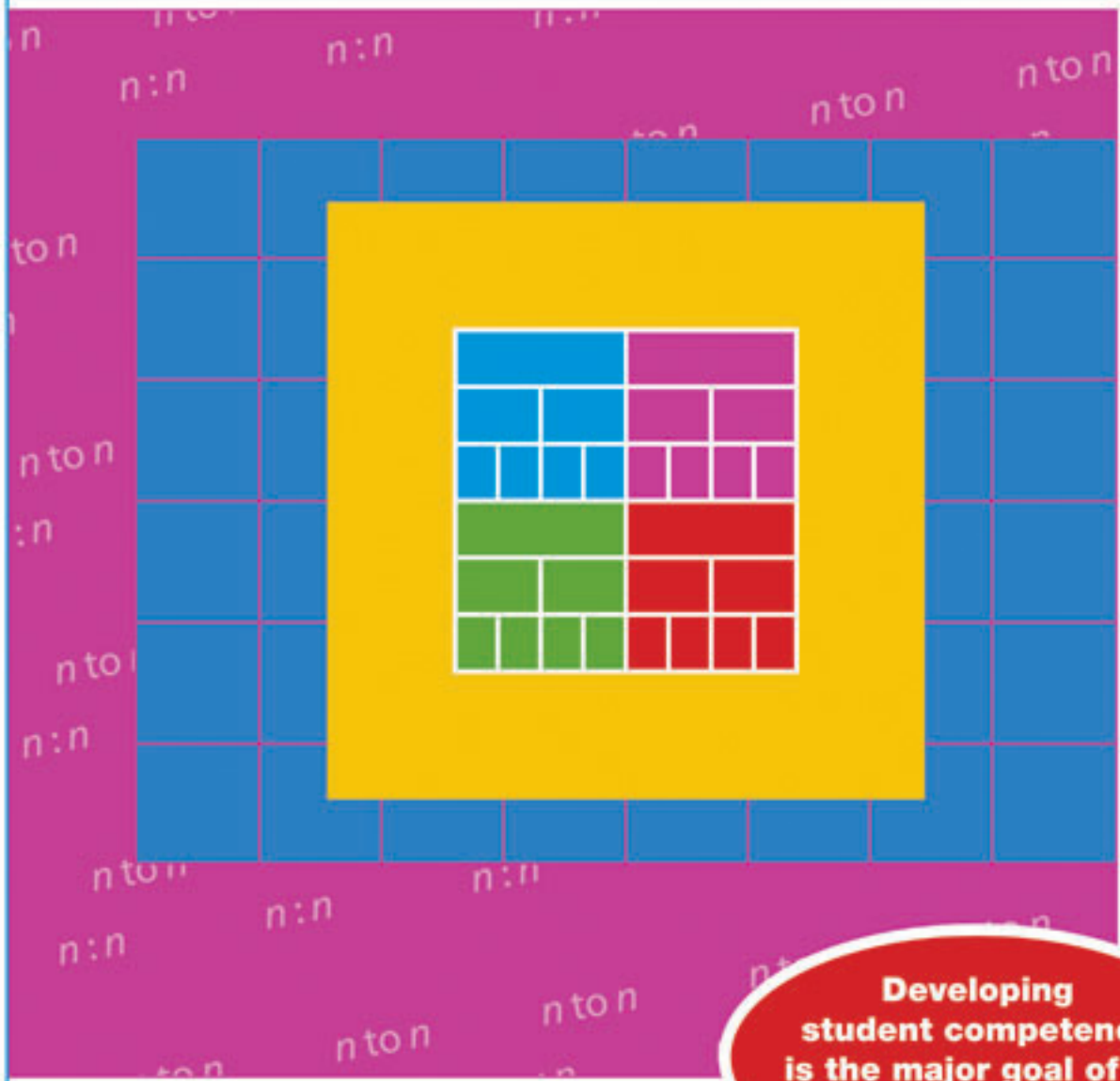
interact
Learn Through Experience

www.teachinteract.com

SQUARED AWAY

RATIOS, RATES & PROPORTIONS



Developing student competency is the major goal of all *Squared Away* units

Ratios, Rates, and Proportions:
A Squared Away Unit

v1.02

About the author

Fran Lyons Sammons completed her undergraduate and graduate work at the University of Rhode Island. After 30 years as a classroom teacher in Jamestown, RI, she is now a freelance writer and consultant to school districts, helping them implement standards-based curricula. She has written, co-authored, and contributed to several Interact titles, including *Election*, *House Design*, *Personal Finance*, *Branches of Government*, and several Squared Away titles. When she's not busy promoting active learning, she can be found sailing with her husband Jim, who is a science writer and retired science teacher.

©2010 Interact
10200 Jefferson Blvd. • P.O. Box 802 Culver City, CA 90232
Phone: (800) 421-4246 • www.teachinteract.com
ISBN# 978-1-56004-593-9
Product Code: INT855 v1.02

Introduction to *Squared Away* Units

"Squared away" was originally a nautical term used to announce that the sails of a square-rigger sailing ship were correctly set. The navy came to use the phrase to describe sailors who completed a task with competency, as in, "He was right squared away!" We have adopted the term to describe students who demonstrate competency in specific content and skills.

> Developing student competency is the major goal of all *Squared Away* units.

Each *Squared Away* unit allows both teachers and students to concentrate on basic concepts that can be mastered in a relatively short period of time. The basic subconcepts are taught in four instructional blocks. The daily activities are interactive, exploratory, and reflective—all best practices to maximize student learning. By the end of each block, students must demonstrate mastery of the subconcepts. After completing four blocks, students may be considered Squared Away. However, to earn a Golden Square, students must go beyond the basic level indicating that they achieved an exemplary score on a final test/project or mastered a final task requiring higher-level thinking skills.

Levels: The units are designed as complete, stand-alone lessons. Although written for either grades 2–4 or 5–8, the content may be used for instruction, enrichment, or remediation.

Differentiation: Teachers are encouraged to reteach and scaffold the learning so that all students master the concepts. Investigations take place in cooperative group settings that allow for peer teaching and support for students with learning difficulties. An extensive list of optional extra activities follows each Instructional Square and provides opportunities for independent or group investigations. The "Golden Square" tasks and an extensive list of optional extra activities will provide opportunities for group investigations or independent challenges for your more talented students.

Student grouping: Students may work in *Squared Away* units as individuals, in pairs, or in heterogeneous teams of three or four. When working in groups, students are responsible for their own learning and for supporting the learning of their teammates. All units provide Cooperative Group Work Rubrics.

Lessons: The lessons begin with a list of concepts to be taught, materials needed, and a lesson-plan schedule. Each lesson is combines whole class instruction, group work, and independent tasks designed to build understanding. Each lesson includes Stop/Think/Draw/Write activities that cause students to synthesize their learning and to share what they know.

Assessments and Rubrics: All units include a pretest/posttest to be administered before starting and after completing the unit. You also assess students daily to check mastery of content and to determine points of confusion. Part of the assessments requires students to explain orally or in writing what they understand. Students may retake assessments until they achieve mastery. The units provide quizzes, tests, and rubrics. There are many opportunities in the daily lessons, optional activities, and assessments for students to demonstrate Gardner's Multiple Intelligences.

Timeline: The lesson plans address four basic instruction blocks and one block to achieve a Golden Square. These may take five or more days depending on the instructional time available and/or your students' grade level and prior knowledge.

Table of Contents

Table of Contents

Ratios, Rates, and Proportions

A Squared Away Unit

Purpose

This Squared Away unit will help your students to become very proficient in using ratios, rates, and proportions to solve many different word problems—not only those they find on standardized tests, but also those they routinely meet in their life experiences. Working in teams, students will review what they know about equivalent fractions and apply this knowledge to solving problems involving equivalent ratios (proportions). These problems include understanding ratios, creating proportions, determining unit rates and understanding problems involving time, rate, and distance. They will also have the opportunity to reflect on their problem solving skills and to create word problems that challenge their classmates. In a relatively short period of time, your students will see themselves as very competent problem solvers.

This Square Away unit was designed with principles promoted by the National Council of Teachers of Mathematics:

> "Research suggests that an important difference between successful and unsuccessful problem solvers lies in their beliefs about problem solving, about themselves as problem solvers, and about ways to approach solving problems (Kroll and Miller 1993)," page 259.
>
> "Teachers should regularly ask students to formulate interesting problems based on a wide variety of situations, both within and outside mathematics. Teachers should also give students frequent opportunities to explain their problem-solving strategies and solutions and to seek general methods that apply to many problem settings. These experiences should engender in students important problem-solving dispositions—an orientation toward problem finding and problem posing; an interest in, and capacity for, explaining and generalizing; and a propensity for reflecting on their work and monitoring their solutions.
>
> "For several reasons, students should reflect on their problem solving and consider how it might be modified, elaborated, streamlined, or clarified: Through guided reflection, students can focus on the mathematics involved in solving a problem, thus solidifying their understanding of the concepts involved. They can learn how to generalize and extend problems, leading to an understanding of some of the structure underlying mathematics. Students should understand that the problem-solving process is not finished until they have looked back at their solution and reviewed their process."
>
> © 2000 *Principles and Standards for School Mathematics*, pages 256–261

Educational standards

This unit addresses NCTM Standards for School Mathematics**.**

Number and Operations Standard for Grades 6–8

In grades 6–8 all students should:

Understand numbers, ways of representing numbers, relationships among numbers, and number systems

- understand and use ratios and proportions to represent quantitative relationships;
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;

Compute fluently and make reasonable estimates

- develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use;
- develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Problem Solving

Instructional programs from prekindergarten through grade 12 should enable all students to:

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Communication Standard for Grades 6–8

Instructional programs from prekindergarten through grade 12 should enable all students to:

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;

- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Representation Standard for Grades 6–8

Instructional programs from prekindergarten through grade 12 should enable all students to:

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

NCTM Technology Principle

- Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.
- Calculators and computers are reshaping the mathematical landscape, and school mathematics should reflect those changes. Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction.
- Technology also offers options for students with special needs. Some students may benefit from the more constrained and engaging task situations possible with computers. Students with physical challenges can become much more engaged in mathematics using special technologies.

Knowledge, skills, and attitudes

Knowledge—*Your students will*:

- Understand that fractions are numbers that can be written as $\frac{a}{b}$ (b ≠ zero).
- Understand that the top number of a fraction is called the numerator and the bottom number is the denominator.
- Understand that when the numerator and the denominator are equal, then the fraction describes a whole or "1".
- Understand that the number "1" can be written as and fraction where the numerator and denominator are the same ($\frac{n}{n}$).
- Understand that ratios are descriptions comparing two items or measures.
- Understand that the order of numbers is important in a ratio.
- Understands that a proportion is a statement that two ratios are equal.
- Understand that all math strategies used with equivalent fractions may be applied to ratios and proportions.
- Understand that the "whole" is revealed when the numbers of a ratio are added together.
- Understand part to part and part to whole ratios.
- Understand that all rates are ratios, but not all ratios are rates.
- Understand that a rate always has two very different labels. *Examples: mph, price per pound, cost per hour, etc.*
- Understand that a rate often contain the word "per" and implies per 1, as in $10/hour means $10 per 1 hour.
- Understand that rates of speed involve time and distance.
- Understand that rates of speed are generally expressed as mph and mean miles per one hour or miles per 60 minutes.
- Understand that rates can be translated into smaller time units, such per hour can be per 60 minutes, per day can be per 24 hours, etc.

Skills—*Your students will learn how to*:

- Create equivalent fractions by creating a table.
- Create equivalent fractions by multiplying or dividing by "1" written as a fraction.

- Determine if two fractions are equivalent by cross-multiplying and comparing the cross-products.
- Determine an equivalent fraction by cross-multiplying and solving for *n*.
- Write a ratio from a statement using 3 formats: __ to __. __ : __, and the ratio in fraction form.
- Create equivalent ratios by creating a table.
- Create equivalent ratios by multiplying or dividing by "1" written as a fraction.
- Simplify ratios.
- Determine if two ratios are equivalent by cross-multiplying and comparing the cross-products.
- Determine a missing term of a proportion by cross-multiplying and solving for *n*.
- Write a proportion from a given statement.
- Determine what constitutes the "whole" of a group from a ratio.
- Use proportions to solve word problems involving ratios and proportions.
- Recognize rates in word problems.
- Solve rate problems using ratios and proportions.
- Determine unit prices.
- Convert rates of speed to ratios in word problems.
- Solve word problems involving time, rate, and distance by using proportions.
- Score their classmates' and their own answers using a rubric.
- Use and understand how calculators work to solve ratios and proportions.

Attitudes—*Your students will appreciate that:*

- Breaking down the elements of a word problem (context, numbers, labels, and questions) will help them to understand what they know and what they are trying to find out.
- Building on what they know about equivalent fractions will help them develop new skills to solve *Ratios, Rates, and Proportion* problems.
- Using *Ratios, Rates, and Proportion*, will greatly improve their ability to solve even difficult word problems.

- Realizing that personal experiences can be described in word problems will help them to solve word problems they meet in textbooks.
- Evaluating their classmates' work using a rubric will improve their own work.
- Checking their answers, even when using a calculator, will ensure that their answers are reasonable.

Content overview

This *Squared Away* math unit is designed to be concept specific and is, therefore, grade level independent. It is very effective as a concept introduction/reinforcement unit and can also serve as an efficient remediation unit through grade 8. Although students may work individually, the lesson plans direct students to work in teams as they complete word problems. These team activities will not only reinforce students' problem solving skills, but also their ability to communicate their understanding to classmates.

Square One concepts—(blue square) Students review how to recognize and generate equivalent fractions. If your students are proficient manipulating equivalent fractions, then give the **Square One Test** and move directly to Square Two.

Square Two concepts—(red square) Students learn what a ratio is and how to create Proportions. They begin to apply their understanding of Ratios and Proportions to solve word problems.

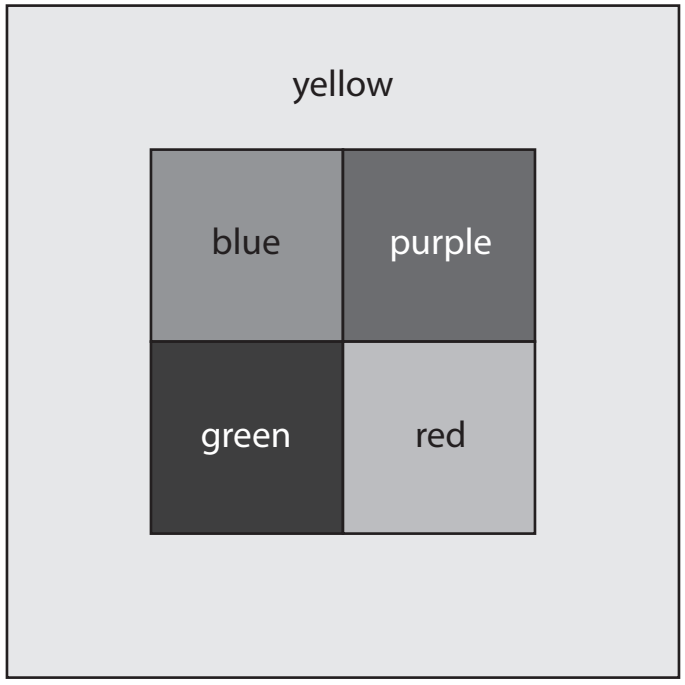
Square Three concepts—(green square) Students learn about rates, including unit rates and apply their understanding to solve word problems.

Square Four concepts—(purple square) Students learn to use rates and proportion to solve word problems involving time, rate, and distance.

Golden Square concepts—Students learn to apply their understanding of proportions to solve real life tasks involving scale and similarity.

Making the individual award squares

Award squares can be created from four different colored pieces of construction paper. Cut the paper first into one-inch strips, and then cut those strips into one-inch squares. The four colors are only suggestions; however, use yellow paper only for the Golden Square. Cut a $2\frac{1}{4}$-inch, plain white square to form the backing. Students will paste their earned squares on the backing as they complete each instruction block. Make a $3\frac{1}{2}$-inch yellow square for each Golden Square. (You may choose to trim the backing before affixing the four-square to the Golden Square.)



Teaching tip
Attach the background squares to the team folders so students do not lose track of them.

Unit Time Chart

Depending on the students' ages, prior knowledge (specifically how fluently they can work with equivalent fractions), and the length of the math period, you may complete one group of concepts per instructional block. Block lesson plans are designed for a math period of 60–90 minutes. If a class is made up of younger students, disabled students or the assessments indicate students need more instruction, then divide the large block lessons into smaller, more workable time periods. Students with stronger math skills may complete the basic instructional block in 45 minutes. At the end of Instructional Blocks two, three, and four are lists of optional activities.

Instruction Block One

- Pretest
- **Square One concepts** (blue)
- Review Equivalent Fractions
- Individual Square One Test

Instruction Block Two

- **Square Two concepts** (red)
- Intro to Ratios and Proportions
- (Optional activities)
- Individual Square Two Test

Instruction Block Three

- **Square Three concepts** (green)
- Ratios and Rates, including unit rates
- (Optional activities)
- Individual Square Three Test

Instruction Block Four

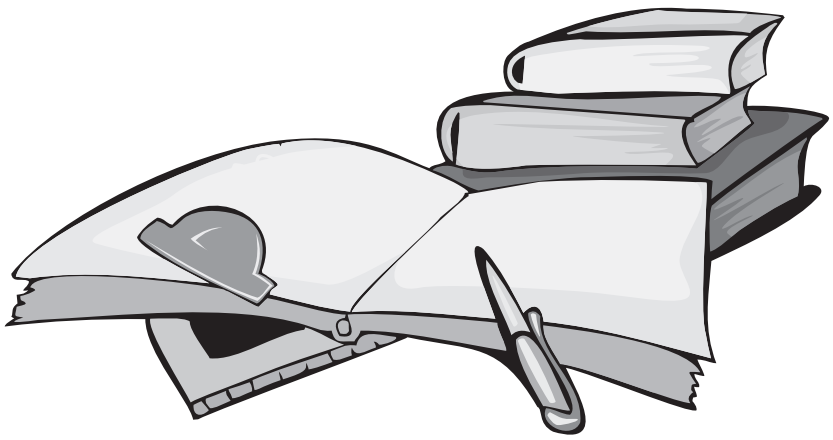
- **Square Four concepts** (purple)
- Solving Time, Rate, and Distance problems with proportions
- (Optional activities)
- Individual Square Four Test

Instruction Block Five

- **Golden Square concepts** (yellow)
- Scale, similarity and Proportions
- Individual Golden Square Challenges

Assessment Block

- Final review
- Posttest (one week later)
- Awards ceremony for Four Squares and Golden Squares



General Directions

Ratios, Rates, and Proportions

Instruction blocks

This unit is divided into four Instruction Blocks that address specific instructional objectives related to equivalent fractions, ratios and proportion, ratios and rates, and time, rate, and distance problems. Each block is sequential and builds on the knowledge and skills learned in the block before it. Always evaluate the tests of one block before going on to the next.

Student grouping

Students may work in *Squared Away* units as individuals, in pairs, in trios, or in groups of four. Create your teams before the first lesson. Generally the most successful teams are mixed by gender, math sense, and study skills.

Teaching tip
With math topics, trios are preferred because the groups are small enough to ensure maximum participation during problem solving without burdening the teacher with too many groups.

Student Roles rotate only after taking a **Square Test**. When working in pairs, combine the roles of Leader and Manager. If you must make a team of four, separate the Reader-Recorder into two roles.

Leaders: organize the team and delegates/direct team members as needed. He/she checks that the day's assignments are complete, and makes sure teammates submit all assignments. The Leader is the official team time keeper who keeps the team on task. He/she maintains the team folder, organizing it and handing it into the teacher at the end of each day.

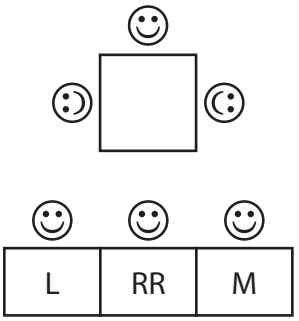
Reader-Recorder: reads handouts and/or activity directions. The Reader clarifies and repeats as necessary, and writes the team's responses.

Manager: collects and returns supplies, handouts, and materials needed for the team's daily work.

Bright Idea
Make a class chart of the role assignments for each team and post it. Briefly review the duties for each role before beginning the lesson.

Classroom arrangement

Organize student desks to accommodate pairs, trios, or groups of 4 with the Reader-Recorder in a center seat. Allow as much space as possible between groups so that students can converse among their teammates without distractions.



Supplies and duplication

Students will need individual simple calculators. At the beginning of each lesson is a list of the duplication you need to prepare. Consider duplicating all the handouts ahead of time, making the instructional packs, and storing

them in order in a hanging file. Be certain to make extra copies of the handouts for teacher aides or resource teachers who may be working with your students. Also make several extra copies of the Square Tests for students who need to retest.

Student folders

Create a folder for each team listing the names of team members on the front. Attach inside one copy each of the **Cooperative Group Work Rubric** and the **Concept Content Rubric**. Also include a copy of the Squared Away Unit Introduction. On the outside cover, paste a blank background square for every team member, and make a role assignment chart. Review the duties of each role before beginning an Instructional block. Decide where in the classroom you want students to store and retrieve folders each day.

Lesson plans and timeline

Squared Away lesson plans begin with a concepts list and materials. There is a lesson schedule that matches the headings within the numbered lesson steps. Always read through the day's lesson ahead of time to familiarize yourself with how the lesson should proceed and how the concepts are developed. Specific concepts are introduced in a special order to minimize confusion. The lessons are arranged in Instructional Blocks that will take about 60 minutes or more depending on your students' prior knowledge. The five instructional blocks may be broken into ten lessons if students need more time to process the concepts. This time arrangement will give you more class time to re-teach students who are having difficulty while those who are squared away may work on extension activities. You may choose to use more days and supplement with other materials.

Online resources

Consider looking at web pages such as the NCTM.org and teachersdomain.org for math videos that your students may watch or additional problems they may solve on the classroom computer or on a Smart™ board.

Team competitions

One goal of this unit is for all students to work cooperatively to learn the content. However, awarding points for good team work and appropriate student behavior may add some incentive for better effort. When grading classwork and teamwork, use the rubrics provided and award 4, 3, 2, or 1 points. Consider giving points for the neatest folder, being on task first, finishing on time, etc. Keep a daily running tally to encourage students to do their best.

Stop/Think/Draw/Write

In every lesson, students are asked to stop, think, draw, and write about something they have been studying. Research supports greater understanding and recalls when students are asked to process new information in writing. If your students maintain math notebooks, you may decide to have students add these pages to their notebooks. Otherwise, have students keep their pages in the folder. Ask them to put their names and team name on each sheet. They should paperclip new pages to existing pages so that all their work is together. Check these drawings and writings daily as quick informal assessments. Award team points if you are using team competition as an incentive. There is a blank copy of **Stop/Think/Draw/Write** without specific prompts on page 64 of this guide. These responses can also be used as portfolio entries.

How to create word problems

For Squares Two, Three, and Four, there is an **Optional Extra Activity** called Round Robin Problem Solving that tells students to write their own word problems to challenge their classmates. Your students will do much better with this exercise if you teach the handout called **How to Create Word Problems** on page 65. Writing interesting problems is not all that easy to do. If your students have language difficulties, allow them to just rephrase one of the problems given in the unit using a different context and different numbers. Being aware of how word problems are structured will take away some of the anxiety students feel when faced with word problems.

Assessments

The Pretest and Posttest are identical and are administered at the beginning and at the end of the unit. Administer the **Pretest** before Instruction Lesson One. The pretest will reveal student prior knowledge so that you can be more confident about the pace of your lessons. The questions are arranged in the order of the Instructional Blocks. (Questions 1–5 assess Square One.)The **Posttest** will let you know how competent your students have become with the content and skills presented in the unit. Give the posttest at least one week after you complete the unit. The answer key is on page 19.

Cooperative Group Work Assessment is ongoing and important to the success of this unit. Students need to work together, being responsible for their own learning and helping their team members to succeed. Post the **Cooperative Group Work Rubric** and tape one copy inside each team folder. During the Instruction Block, occasionally refer to the Rubric and comment on how the teams are working. Praise good work and, if necessary, point out where teams could improve. At the end of each block or at the end of a teaching day, assess every team member using the Cooperative Group Work

Rubric as your guide. At the end of the unit, give students a copy of the rubric with an overall assessment of his/her group work.

Teaching tip

Sometimes some students "hide" during group work allowing the more vocal students to answer all the questions. To make certain you are interacting with every student, create a class team seating chart for each day of instruction. When you call on a student, make a check mark near his/her name. If the student seems confused, circle the check mark.

Concept Content Rubric is provided for assessing student understanding when they draw and/or explain their answers. Tape one copy of this rubric inside the team folders. Use this rubric to provide maximum feedback to the students and to allow you to check off items on the class **Content/Skills Checklist**. Encourage students to evaluate their own and their teammates' classwork using this rubric. Look at pages 16–17 for exemplars of each score.

Informal Assessments are also on-going and important. At different times during the lesson, take a moment to ask students individually or as teams to explain what they understand or to demonstrate something they have just learned. You can ask them to respond orally, or ask them to quickly write an explanation to share. If you detect confusion in one student, it may indicate a general confusion. Use the available **Content/Skills Checksheet** on pages 62–63 daily to keep track of skills students have learned or still need to learn.

Quick Team Quizzes take about five minutes to administer, but they allow you to make a final check before distributing the individual tests. Each team member gets his own paper to complete, but when all the team members have finished, they compare answers. If there is a discrepancy among the answers, encourage peer teaching and ask the team discusses the different answers and what the team members were thinking. You may also pair teams to compare answers. Always remind students of the **Cooperative Group Work Rubric** and their responsibility to be kind and supportive. After 3–5 minutes, go over the correct answers with the whole class addressing any questions they may still have.

Individual Square Tests are given at the end of each Instruction Block. You may administer them at the end of an Instructional class period or at a later time. The second, third, and fourth Individual Tests contain questions from the previous instructional blocks so that the Fourth Test may be considered a final exam for the four blocks. Separate the desks and insist that students work individually on these assessments. Before beginning a new Instruction Block, evaluate student answers for the Square test to determine the need to re-teach some concepts. Generally only a few students are confused about one or two test questions. Create ad hoc groups, review, and retest. The retest should be a new copy of the same test with those items he/she must redo circled or highlighted. Follow this procedure for each Square Test.

Answer Keys for all activities and assessments are found at the end of the lesson plans for each Instructional block.

The Gold Square Challenge is a fifth assessment given after the class has completed Blocks 1–4 and is Squared Away. Not all students are

expected to earn the fifth, golden square. In this unit, the students use their understanding of proportions to solve problems and complete tasks involving scale and similar triangles. The **Golden Square Challenge** is an individual challenge, although you may choose to allow teams. You can assign the challenges for homework, but expect that parents will help. Therefore, insist that students explain in writing what they did and how they completed the task. You may choose instead to give the two **Golden Square Challenge Word Problems**. These are quite difficult, but you can assign them as an in-class assessment.

Reinforcing and reteaching concepts

The timeline for this unit depends on your students' prior knowledge and the length of your math period. It is also often difficult to find time to re-teach students who do not grasp concepts on the first round. However, taking the time to work with struggling students before they are overwhelmed is essential. Consider stopping after Instruction Block Two and again after Block Three to work with students who still need help. Allow students who have mastered concepts to work on the **Optional Extra Activities** while you work with those who need more instruction.

Homework and optional activities

As part of the Optional Activities, there are worksheets that reinforce problem solving for each Instructional block. Or if your students need more practice, you might assign problems from their math book or from sites online. There is also a list of activities that students can complete as individuals, in pairs or teams. When you assign homework, award extra points to those teams who all complete the work. On the day after an assignment, you might also consider asking you students to quickly and accurately solve some of the homework problems or similar problems in a "race" or other competition with other teams. Students who have practiced with homework will solve the problems faster and more accurately.

Calculators

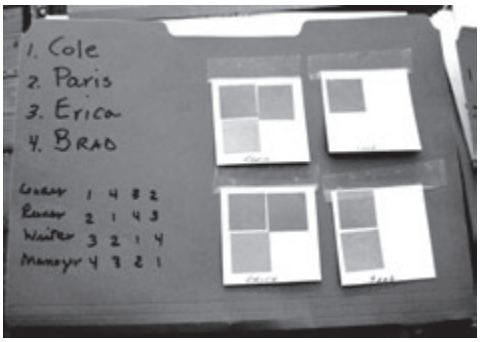
In order to complete this unit in a relatively short amount of time, you must allow students to use calculators. NCTM supports appropriate calculator use in combination with estimation. Using calculators, however, does not mean no paper and pencils. Require your students to write their proportions with labels. Remind them to circle the numbers they are cross multiplying so they can be certain they are entering the correct numbers. They can and will get lost if they just enter numbers on the calculators and will otherwise have no way to check where they may have made an error. Reinforce recording by only giving partial credit (a 2 out of 4) to papers that list a correct answer without records of computations. Insist that they reread the questions and

Teaching tip

"Research has also shown that calculators can aid in 'stimulating problem solving, in widening children's number sense, and in strengthening understanding of arithmetic operations' (Campbell and Stewart 1993)." NTCM

look back at their answers to ensure that they are reasonable.

Awarding squares



The most efficient way to award the squares is to glue them onto team folders and announce the names when you distribute the folders before beginning a new Instructional Block. If possible, notify students ahead of time why they are not going to earn a square that day. Stress, though, that they will eventually earn their squares and arrange a time to re-teach and re-test them. Consider giving students a portion of a square to indicate that they have mastered some of the content. Pasting the squares on the outside of folders allows you to quickly see how groups are doing and to check on teams that are falling behind.

Your classes may be too "grown-up" to respond well to collecting squares. In that case, you may choose to acknowledge achievement in other ways. Consider posting a wall chart with check marks, offering class privileges (free time, prizes, etc) or whatever else will help motivate your students.

Final award celebration

With the successful completion of the Square Four Test, your students are considered "Squared Away." However, consider waiting until you run the Golden Square Challenge Activities to celebrate. This will give you more time to work with those students who are close to finishing the four squares. The Golden Square Challenges are just that—challenges, and not essential to be considered "Squared Away." However, give special recognition to all those who do achieve a Golden Square.

Design a celebration appropriate to your students' age, and your available time and resources. Give one of the two Special Award Certificates on pages 109 and 110 to acknowledge each student's achievement.

Using the Content Concept Rubric for Math

Concept Content Rubric

4 **Exemplary**—You demonstrated a *clear* understanding of the math problem(s). You *accurately* and *completely* described/drew the concept in detail using *correct labels*. You communicated your understanding clearly with few, if any, spelling or grammatical errors.

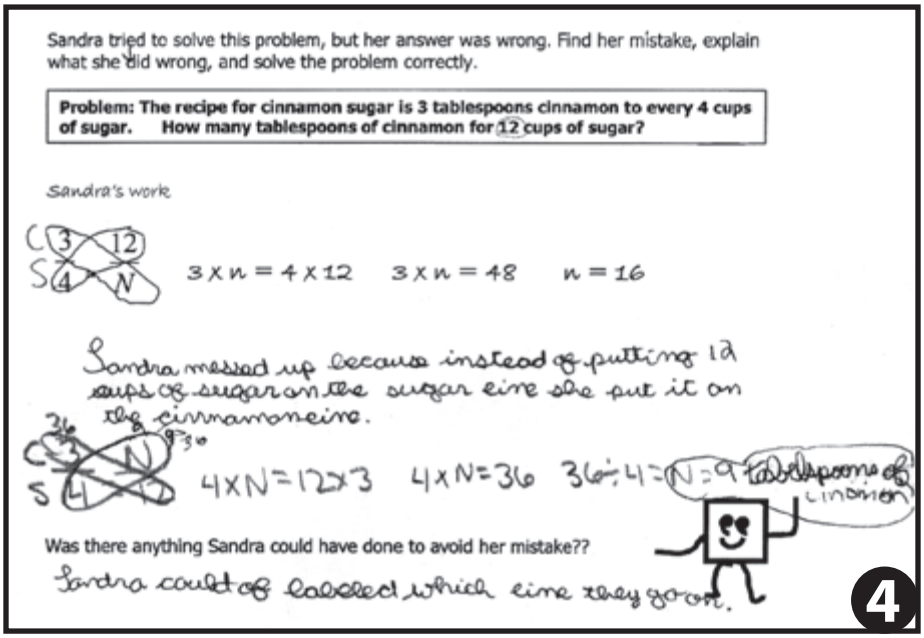
3 **Expected**—You demonstrated a *good understanding* of the math problem(s). You *accurately* described/drew the concept using *some detail* and *correct labels*. You communicated your understanding clearly with few, if any, spelling or grammatical errors.

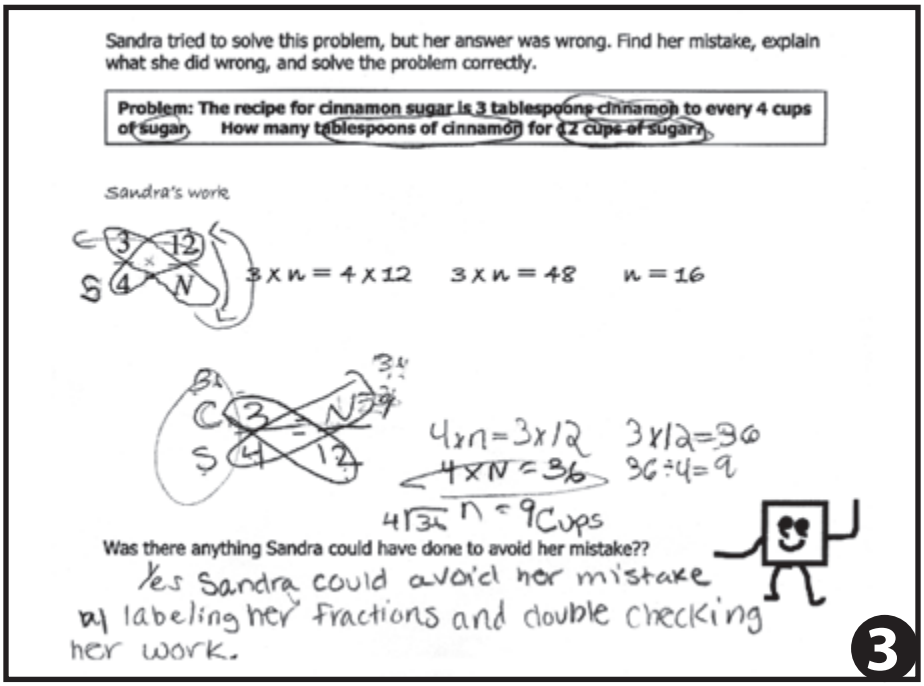
(If your evaluation is Level 2 or 1, strive to correct your work at least to Level 3.)

2 **Almost There**—You demonstrated *some* understanding of the math problem(s). However, you did not describe/draw it accurately and your answer was not complete and/or some of your labels were incorrect. Or you did not communicate your understanding clearly because of spelling or grammatical errors. You may need to meet with your team or teacher to learn the concept more completely, or you may need to redo your work, correcting the errors.

1 **Incomplete**—You demonstrated *little or no understanding* of the math problem(s), so you could not describe/draw it. You need to meet with your team or teacher to relearn the material.

Sample student work





Sandra tried to solve this problem, but her answer was wrong. Find her mistake, explain what she did wrong, and solve the problem correctly.

Problem: The recipe for cinnamon sugar is 3 tablespoons cinnamon to every 4 cups of sugar. How many tablespoons of cinnamon for 12 cups of sugar?

Sandra's work

$3 \times n = 4 \times 12 \quad 3 \times n = 48 \quad n = 16$

She set up her problem wrong, 12 goes on the bottom.

Was there anything Sandra could have done to avoid her mistake??

Set up the problem right

2

Sandra tried to solve this problem, but her answer was wrong. Find her mistake, explain what she did wrong, and solve the problem correctly.

Problem: The recipe for cinnamon sugar is 3 tablespoons cinnamon to every 4 cups of sugar. How many tablespoons of cinnamon for 12 cups of sugar?

Sandra's work

$3 \times n = 4 \times 12 \quad 3 \times n = 48 \quad n = 16$

$3+12=15$

$4\overline{)15}$ 3 cups is on

12

3

Was there anything Sandra could have done to avoid her mistake??

She should do adding not divide

1

Before Starting This Unit

1. Read through the **Teacher's Guide** to familiarize yourself with the content and materials. You will need a class set of simple calculators.
2. Organize and prepare the **team folders**. See pages 7 and 14. If you decide to award squares, prepare the awards background square and the colored squares. If you decide the use a chart, prepare the chart and introduce it when you explain the unit.
3. Decide how you will arrange your class and where the folders will be stored.


Individual

4. **Pretest**

   Administer the **Pretest** to all students individually before starting this unit. Remind students they should *not* guess on the pretest, and assume if there is no written work in the space to the right that they did guess. Obviously having a general idea of student pre-knowledge will help you to pace and/or supplement your unit.

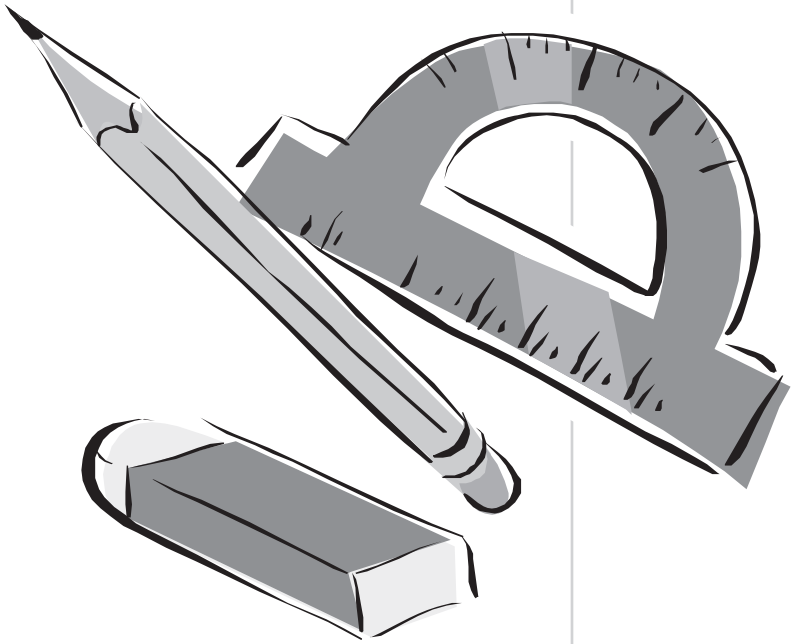
5. **Duplicate** the handouts for Instruction Block One. Those marked *one per team folder* should be put in the team folders ahead of time. Those marked *one per student* will be distributed during the instructional period. Gather class sets of calculators and any other necessary materials before beginning.
6. If your students need time to form cohesive teams, announce the team groups earlier and allow each team to choose a name before the first day. Simpler is better and asking students to include a math term in the name keeps it simpler still. (Examples: Flying triangles, the Dividends, Red Rhombi, etc.)

Answer key for *Pretest/Posttest*

For post test, look at the exemplars on pages 16–17 to see how to use the **Concept Content Rubric** to evaluate answers.

1. A and D
2. C and D
3. B
4. D
5. A
6. T
7. B
8. A
9. B
10. B
11. D
12. B
13. C
14. A
15. C
16. $\frac{58 \text{ miles}}{1 \text{ hour}}$
17. $\frac{7 \text{ miles}}{60 \text{ minutes}}$
18. B
19. C

Challenge: 24 by 32 by 40



Instruction Block One

Introduction to Equivalent Fractions

***Square One Concepts**—Students will:*

- Understand that fractions are numbers that can be written as $\frac{a}{b}$ where $b \neq$ zero.
- Understand that the top number of a fraction is called the numerator and the bottom number is the denominator.
- Understand that when the numerator and the denominator are equal, then the fraction describes a whole or "1".
- Understand that the number "1" can be written as any fraction where the numerator and denominator are the same ($\frac{n}{n}$).
- Create equivalent fractions by creating a table.
- Create equivalent fractions by multiplying by "1" written as a fraction.
- Create equivalent fractions by dividing by "1" written as a fraction.
- Determine if two fractions are equivalent fractions by cross-multiplying and comparing the cross-products.
- Use cross-products to create an equivalent fraction when only 3 of the four terms are known.
- Score their own or group answers using the Concept Content Rubric.

Materials

- Calculators—*one per student*
- large paperclips—*one per student*
- Fraction manipulatives as needed

Duplicate

- Pretest/Posttest—*one per student* (file the second for a Post test)
- Cooperative Group Work Rubric—*one per team folder, one copy to display*
- Concept Content Rubric—*one per team folder, one copy to display*
- Introduction—*one per team folder*
- Equivalent Fractions—Parts 1, 2, 3 stapled into a pack—*one per student*
- Worksheet to Reinforce Square One Concepts (optional)—*one per student*
- Quick Team Quiz One—*one per student*
- Square Test One—*one per student*

Lesson plan schedule

- Pretest on an earlier day
- Equivalent Fractions—Part 1
- Stop/Think/Draw/Write 1
- Equivalent Fractions—Part 2
- Stop/Think/Draw/Write 2 and 3
- Equivalent Fractions—Part 3
  - Stop/Think/Draw/Write 4
- (Optional) Worksheet to Reinforce Square One Concepts
- Quick Team Quiz One
- Square One Test

Teaching tip
It is better to divide the lesson into separate parts that end with a **Stop/Think/Draw/Write** activity. Students usually need time and opportunity to process the new information and consolidate learning.

Lesson Plan

Introduction to Equivalent Fractions

1. Pretest


Individual

- Before you begin the unit, administer the **Pretest** and correct it.

2. Introduction


Small group

- On the day you begin the unit, arrange the room. Announce the teams. Allow 2 minutes for students to come up with team names.
- Have students read along with the folder copy of the **Introduction** as you read it to the class.
- Assign student roles. Review the duties of each role and the **Cooperative Group Work Rubric**. (See page 59–60 of this guide.) Direct students to the schedule of role changes written on their team folder. Remind them that they will rotate roles only after they take a Square Test.


Whole class

- Give Managers enough **Equivalent Fractions** packets for each of their team members. Read through Part 1 as a *whole* class activity. Depending on the **Pretest** outcome, review this material quickly or with more detail. Make certain students understand any vocabulary introduced in the essay: *numerator, denominator, equivalent, cross multiplication, cross-products.*

3. Stop/Think/Draw/Write 1

Teaching tip
Remind students that when asked to "draw," they should work quickly using simple sketches. Labels should be clear and spelled correctly.

- To be certain all students understand equivalency, direct them to **Stop/Think/Draw/Write**. Have students fill in their names and team names and complete the task. Allow teams 2–3 minutes to complete their work. Walk around as students work encouraging them to work quickly, but neatly to make their points.
- Have students first share what they wrote or drew with their team. Then ask teams to choose one teammate's explanations and drawings to share with the whole class.
- Using the **Concept Content Rubric**, have the whole class assess team answers. Reinforce correct explanations and correct any confusing or inaccurate answers. Summarize the explanations and model the correct explanations/drawings on the board.
- Send students back to their teams to correct all their papers. If necessary, have them rewrite what you have modeled.

4. Equivalent Fractions—Part 2


Whole class

or


Small group

All this information should be review. Again let the **Pretest** determine the teaching pace and the need for more practice.

Go over Part 2 as a *whole class* activity or a t*eam* activity. (Review roles if assigning a team activity.)

Make certain students understand the concept that numbers multiplied by one do not change in value, only in the way they appear. You may ask students, "What did "1" appear like in this problem?"

5. Stop/Think/Draw/Write 2 and 3

- Allow students 2–4 minutes to explain how to generate equivalent fractions by multiplying by 1 and then by dividing by 1.
- Reinforce using the **Concept Content Rubric**. When a team finishes, ask them to look at each other's papers and use that rubric to assign a score for each team member's explanation. As they work, walk around looking at their evaluations and comment on how closely they would have matched your evaluations.
- When all the teams are finished, take the opportunity to highlight answers that exceeded the standard (4). Point out why other pages would earn a lower score.

6. Equivalent Fractions—Part 3

- Have the Managers distribute the last **Equivalent Fraction** page. Although this, too, can be done as a team activity, it may be wiser to conduct a whole class activity. This page is the essential teaching page necessary to do the other squares.
- Stress the four steps, and the importance of writing them down, especially if they are using calculators. On step four, students may just compute 120 ÷ 8 or you can write the equation algebraically dividing both sides by 8.
- Allow students to work independently for 3–5 minutes. Walk around the room as students work. When they finish they should compare answers within their team. Teammates may offer help to any member of their team who has made an error.

$$\frac{8N = 120}{8 \quad 8}$$

$$\begin{array}{r} \quad\quad 40 \\ \cancel{8}N = \cancel{120} \\ \hline \cancel{8} \quad \cancel{8} \end{array}$$

$$N = 40$$

7. Stop/Think/Write/Draw 4


Individual

- When teams have finished with the three practice problems, students should work individually to answer the last **Stop/Think/Draw/Write 4** prompt, but on the back of their papers. Remind them to put their name

and team name at the top before they begin.

- While they work, walk around and check that students who have finished "early" have included all they know. Remind them of the **Concept Content Rubric**. Tell students to check their spelling and work neatly.


Small group

- After 3–4 minutes, ask students to first share what they wrote or drew with their team. Then ask teams to choose the best explanation representing their team to share with the whole class.

Teaching tip
Regarding Team Competitions: Use the **Concept Content Rubrics** or **Cooperative Group Work Rubrics** to award points to teams. Keeping track of team points sometimes helps students stay on track with their responsibilities.

- Use the **Concept Content Rubric** to assess team answers. Reinforce correct explanations and correct those that are confusing or inaccurate. Summarize the explanations and model the correct drawings on the board. Ask students to correct their papers if their explanations were incorrect or confusing. They may copy your model from the board. Consider awarding and keeping track of team points for team explanations

8. Worksheet to Reinforce Square One Concepts

- You can use this worksheet before or after you give the **Quick Team Quiz One**. It makes sense to have students complete it before they take the **Square One Test**. Send it for homework or allow students to work on it while you work with those students who need more instruction on Square One.

9. Quick Team Quiz One


Individual

- Ask the Managers to come to you for copies of **Quick Team Quiz One**. Students should work as individuals, not teams. While students are working, walk around the room clarifying and instructing.


Small group

- When all the individuals in each team have finished **Quick Quiz One**, the team leader should help his/her team to correct the papers by comparing answers. Team members should kindly and supportively help other team members who made errors.
- Present the correct answers on the board and answer any questions.
- Tell Team Leaders to collect all the **Quick Quiz One** papers neatly in the team folder. Give each Manager a paperclip for each team member. The Leader should ask the Team members to clip their **Quick Quiz One** paper on top of their packets. All papers should be placed in the folders and the Leader should give the folders to you.

10. Square One Test.


Individual

- Administer the **Square One Test** to *individuals*, not teams. Separate student desks for privacy. Collect tests when students have finished.

- While students are taking their tests, use the time to assess the students' cooperative group work. Use the **Cooperative Group Work Rubric** as your guide. Assess each student and assign a number from 1–4 to describe their cooperative behavior. Before beginning the next class, let students know how well they are meeting your expectations and, if necessary, what specifically they can do to improve.

11. Awarding Squares

- Before the next class period, check the students' folder work as an informal assessment. If the pages reveal that someone is confused, try to create an opportunity before the next class to re-teach the concept. If you administered a Square Test, correct the individual tests and evaluate your students' mastery of concepts in Instruction Block One before starting Instruction Block Two. Make a list of those students who have earned a square. Re-teach and retest if necessary. Allow other students to complete one of the **Optional Extra Activities** on page 27. Keep the Square Tests in a separate teacher folder until the end of the unit, thereby allowing you to use the same Square Test as a retest for those who did not pass.
- Paste the earned Squares onto the team folders. If someone has nearly passed the **Square One Test**, then award a piece of the square.

Teaching tip
You may decide to move a student who has earned a piece of a square along rather than stopping to re-teach. Every new square reinforces previous content and skills. Students will have the opportunity to show they have mastered a skill from Square One when they take the **Square Two Test**. Award the squares as soon as you feel that the student has demonstrated his mastery.

Square One Answer Key

Stop/Think/Draw/Write 1

To earn a +3 or +4, students need to show a shape like the rectangle described on handout **Equivalent Fractions—Part 1** with an explanation.

Practice: 864 and 864, 540 and 570. 3240 and 2430, 312 and 312.

Stop/Think/Draw/Write 2 and 3

Answers will vary. Students must show steps and give examples for a +3 or +4 score.

Top: Multiply by $\frac{n}{n}$. Bottom: Divide by $\frac{n}{n}$

Equivalent Fractions Part 3

Stop/Think/Draw/Write 4
$n = 28$. Having the correct answer earns +2; students need numbered steps to earn a +3 or +4.

Worksheet to Reinforce Square One Concepts

1. Students should color in $\frac{2}{3}$ of the block and all the blocks under $\frac{2}{3}$.
2. a. 30, 30 b. 72, 72 c. 180, 240 ≠ d. 252, 168 ≠
3. 3, 6, 9, 12, 15 and 5, 10, 15, 20, 25.
4. a. 16 b. 30 c. $\frac{10}{10}$ d. $\frac{4}{4}$
5. a. 5 b. 6 c. $\frac{5}{5}$ d. $\frac{25}{25}$
6. a. 27 b. 10 c. 21 d. 15

Quick Team Quiz One

1. Color 3 of 5 pieces in the first rectangle; divide the second rectangle into 10 pieces and color 6; divide the third rectangle into 15 pieces and color 9.
2. Complete table 10, 20, 30, 40, 50 over 25, 50, 75, 100, 125.
3. Answers will vary, but one is always $\frac{n}{n}$
4. Answers will vary, but one is always $\frac{n}{n}$
5. 180=180, 400 ≠ 600, 98 ≠ 72
6. $n = 9$ and $n = 22$

Square One Test

1. A and C
2. B and C
3. C
4. D
5. C
6. $n = 12$
7. $n = 45$

Optional Activities One

1. **Worksheet to Reinforce Square One concepts**

   - Assign this as a homework assignment, a team assignment, or as a whole class assignment. Correct in teams or as a whole class. Consider giving recognition or awarding points to teams where all students completed the assignment on time.

   - Run a competition at the beginning of the next class and use some of the same problems that were in the homework. Write one problem on newsprint or on the board ahead of time. Start all the students at the same time and tell them to raise their hands when they finish. Recognize each team when all members of their team have raised their hands. Check the first team that finished and award 3 points if all the team's answers are correct. If any member of the team has made an error, the team gets no points. Move to the second team and repeat. Award points only to the top three teams: first team 3 points, second team 2 points, and third team 1 point.


Whole class
or

Small group
or

Individual


Small group

2. **Journal writing** is always an excellent way for students to reinforce their own learning. Below are 2 prompts you can give individual students, teams, or the whole class. Allow student to write for at least three minutes. Direct them to first share what they wrote with their team. Ask for volunteers to share with the whole class. Look for common comments and strategies that students write.

   - **Prompt 1:** Jose, Jen, Mario, and Bob shared two pizzas. The first pizza was cut into 8 pieces. The second pizza was cut into 12 pieces. If the four friends shared the two pizzas equally, describe how many pieces each one ate. With drawings or math, explain how the shares were equivalent.

   - **Prompt 2:** What part of this equivalent fractions lesson was the easiest for you? What part of the equivalent fractions lesson was the most difficult for you? Explain your opinion.


Whole class
or

Small group
or

Individual

3. **Real life situations**

   - Ask students to go into their kitchen and find at least five different equivalent ways they can measure 1 cup of sugar using the different measuring tools (cups, teaspoons, tablespoons, etc.)
     They should write the equivalent measures. (*Example: 1 c = four* $\frac{1}{4}$ *cup measures.*)


Individual

Instruction Block Two

Ratios and Proportions

Square Two Concepts—*Students will*:

- Understand that ratios are descriptions comparing two items or measures.
- Understand that a ratio can be written three ways: $\frac{x}{y}$ ___ to ___, or ___ : ___.
- Understand that the order of a ratio is important and ratios must be written as a sentence or with labels so that the description is clear. *Example: The ratio of red to blue is 4 to 3.*
- Understand that a proportion is an equation stating that two ratios are equal.
- Write ratios from a given statement.
- Write proportions from a given statement.
- Simplify ratios.
- Solve problems involving ratio and proportion by applying skills learned with equivalent fractions.
- Understand how to interpret the whole group from a ratio. *Example: 5 : 1 means there are 6 in the whole group.*

Materials

- Calculators—*one per student*

Duplicate

- Ratios and Proportion—*one pack per student*
- Writing Ratios—*one per student*
- Worksheet to Reinforce Square Two concepts—*one per student*
- (Optional) How to Create Word Problems—*one per student or team*
- Practice and More Practice—*one per student*
- Quick Team Quiz Two—*one per student*
- Square Test Two—*one per student*

Lesson plan schedule

- Award squares for Instruction Block One
- Ratios and Proportions
  - Writing ratios
  - Simplifying ratios
  - What's a proportion?
  - Solving problems with equivalent ratios and proportion
  - Stop/Think/Draw/Write 5
  - Finding the ratio
  - Finding the "whole"
  - Stop/Think/Draw/Write 6
  - Keeping track of the labels in the question
- (Optional) Worksheet to Reinforce Square Two Concepts
- (Optional) Practice
- (Optional) More Practice
- Quick Team Quiz Two
- Square Two Test

Lesson Plan

Ratios and Proportions

Small group

Teaching tip

Explain to students who have not yet mastered the first concepts that they will shortly and you will award them their squares as soon as they do. Consider awarding parts of squares to recognize content these students have mastered.

Teaching tip

Student may be reluctant to write labels saying *they* know what they are saying/ doing, but when they work with calculators, they tend to "forget" where they were and what they have entered in the calculator. Insist that they write at least some label (*ex. c-cow, ck= chicken*)

Awarding squares

Arrange the room and send students into teams. Students will rotate their roles today. If you have not already done so, announce/award the First Squares (blue) to students who have mastered the first concepts in Instruction Block One.

1. Writing ratios

- Give enough packs of **Ratios and Proportions** to each team Manager for every member of his/her team. Because this square is the basis for the rest of the unit, you should probably review the handouts as a whole class, rather than team activity.
- Introduce #1 **Writing Ratios** and introduce the three different ways to write a ratio. Practice how to read a ratio. If needed, have students create ratio sentences so they become familiar with the format:

  "The ratio of x to y is __ to __."

  Have students note that ratios always have <u>two different</u> labels. Allow students time to draw the clock and write the ratio of 60 to 1 (minutes to hours). Encourage them to offer the correct labels for the ratio of 1:60.
- **Writing Ratios** handout (page two of **Rates and Proportions**). Have students count the figures in each box and write the appropriate ratio. Reinforce the idea that these ratios can be written three different ways. Note that when a ratio is written as a fraction, that they can put either part in the numerator or denominator as long as it is labeled correctly.

2. Simplifying ratios

- Remind students that they already know how to simplify ratios because they already know about simplifying fractions. For this unit, the fractions have easily recognizable GCFs. If want your students to review simplifying more rigorously, choose problems from other texts. Go back to the **Practice** sheet for writing ratios and have students simplify the ratios they wrote after counting.

3. What is a proportion?

- Introduce proportions. Go over the definition comparing it to the definition of a ratio. Stress that working with proportions is not really new material, but something they already know how to do from their

work with equivalent fractions. Unlike fraction worksheets, proportions have actual application in word problems.

4. Solving problems with ratios and proportions


Whole class

- For parts 4a, b, and c, quickly review the skills for equivalent fractions as they apply to a proportion. The big difference is the need for *labels*. Show students that for their computations, they often need only initial letters. Keep an eye on students who have not yet passed the **Square One Test**. They may need more instruction.


Individual

- **Stop/Think/Draw/Write 5**
  Have students work individually to complete the prompt. This prompt will show them what can happen if they don't keep track of labels. Walk around and check that students who have finished "early" have included all they know. Tell students to check their spelling and work neatly. Be certain that they have written a label on their final answer.


Small group

- After 2–3 minutes, have students first share what they wrote or drew with their team. Then ask teams to choose one teammate's explanations and drawings to share with the whole class.

5. Finding the ratio

Teaching tip
This is a ***very*** important concept. If you feel your students need more concrete practice, use math manipulatives or pieces of black and white paper on their desks to show finding the ratio and determining the "whole."

- Sometimes students just grab numbers and force them into a proportion. It's important, however, for students to realize that a ratio of 5 to 2 implies a total of 7 items, and that when they see a fraction of $\frac{2}{3}$, this implies a ratio of 2 to 1. If they confuse the ratio with the fraction, they will set up incorrect proportions. Drawings help students to see the difference.

6. Finding the "whole"

Teaching tip
Students may not have thought about what "out of" means. Take a moment to discuss concrete examples within the classroom. For example: 5 out of 6 windows are open, 3 out of 25 students are absent, 15 out of 28 students are males, etc. Also point out that the second number in an "out of" statement represents the "whole."

- This skill is very important to setting up proper proportions. Adding the numbers of a proportion will reveal the "whole." If the statement is already given as a fraction, the denominator is the "whole."


Small group

- **Stop/Think/Draw/Write 6**
  Have students work individually to complete the prompt. Walk around and check that students who have finished "early" have included all they know. Tell students to check their spelling and work neatly.

- After 2–3 minutes, have students first share what they wrote or drew with their team. Then ask teams to choose one teammate's explanations and drawings to share with the whole class. Correct responses should note that the sum of the two numbers in the ratio equal the total number. When writing the fraction, the total number is always in the denominator.

7. Keeping track of labels in the question


Individual

- Students may find it difficult to know when they need a "whole" when they are stetting up their first ratio. The questions will give them the labels they need. Have students practice underlining labels in a word problem. Go through problems 7a and b carefully. If students need additional practice, see **Worksheet to Reinforce Square Two Concepts** on page 83 for practice, or **Practice** and **More Practice** on pages 84–85. Assign these for homework or allow students to work on them while you work with those students who did not earn a Square One blue square. Insist that students reread the questions and look at their answers to ensure that they are reasonable.

8. Quick Team Quiz Two


Individual

- Ask the Managers to come to you for the **Quick Team Quiz Two**. While students are working individually, walk around the room clarifying and instructing.


Small group

- When the individuals in each team have finished **Quick Team Quiz Two**, the teams should work together to correct their papers. Team members should help other team members who made errors. Remind students of the **Cooperative Group Work Rubric**.


Whole class

- As a whole class, present the correct answers on the board and answer any questions. Tell Leaders to put all the **Quick Team Quiz Two** papers neatly in the team folder and give the folders to you.

9. Square Two Test


Individual

- Before starting Instruction Block Three, administer the **Square Two Test** to individuals, not teams. Separate student desks for privacy. Collect tests when students have finished.
- While students are taking their tests, consider using the time to assess the students' overall cooperative group work during Instruction Block Two. Use the **Cooperative Group Work Rubric** as your guide, assess each student and assign a number from 1–4 to describe their cooperative behavior. Before beginning the next class, let students know how well they are meeting your expectations and, if necessary, what specifically they can do to improve.
- Correct the individual tests and evaluate your students' mastery of concepts in Instruction Block Two. Re-teach and retest if necessary. Doing the **Worksheet to Reinforce Square Two Concepts** or the **Practice** handouts a second time may help students who need more practice. Those who have passed the Squares can do the **Optional Extra Activities** on page 36.


Individual

Square Two Answer Key

Ratios and Proportions:

1. 2 to 15 left-handers to right-handers, 2 : 4 cups fruit to cups of juice, $\frac{\text{sparrows } 14}{\text{robins } 3}$

   The ratio of minutes to hours is 60 to 1

   a. 5 to 13 b. 8 : 6 c. 8 to 12 d. $\frac{\text{10 white}}{\text{15 black}}$

2. a. $\frac{1}{2}$ $\frac{3}{4}$ $\frac{2}{3}$ b. Answers will vary
   c. Answers will vary and 4 to 5

4. a. $16 b. 42 blue marbles c. 24 cups of tea
   d. 12 Canadian stamps.

Stop/Think/Draw/Write 5
Sandra set up her ratio wrong, putting the cinnamon on the sugar line. The correct answer is 9 tsp of cinnamon. Sandra would not have made this mistake if she had labeled her lines.

5. The ratio of black to white marbles is 2 to 5. $\frac{\text{black marbles } 2}{\text{marbles } 7}$

6. a. 5 to 1. 40 players rode the bus. b. 3 to 2, whole = 5, 105 diners

Stop/Think/Draw/Write 6
8 to 2, whole = 10 (They say it was given in the statement, but remind them that they should add the numbers in the ratio.) 560 bought tickets online.

7. a. 12 French stamps b. 14 Spanish stamps

Worksheet to Reinforce Square Two Concepts:

1. a. 7 to 4 b. 4 to 7

2. a. 4 : 5 b. 2 to 5 c. $\frac{2}{9}$

3. a. 125 to 25 b. 5 to 1

4. a. 2,400 to 1,800 b. 4 to 3

5. Yellow 8, 16, 24, 32; blue 3, 6, 9, 12
   Answer: 12 blue

6. 54 cats

7. 56 spoons
8. 2 cows
9. 24 ducks
10. 5 to 2, 60 cats
11. 12 red ants

Practice:

1. a. high schoolers and students b. 9 HS to 5 MS, 14
   c. $\frac{9 \text{ high schoolers}}{14 \text{ students}} = \frac{n}{560}$. 360 HS students were at the game.
2. a. $\frac{T7}{J3}$
   b. $\frac{T7}{J3} = \frac{1{,}015}{n}$ $7 \times n = 3 \times 1{,}015$ $n = 435$ voted for Jon
3. 3366 voters

More Practice:

3. a. 14 to 2 b. $\frac{\text{apples } 14}{\text{pears } 2} = \frac{588}{n}$ c. 84 pear trees
4. a. $\frac{\text{cherries } 6}{\text{peach } 4}$ simplify to $\frac{3}{2}$ b. $\frac{3}{5}$ or $\frac{6}{10}$ trees are cherry
   c. $\frac{3}{5} = \frac{936}{n}$ d. 1,560 trees
5. 1,080 cucumber plants
6. 240 drivers forgot

Challenge: Answers will vary.

Quick Team Quiz Two:

1. a. $147 \neq 105$ b. $504 = 504$ c. $900 = 900$
2. The ratio of photos to paintings is 23 to 5, 23 : 5, $\frac{23 \text{ photos}}{5 \text{ paintings}}$
3. a. $\frac{1}{3}$ b. $\frac{2}{3}$ c. $\frac{3}{50}$
4. Peanuts 3, 6, 9, 12, 15, 18; popcorn 11, 22, 33, 44,55, 66
   Answer: 66 popcorn
5. 88 guppies
6. 168 foreign cars
7. a. 2 to 3 b. 14 black boxes c. 16 black boxes

Square Test Two:

1. 600 = 600
2. 3 to18,
3. The ratio of squares to triangles is 7 : 9 $\frac{7 \text{ squares}}{9 \text{ triangles}}$.
4. The ratio of hours to days is 24 to 1
5. 3 to1
6. Books 4, 8, 12; days 14, 28, 42
   Answer: 12
7. 72
8. a. 2 to 1 b. $\frac{2}{3}$ c. 21 white d. 64 red

Optional Activities Two


Individual

1. **Worksheet to Reinforce Square Two Concepts**, **Practice**, and **More Practice**. Assign these for homework, review, or for seatwork while you help students who are still struggling. Tell students that you will be running competitions on problem solving at the beginning of the next class and that you will be using the same problems that were in the homework. Recognize each team as they finish and raise their hands. If there are 6 teams, award the first team that finishes with the correct answer 6 points, the second team earns 5 points, etc.


Whole class

or


Small group

or


Individual

2. **Journal writing** is always an excellent way for students to reinforce their own learning. Below are 2 prompts you can give individual students, teams, or the whole class. Allow students to write for at least three minutes. Direct them to first share what they wrote with their team. Ask for volunteers to share with the whole class. Look for common comments and strategies that students write.

   - **Prompt 1:** If the ad says 5 out of 7 girls prefer lip gloss to lipstick, why isn't the ratio 5 to 7? Explain your answer with words and drawings.
   - **Prompt 2:** What does it mean to "simplify" ratio and why might that be a good idea? Explain your answer with examples.


Individual

3. **Real life situations**

   a. Ask students to cut pictures from magazines to use to create different ratios. (For example, an ad for nail polish might show two hands with painted nails. The ratio of nails to hands is 10 to 2, simplified to 5 to 1.)

   b. Ask students to count items in the room and write ratios. (For example, the ratio of desks to chairs if 28 to 34.) Have students simplify the ratios if possible.

   c. Ask students to go home and count items in their houses and write at least 7 new ratios. For example: windows to outside doors, carpeted rooms to rooms with wooden floors, bathrooms to bedrooms, etc.

   d. Have students go into their kitchen to make 6 new ratios. (For example, forks to spoons, saucepans to lids, canned goods to packages, boxes of pasta to boxes of cereal.

4. **Three-part ratios.** Point out that there can be 3 comparisons in a ratio. The ratio of red, blue, and yellow hats is 2 to 3 to 5. If they know how many red hats, they can determine how many blue and yellow hats there are by solving one ratio and then the next. Solve the following:

   a. Ratio of red to blue to yellow is 2 to 3 to 5. There are 4 reds, how many blues and how many yellows?

   b. Ratio of daffodils to pansies to lilies is 4 to 8 to 10. There are 6 daffodils. How many pansies and lilies?

   c. The ratio of cats to dogs to goldfish is 2 to 4 to 6. If there are 30 goldfish, how many cats and how many dogs?

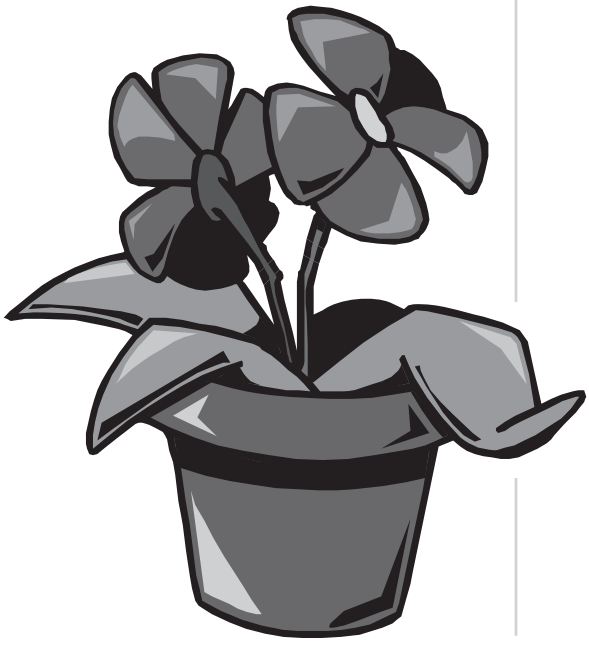
Whole class

5. **Round Robin Problem Solving.** Distribute **How to Create Word Problems** on page 65 and go over the steps. Have students work in teams or individually to write original ratio word problems. Students pass their word problems to the team to their right, which solves the problems on a separate piece of paper. After a couple of minutes, teams pass the problems again to the team on their right. Continue until all teams have solved all problems.

Small group

or

Individual



Instruction Block Three

Unit Rates

***Square Three Concepts**—Students will:*

- Understand that all rates are ratios, but not all ratios are rates.
- Understand that a rate always has 2 very different labels (ex. mph, price per pound, cost per hour, etc.)
- Understand that a rate often contain the word "per" and implies per 1, as in $10/hour means $10 per 1 hour.
- Recognize rates in word problems.
- Convert rates of speed into ratios to solve word problems.
- Solve rate problems using proportions.
- Determine unit prices.

Materials

- Calculators—*one per student*

Duplicate

- Rates and Ratios—*one pack per student*
- Worksheet to Reinforce Square Three Concepts—*one per student*
- (Optional) How to Create Word Problems—*one per student or team*
- Quick Team Quiz Three—*one per student*
- Square Three Test—*one per student*

Lesson plan schedule

- Award squares for Instruction Block Two
- Unit Rates
  - Ratios and rates
  - Solving rate problems
  - Practice
  - Finding unit rates
  - Two ways to finding a better buy
  - Practice finding the better buy
  - Stop/Think/Draw/Write 7
- (Optional) Worksheet to Reinforce Square Three Concepts
- Quick Team Quiz Three
- Square Three Test

Lesson Plan

Unit Rates

Bright Idea
Explain to students who have not yet mastered the first or second concepts that they will shortly and you will award them their squares as soon as they do. Consider awarding parts of squares to recognize content these students have mastered.

Whole class

or

Small group

Awarding squares

Arrange the room and send students into teams. Students will rotate their roles again today. If you have not already done so, announce/award the Second Squares (red) to students who have mastered the concepts in Instruction Block Two.

1. Understanding the difference between a ratio and a rate

- Decide if you are going to run this as a whole class or as a team activity. Review roles if doing as a team activity.

  a. Student should add the numbers to the labels for each of the examples given $\frac{\text{gallons } 20}{\text{wash } 1}$ $\frac{\text{oz } 7}{\text{day } 1}$ $\frac{\text{inch } 1}{\text{month } 2}$

  b. $\frac{\text{cans } 4}{\$6}$ $\frac{\text{cans } 3}{\$7}$ $\frac{\text{lemons } 3}{\$2}$ $\frac{\text{pounds } 1}{\$.50}$ (Note pounds, *not* watermelons!)

Teaching tip
Note that having the labels on the far left makes working with the proportions easier.

2. Solving rate problems

- Some students may find that they can figure out what numbers to divide or multiply without creating proportions. However, teach them how to use the proportions so that they will have a method to fall back on if they are ever confused. If they do not use proportions, however, insist that they tell you what they are doing on their calculators by writing the equations.

Teaching tip
Remind students that calculators write in the simplest decimal form. When they see 3.1 in the answer screen for a dollar amount, they must add the zero to make $3.10.

3. Practice

- Direct students to show how they solve these problems and to include the correct labels. Insist that they look back at their answers to ensure that their answers are reasonable.

- If necessary, take the time to discuss remainders. In part e, there is a remainder. Remind students that they need to interpret the meaning of remainders. 2.5 candy bars means two whole candy bars and one-half of another. When an item can be divided into pieces that are less than one, then remainders are appropriate. If the answer is 2.5 buses are needed to take the students to camp, this really means that they need 3 buses, because you cannot send one-half of a bus down the road. In part e, they can only buy 5 pairs of glasses.

4. Finding unit rates

Teaching tip
To help students remember the meaning of unit price, askthem to think of a unicycle with *one* wheel.

- Be certain students understand the term "unit price" because the assessments use that term. Have students recognize that *each* often signals a unit price. If necessary, go over how to change a number like 2.4 into dollar notation, $2.40. Also review, if necessary, how to round to the nearest penny.

5. Two ways to finding the better buy

- Go over the two methods one at a time. Be certain that students understand in the first method, they set up the first part of the proportion using the information given in the first offer. They then insert the number from the second offer into the second part of the proportion. This will give them the cost of a given number of items at the rate of the first offer. They can compare the that number to the second offer to determine the better buy.
- The second method does not use proportion, but determines unit prices by division.

6. Practice finding the better buy

- Review the longer way to solve problems using proportions. Next show them the quicker way by determining and comparing unit prices. The proportion is a better idea if students seem to forget which number to enter in the calculator first. Regardless, student should show their work by either writing the proportion or the equations to show which buttons they pushed. Take time to discuss the answers because some students do not realize that the lower price is the better buy.
- **Stop/Think/Draw/Write 7**
  Have students work individually to complete the prompt. Walk around and check that students who have finished "early" have included all they know. Tell students to check their spelling and work neatly.


Individual

- After 2–3 minutes, have students first share what they wrote or drew with their team. Then ask teams to choose one teammate's explanations and drawings to share with the whole class. This is a great opportunity to reinforce all the steps of the process for determining unit price. Remind students about rounding to the nearest penny and translating the calculator numbers to dollars notation. (3.1 = $3.10) Use the **Concept Content Rubric** to assess each team's response. You can make a ratio of student use the first way to those who use the second way.


Small group

- Have the team Leaders put all the pages into the team folder. Check the students' work each night as an informal assessment. If the pages reveal that someone is still confused on one of the concepts covered, try to create an opportunity before the next class to re-teach the concept.

7. Worksheet to Reinforce Square Three Concepts


Individual

- You can use this worksheet before or after you give the **Quick Team Quiz Three**. It makes sense to have students complete it before they take the **Square Three Test**. Send it for homework, allow students to work on it while you work with those students who did not earn a Square Two red square or who need more instruction on Square Three.

Teaching tip

You may decide to move a student along rather than stopping to re-teach. Every new square reinforces previous content and skills. Students will have the opportunity to show they have mastered a skill from Square Two when they take the **Square Three Test**. Award the squares as soon as you feel that the student has demonstrated his mastery.

8. Quick Team Quiz Three

- Ask the Managers to come to you for the **Quick Team Quiz Three**. While students are working individually, walk around the room clarifying and instructing.
- When the individuals in each team have finished **Quick Team Quiz Three**, the teams should correct their papers as a group. Team members should help other team members who have made errors.
- Present the correct answers on the board and answer any questions. Tell Leaders to put all the **Quick Team Quiz Three** papers neatly in the team folder and give them to you.

9. Square Three Test


Individual

- Before starting Instruction Block Four, administer the **Square Three Test** to individuals, not teams. Separate student desks for privacy. Collect tests when students have finished.
- While students are taking their tests, consider using the time to assess the students' overall cooperative group work during Instruction Block Three. Use the **Cooperative Group Work Rubric** as your guide, assess each student and assign a number from 1–4 to describe their cooperative behavior. Before beginning the next class, let students know how well they are meeting your expectations and, if necessary, what specifically they can do to improve.
- Correct the individual tests and evaluate your students' mastery of concepts in Instruction Block Three. Re-teach and retest if necessary. Re-doing the **Worksheet to Reinforce Square Three Concepts** may help students who need more practice.


Individual

- Allow those who have earned a square to work on the **Optional Extra Activities** on page 44.

Square Three Answer Key

Unit Rates

3. a. $65 b. 6 qts. c. 8 cans
   d. 8 pizzas e. 5 sunglasses
4. c. $1.88
5. c. 16 pens for $19.20
6. a. 8 tacos for $16 b. 16 oz for $2.56 c. same price

Stop/Think/Draw/Write 7
Answers will vary. Make sure the steps are there.

Worksheet to Reinforce Square Three Concepts:

1. The ratio of vans to cars is 3 : 5 or 3 to 5 *or* $\frac{3 \text{ vans}}{5 \text{ cars}}$
2. 45 dogs
3. 16 goats
4. 36 forks
5. a. $\frac{\text{gallons } 40}{1 \text{ day}}$ b. $\frac{300 \text{ pounds}}{2 \text{ days}}$ c. $\frac{1 \text{ spin}}{24 \text{ hours}}$ d. $\frac{1 \text{ revolution}}{30 \text{ days}}$
6. $10.40
7. $13.58
8. $120.63
9. $113.20
10. 9 pounds for $6.50

Quick Team Quiz Three:

1. a. 4 to 9 b 1 to 2.
2. $\frac{3}{8}$ $\frac{5}{7}$ $\frac{1}{3}$
3. 240 pansies
4. 60 pansies
5. a. $0.26 b. 8 pens c.$0.28, 9 for $12

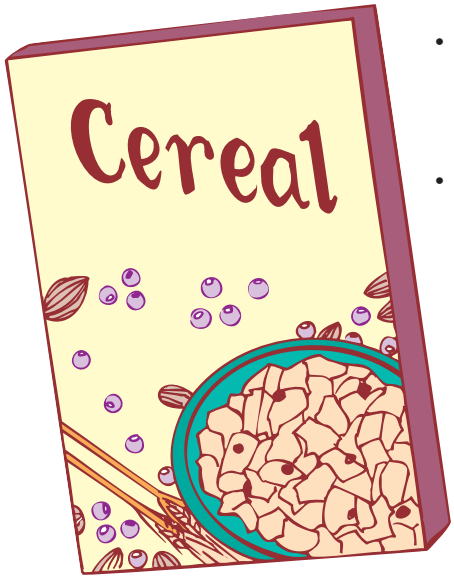
Square Test Three:

1. a. 60 to 1 b. 24 to 1
2. 4 to 1
3. $\frac{1}{3}$ and $\frac{1}{4}$
4. 10 hawks
5. 21 green frogs
6. 84 bars of soap
7. a. $2.50 b. $17.20
8. 12 hats for $53

Optional Activities Three

Individual

1. **Worksheet to Reinforce Square Three Concepts.** Assign the worksheet (page 92) for homework, review, or for seatwork while you help students who are still struggling.
2. **Graphing.** Have students make a Venn diagram comparing a ratio to a rate. Write a short paragraph explaining how/where they overlap.
3. **Research**
   - Ask students to use the Internet or encyclopedias to compare the daily rate of food consumption for 6 different animals from smallest to largest. (*Example: grasshopper, mouse, owl, dog, buffalo, stegosaurus.*)
   - Ask students to use the Internet or encyclopedia to determine the rate of revolutions for planets revolving around the Sun.
   - Ask students to use the Internet to find the hourly rate of pay for various professions. If they find a yearly rate, then can convert it to hourly by assuming a 40 hour week and 52 weeks per year.
   - Ask students to determine the salary rate per game of some professional athlete.
4. **Real life situations**
   - Look at newspaper ads for items selling as 2 for $$ or buy one, get one free. Use them to find a unit price (*ex.* $\frac{5}{\$7} = \$1.40$), or choose an item that already has a unit price and determine how many they could afford for a given amount of money. (*Example: $25 each, how many for $375?*)
   - Ask students to go to the grocery store, look at a grocery store flyer, or look at an online grocery web page like peapod.com and use the sales promotions to determine unit costs and/or best buys.
   - Give students four different kinds of empty cereal boxes with their unit prices. Have them initially determine the best buy by price alone. Next have them determine how many servings each box provides and then determine the cost per serving. Ask them to re-evaluate their selection of best buy.



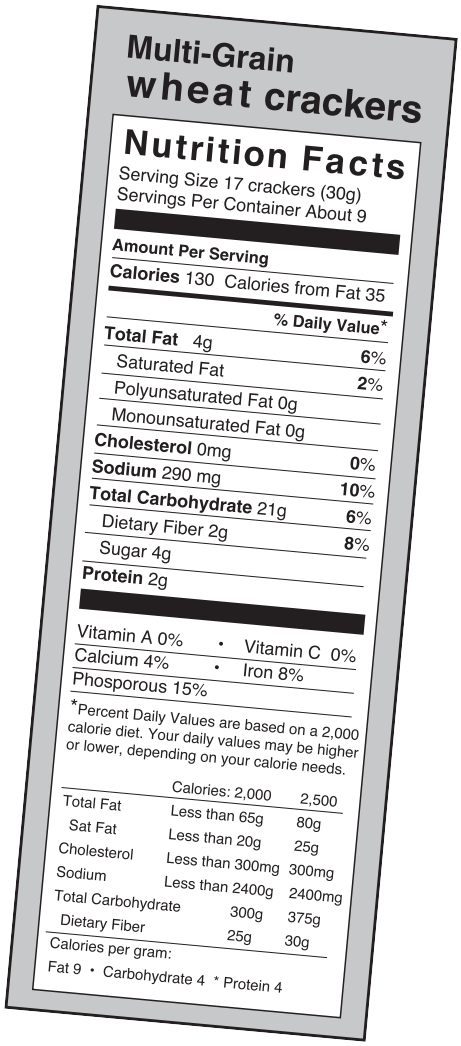
- School-age boys and girls require about 1,600 to 2,200 calories each day. Ask students to look at the nutrition facts labels on 5 different packaged foods that they eat. Have them make proportions to determine (when looking at calories alone) how many servings of only this food could they eat to reach their total calories per day. (Depending on their age and activity level, they should choose either 1800 or 2000 calories/day.)

5. **Round Robin Problem Solving.** Distribute **How to Create Word Problems** on page 65 and go over the steps. Have students work in teams or individually to write original rate problems. Students pass their word problems to the team to their right, which solves the problems on a separate piece of paper. After a couple of minutes, teams pass the problems again to the team on their right. Continue until all teams have solved all problems.


Small group

or


Individual



Multi-Grain wheat crackers

Nutrition Facts

Serving Size 17 crackers (30g)
Servings Per Container About 9

Amount Per Serving	
Calories 130 Calories from Fat 35	
	% Daily Value*
Total Fat 4g	**6%**
Saturated Fat	**2%**
Polyunsaturated Fat 0g	
Monounsaturated Fat 0g	
Cholesterol 0mg	**0%**
Sodium 290 mg	**10%**
Total Carbohydrate 21g	**6%**
Dietary Fiber 2g	**8%**
Sugar 4g	
Protein 2g	

Vitamin A 0% • Vitamin C 0%
Calcium 4% • Iron 8%
Phosporous 15%

*Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower, depending on your calorie needs.

	Calories: 2,000	2,500
Total Fat	Less than 65g	80g
Sat Fat	Less than 20g	25g
Cholesterol	Less than 300mg	300mg
Sodium	Less than 2400g	2400mg
Total Carbohydrate	300g	375g
Dietary Fiber	25g	30g

Calories per gram:
Fat 9 • Carbohydrate 4 * Protein 4

Instruction Block Four

Rates of Speed

***Square Four Concepts**—Students will*:

- Understand that rates of speed involve time and distance.
- Understand that rates of speed are generally expressed as mph or miles per one hour.
- Understand that rates can be translated into smaller time units, such per hour can be per 60 minutes, per day can be per 24 hours, etc.
- Recognize rates of speed in word problems.
- Solve word problems involving time, rate, and distance using proportions.

Materials

- Calculators—*one per student*

Duplicate

- Rates of Speed—*one pack per student*
- Worksheet to Reinforce Square Four Concepts—*one per student*
- (Optional) How to Create Word Problems—*one per student or team*
- Quick Team Quiz Four—*one per student*
- Square Four Test—*one per student*

Teaching tip
Look carefully at student mistakes when deciding if they earned a square. Sometimes their errors reflect a misunderstanding of another math skill, such as rounding. This should not disqualify a student from earning a square on rates and ratios.

Lesson plan schedule

- Award squares from Instruction Block Three
- Rates of Speed—Part 1
- Stop/Think/Draw/Write 8
- Rates of Speed—Part 2
  - Rate and Time
  - Practice
  - Stop/Think/Draw/Write 9
- Worksheet to Reinforce Square Four Concepts
- Quick Team Quiz Four
- Square Four Test

Lesson Plan

Rates of Speed

Awarding squares

Arrange the room and send students into teams. Students will rotate their roles again today. If you have not already done so, announce/award the Third Squares (green) to students who have mastered the concepts in Instruction Block Three.

1. Understanding rates of speed

Whole class

or

Small group

- Decide if you are going to run this as a whole class or as a team activity. Review roles if doing as a team activity. Students may be very familiar with solving rates of speed problems. However, it is not necessary to introduce the formula of $D = r \times t$, or $r = \frac{D}{t}$. By setting up the ratios and proportions, students automatically set up the formula.

    a. Without some practice, students may have trouble understanding how to write a rate when just given "mph." Practice a will reinforce that the rate when written in a proportion is in miles per 1 hour or miles per 60 minutes.

    b. Some students also need to visualize what a rate tells them. Practice b. causes them to think about a car or plane moving from one place to another and thinking about how far they will travel in 1, 2, or 5 hours. This will also help students to evaluate if the answers that appear on their calculator screens are reasonable.

Teaching tip
Remind students that all rates of speed are actually *average* rates of speed because cars and planes do not accelerate from 0 to 60 in an instant, nor do they stop in an instant. And generally over the time of a trip, they go a little faster or a little slower.

2. Solving rates of speed problems

- At this point student know they need three numbers to solve a proportion. In time, rate, and distance problems, they may not recognize the third number is part of the mph. (mph = miles per 1 hour or miles per 60 minutes.)
- Some students may find that they can figure out what numbers to divide or multiply without creating proportions. However, teach them how to use the proportions so that they will have a method to fall back on if they are ever confused. If they do not use proportions, however, insist that they tell you what they are doing on their calculators by writing the equations.
- It is always important to have students evaluate their answers to be certain they are reasonable. Remind them also, that their answers may not always be whole numbers. They should round to the nearest tenth.

3. Stop/Think/Draw/Write 8

Individual

- Have students work individually to complete the prompt. Insist that they

Teaching tip
Tell students to add a line that explains mph in their answer. (*Example: I realized that mph meant miles per 1 hour.*) Scorers generally score answers where the students add insights such as this to test answers higher than those where they don't. You can use the rubric to encourage more detailed answers.

Small group

label the ratio lines. They may not recognize the third element of the proportion because it is hidden in the phrase miles per hour (mph). If many are missing the element, remind them that it is miles per "1" hour. Walk around and check that students who have finished "early" have included all they know. Tell students to check their spelling and work neatly.

- After 2–3 minutes, have students first share what they wrote or drew with their team. Then ask teams to choose one teammate's explanations and drawings to share with the whole class. This is a great opportunity to reinforce all the steps of the process for using a ratio to solve a rate of speed problem. Remind students about labeling their proportions and their answers. Use the **Concept Content Rubric** to assess each team's response.

4. Working with rates and different units of time

- Sometimes a problem gives a rate, but the information is more than or less than one hour. This section of the Square talks about translating units of time into smaller or larger units in order to use the information given in the problem. Students need to practice translating 30 minutes into a decimal 0.5 hours or changing 1 hour into 60 minutes. Point out how to determine the decimal equivalency. For example 0.10 hour equals six minutes, *not* ten minutes. Ten minutes is 0.2 hour when rounded to the nearest tenth. Consider having students memorize 0.1 = 6 minutes, 0.25 = 15 min, 0.5 = 30 min. and 0.75 = 45 minutes.
- It is always important to have students evaluate their answers to be certain they are reasonable. (If they set up their proportion wrong on the fire truck problem, the rate of speed is less than 3 mph!)

5. Stop/Think/Draw/Write 9

Individual

Teaching tip
Tell students to earn a 4 they should add a sentence explaining that they translated 1 hour into 60 min. or 90 min. into 1.5 hours to solve this problem.

Small group

- Have students work individually to complete the prompt. Walk around and check that students who have finished "early" have included all they know. Tell students to check their spelling and work neatly.
- After 2–3 minutes, have students first share what they wrote or drew with their team. Then ask teams to choose one teammate's explanations and drawings to share with the whole class. This is a great opportunity to reinforce all the steps of the process for using a ratio to solve a rate of speed problem. Remind students about labeling their proportions and their answers. Use the Concept Content Rubric to assess each team's response, but give a 4 only to answers that include a full explanation of how they solved the problem.

6. Worksheet to Reinforce Square Four Concepts

Individual

- You can use this worksheet before or after you give the **Quick Team Quiz Four**. It makes sense to have students complete it before they take the

Square Four test. Send it for homework, and allow students to work on it while you work with those students who did not earn a Square Three green square or who need more instruction on Square Four.

7. Quick Team Quiz Four


Individual

- Ask the Managers to come to you for the **Quick Team Quiz Four**. While students are working individually, walk around the room clarifying and instructing.


Small group

- When the individuals in each team have finished **Quick Team Quiz Four**, the teams should correct their papers as a group. Team members should help other team members who have made errors.


Whole class

- Present the correct answers on the board and answer any questions. Tell Leaders to put all the **Quick Team Quiz Four** papers neatly in the team folder and give them to you.

8. Square Four Test


Individual

- This is the last square test that your students will take. It is a cumulative test covering all the four square concepts so it may take a little bit longer to complete. Administer the **Square Four Test** to individuals, not teams. Separate student desks for privacy. Collect tests when students have finished.
- While students are taking their tests, consider using the time to make a final assessment of the students' overall cooperative group work during Instruction Block Four. Use the **Cooperative Group Work Rubric** as your guide. Consider giving special recognition to teams or individuals who had consistently high scores on their **Cooperative Group Work Rubrics**.


Teaching tip
The Square Four test is a cumulative test. If a student completes the **Square Four Test** with few if any errors, you may award all of any missing squares.


Individual

- Correct the individual tests and evaluate your students' mastery of concepts in Instruction Block Three. Re-teach and retest if necessary. Re-doing the **Worksheet to Reinforce Square Four Concepts** may help students who need more practice.
- Allow students who have earned the square to work on the **Optional Extra Activities** on page 52.

9. Are you done yet?

- You may stop at this point having completed the Four Squares of Squared Away for *Ratios, Rates, and Proportions*. However, if you want to challenge your students further, you may assign them the Golden Square activities. These are more difficult and you should expect only 50% or fewer of your students will be able to successfully complete the activities. See the next Lesson Block for **Golden Square Challenges**.


Individual

10. Posttest

- One week after you finish the unit, give the **Posttest**. Take the time to look at the student answers that were incorrect. A student may have an incorrect answer because of a simple arithmetic error, but still have a good understanding of what to do.

11. Awards


Whole class

- With the successful completion of the **Square Four Test**, students are considered "squared away." However, consider waiting until you run the **Golden Square Activities** to celebrate. This will give you more time to work with those students who are close to finishing the four squares. It would be best if all your students successfully completed the four instructional blocks. The **Golden Square Challenges** are just that—challenges, and not essential to be considered "squared away." However, give special recognition to all those who do achieve a Golden Square. Design a celebration appropriate to your students' age, and your available time and resources. Give **Special Award Certificates** on pages 109–110 to acknowledge each student's achievement.

Square Four Answer Key

Rates of speed

1. a. 45 miles per hour, $\frac{\text{miles } 45}{\text{hour } 1}$ $\frac{45 \text{ miles}}{60 \text{ minutes}}$

   100 miles per hour $\frac{\text{miles } 100}{\text{hour } 1}$, $\frac{\text{miles } 100}{\text{minutes } 60}$

   115 mph $\frac{\text{miles } 115}{1 \text{ hour}}$ $\frac{\text{miles } 115}{\text{minutes } 60}$

   250 mph, 250 miles per hour, $\frac{\text{miles } 250}{\text{minutes } 60}$

   b. 30 miles, 60 miles, 150miles
   50 miles, 100 miles, 250 miles
   60miles, 120 miles, 300 miles

Stop/Think/Draw/Write 8
494 mph. Make sure the labels and steps are there.

5. 48 mph. Make sure the labels and steps are there.

6. 36 hours, 45 minutes, 480 hours, 6 minutes, 6 months

Stop/Think/Draw/Write 9
3.3 mph. Make sure the labels and steps are there.

Worksheet to Reinforce Square Four Concepts:

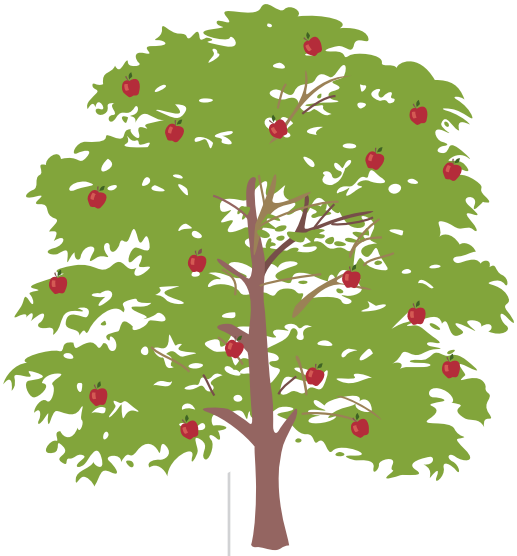
1. a. 20 lollipops b. 16 sourballs c. 60 lollipops
   d. 9 bags
2. a. $\frac{15 \text{ pounds}}{1 \text{ day}}$ b. $\frac{33 \text{ pounds}}{3 \text{ days}}$ c. $\frac{1 \text{ revolution}}{88 \text{ days}}$
   d. $\frac{1 \text{ revolution}}{165 \text{ days}}$
3. $89.88
4. $12.98
5. 5 for $217.50
6. 3 hours 30 min or 3.5 hours
7. a. 180 mph b. 45 minutes
8. a. 106mph b. 6.5 hours or 6 hours, 30 minutes

Quick Team Quiz Four:

1. 126 lions
2. 36 cod
3. a. $79.99 b. 8 chain saws c. $2.20
   d. buy one for $11 and get one free ($5.50 v. $5.67)
4. 6 hours 10 minutes
5. 32 mph
6. 22.2 mph

Square Four Test:

1. a. 9 to 6 b. $\frac{9 \text{ baseball}}{6 \text{ soccer}}$
2. 27 oranges
3. 18 apple trees
4. $19.12
5. a. $0.88 b. $3.95
6. a. $0.22 b. $0.20 36 pills for $7.20.
7. 520 mph
8. 19 hours 30 minutes



Optional Activities Four


Individual

1. **Worksheet to Reinforce Square Four Concepts.** Assign this for homework, review, or for seatwork while you help students who are still struggling.

2. **Graphing.** Have students create a graph comparing the speeds of the following:

jet	600 mph
bullet	3,000 mph
space shuttle	17,000 mph
Earth in its orbit around the sun	66,000

   Their graph should have a range of 0–70,000 mph.

3. Have students research the distance from Earth to the Moon and using a proportion and a rocket speed of 20,000 mph, calculate how long it would take a rocket ship to reach the Moon.


Whole class

or


Small group

or


Individual

4. **Journal writing** is always an excellent way for students to reinforce their own learning. Below is a list of prompts you can give individual students, teams, or the whole class. Allow students to write for at least three minutes. Direct them to first share what they wrote with their team. Ask for volunteers to share with the whole class. Look for common comments and strategies that students write.

   - **Prompt 1:** A recent survey found out that 3 out of 4 people prefer to read their news online instead of in a newspaper. Explain why this expression is not a proportion of 3 to 4.
   - **Prompt 2:** Explain why rates of speed are actually *average* rates of speed.
   - **Prompt 3:** Explain how you would write "45 mph" in ratio. Write a short word problem using "45 mph" and write the proportion you would use to solve it and then solve the problem.

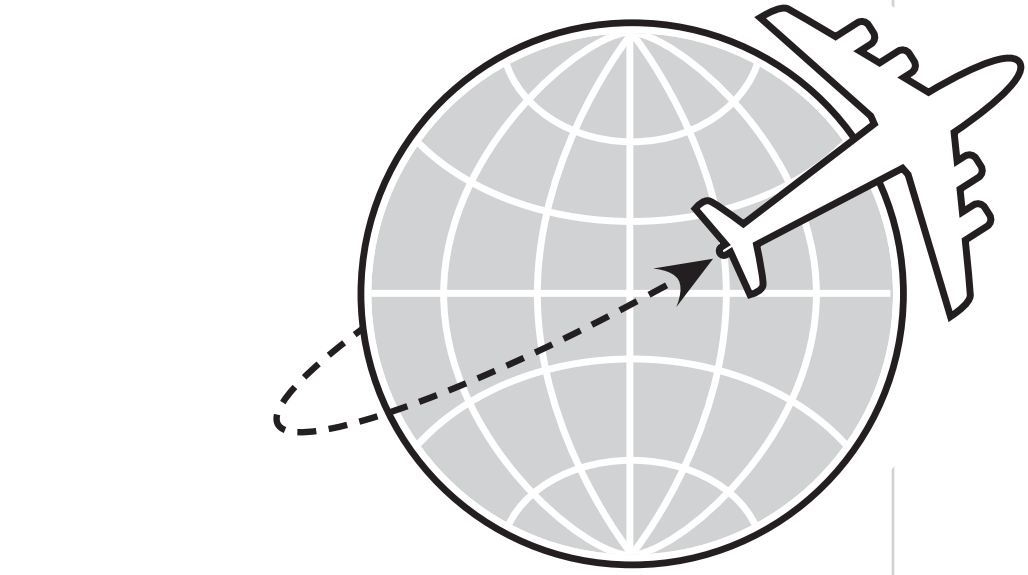
Real life situations

5. Have students research the distance between their nearest major city and five places in the U.S.A. they would like to visit by car. Have them plan five trips and determine how long it would take them to get to the five locations if driving at 60 mph. (They can check their answers with mapquest.com, but insist on seeing their proportions and calculations.)

6. Have students research the distance between their nearest major airport and five places in the world they would like to visit by jet. Have them plan five trips and determine how long it would take them to get to the five locations if flying at 500 mph. (They can check their answers using travel sites, but insist on seeing their proportions and calculations.)

7. Miles per gallon is also a rate. Have students determine the costs of a 200 mile trip using four different cars: a Hybrid (41 mpg), a commuter sedan (31 mpg), a SUV (18 mpg) and a Hummer (11 mpg). Assume all the cars use regular gas and use the current cost of gasoline.

8. **Round Robin Problem Solving.** Distribute **How to Create Word Problems** on page 65 and go over the steps. Have students work in teams or individually to write original problems involving time, rate, and distance. Students pass their word problems to the team to their right, which solves the problems on a separate piece of paper. After a couple of minutes, teams pass the problems again to the team on their right. Continue until all teams have solved all problems.



Instruction Block Five

Golden Square: Scale Drawings and Similar Triangles

Golden Square Tasks—*To earn a Golden Square, students will complete at least two of the five Challenges using ratio and proportion*

- Make a scale map of the route they take from school to home.
- Make a scale drawing of a car, truck, or SUV.
- Make a scale drawing enlarging an actual object by 400%.
- Find the height of a tree, flag pole, etc., using ratios and similarity.
- Solve two difficult word problems using ratio and proportion.

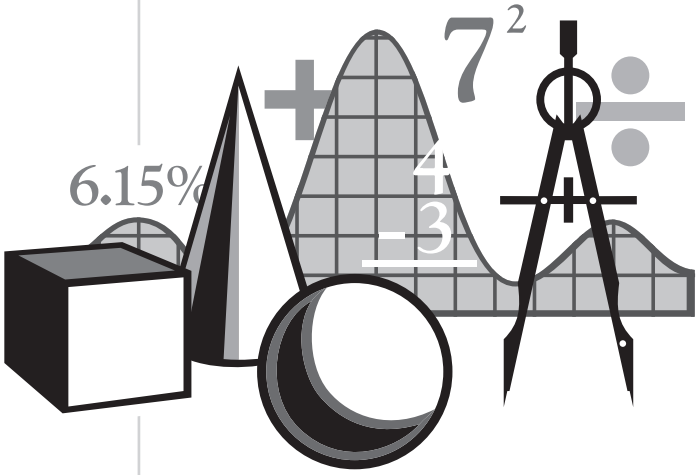
Materials

Depending on which challenges they choose to complete...

- Calculators, measuring tape, ruler, graph paper ($\frac{1}{4}$ inch or centimeter)
- Small items such as paperclips, scissors, and composition paper

Duplicate

- Golden Challenges—*one pack per student*



Lesson Plan

Golden Square: Scale Drawings and Similar Triangles

1. Making decisions regarding the Golden Challenges

Small group

or

Individual

a. Decide whether you want the students to work individually or in teams.

b. Decide if you will do the challenges during class time or as homework.

c. If you want the students to work individually (without parent or team help), give *only* Challenge #5 Amazing Problem Solving during class time and under your supervision.

d. Decide how many challenges students need to complete to earn the Golden Square.

Teaching tip
The author recommends that they complete at least two challenges at level 3 as measured on the **Concept Content Rubric**.

2. Introduce the Challenges to the class

a. Explain that you do not expect that all kids can earn the Golden Square. These challenges are beyond being "squared away."

b. Explain each challenge thoroughly and all the elements that must be present in the student answers in order to earn at least a 3 on the **Concept Content Rubric**.

c. Give a deadline for all projects to be submitted. Students need at least one sunny day to complete Challenge #4. If you run into a spot of dreary weather, extend the deadline. Suggest students work close to noon for shorter shadows.

3. Answers to Challenge #5

A. The Creature Problem—720 creatures with bumpy skin.
B. The Piggybank Problem—$44.28

4. Plan time for sharing

- If students do accept the Golden Challenge, afford them class time to explain what challenges they chose and how they completed them. Be certain to have them read their paragraphs of explanation.

5. Awards

- Prepare **Awards Certificates**. There is one specifically for Squared Away on page 109 and another for Squared Away with the Golden Square on page 110.

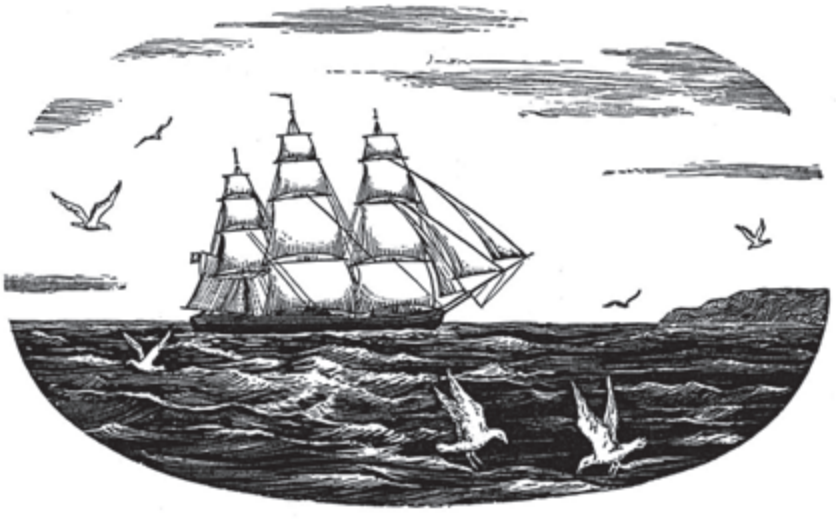
Introduction

"Squared away" was originally a nautical term used to announce that the sails of a square-rigger sailing ship were correctly set. The navy came to use it to describe sailors who completed a task with competency, as in, "He was right squared away!" In this unit, you will learn how to solve word problems using ratios, rates, and proportions. These problems will include creating basic ratios, determining unit rates, and solving problems that include time, rate, and distance. When you can demonstrate competency working with these concepts, you will be considered Squared Away.

This unit is divided into five instructional blocks. At the end of each block you will be tested on specific skills and content. When you have demonstrated you have mastered the material, then you will be awarded a colored square. When you have earned four squares, you will be declared, Squared Away. The fifth square is called the Golden Square. In order to earn a Golden Square, you must apply ratios to the concept of similarity and achieve an exemplary score on challenging tasks that requires higher thinking skills.

In this Squared Away unit you will become outstanding problem solvers. You will be working in teams of 3 or 4 in activities that apply what you already know about equivalent fractions to ratios, rates and proportions. You will discover that when using this strategy, you will solve even apparently difficult math problems more quickly and accurately.

There are practice materials and optional extra activities associated with each day's lesson that reinforce what you have learned. The more consistently you complete these, the deeper your understanding will be. Don't miss the opportunity each day to share what you are learning with your parents.

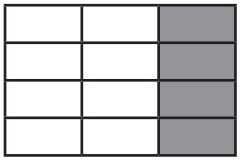


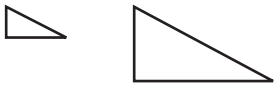
Name: _____ Date: _____

Pretest / Posttest

Use calculators!

Choose the correct answer and write the letter(s) in the space to the left. In the space to the right of each problem, *show* your work or how you used your calculator. If you need more room, use scrap paper, but number your problems. If you don't know, don't guess. Just write a question mark—?

1.	What fraction of this shape is shaded? There may be more than one answer. 	A. $\frac{4}{12}$	B. $\frac{3}{12}$
		C. $\frac{4}{8}$	D. $\frac{1}{3}$
2.	Which of the following equals *one*? There may be more than one answer.	A. $\frac{5}{1}$	B. $\frac{1}{5}$
		C. $\frac{1}{1}$	D. $\frac{5}{5}$
3.	What number must we divide the *denominator* by in order to make an equivalent fraction? $\frac{18}{30} \div \frac{3}{\mathbf{?}}$	A. 6	B. 3
		C. 5	D. 18
4.	What are the cross-products of $\frac{2}{3} = \frac{8}{12}$	A. 16 and 24	B. 6 and 6
		C. 6 and 96	D. 24 and 24
5.	What is the missing numerator? $\frac{8}{20} = \frac{n}{15}$	A. 6	B. 4
		C. 10	D. 13
6.	(T) or (F) True or False	These ratio fractions mean the same thing: $\frac{15 \text{ miles}}{5 \text{ hours}}$ and $\frac{5 \text{ hours}}{15 \text{ miles}}$	
7.	There were 6 bicycle riders for every 18 walkers. What is the ratio of bicycle riders to walkers?	A. 18 to 6	B. 6 to 18
		C. 12	D. 7 : 15
8.	Reduce this ratio: 8 to 24	A. 1 to 3	B. 8 to 3
		C. 4 to 16	D. 1 to 4
9.	There were 15 Sox fans for every 10 Yankee fans. What is the ratio of Sox fans to Yankee fans?	A. 10 : 15	B. 3 : 2
		C. 25	D. 5
10.	If a store sells 6 t-shirts for $25.68, what is the unit price?	A. $3.68	B. $4.28
		C. $19.68	D. $158.08
11.	If a store sells 3 bunches of grapes for $4.68, how much will 2 bunches of grapes cost?	A. $9.36	B. $1.56
		C. $2.34	D. $3.12

12.	What's the best buy?	A. 7 for $8.47 B. 3 for $3.48 C. Buy one for $2.40, get one free D. 2 for $2.54	
13.	If there were 18 daisies for every 8 roses in the garden, how many daisies would you have if you had 28 roses?	A. 56 C. 63	B. 38 D. 54
14.	When the package arrived , the ratio of broken to unbroken bulbs was 2 to 7. There were 10 broken bulbs. How many bulbs were in the whole package?	A. 45 C. 35	B. 24 D. 50
15.	5 out of 9 voters voted for Harry. 72 people voted for Denny. How many people voted for Harry?	A. 45 C. 90	B. 81 D. 144
16.	Write 58 mph as a rate fraction (with labels) in the space to the right. →		
17.	Write a rate fraction (with labels) for 7 mph, but use minutes instead of hours. →		
18.	The bus drove 168 miles at an average speed of 48 mph from Mountain Town to Coastal City. If it didn't stop along the way, how long did the trip take?	A. 3 hours C. 4 hours	B. 3.5 hours D. 4.25 hours
19.	Ted completed his 6-mile paper route in 90 minutes. What was his average speed?	A. 3 mph C. 4 mph	B. 3.5 mph D. 5 mph
20.	NE Airlines flies at a speed of 240 mph from Hartford to Washington. The trip takes 90 minutes. How far is Hartford from Washington?	A. 225 miles C. 325 miles	B. 400 miles D. 360 miles
Challenge	**Similar triangles are proportional**. If the dimensions of a right triangle are 9 by 12 by 15, what would be the dimension of a **similar** triangle whose shortest side was 24?	Challenge:  24 by ___ by ___	

Go back to check if your answers are reasonable!

Cooperative Group Work Rubric

4 **Exemplary**—You *consistently* and *actively* help your classmates to achieve their goals whether you are working in a team or with a partner. You communicate well with others, encouraging them and helping them to understand the lesson. You *willingly* share materials and responsibilities.

3 **Expected**—You *usually* help your classmates to achieve their goals whether you are working in a team or with a partner. You *generally* communicate well with others, encouraging them and helping them to understand the lesson. You share materials and responsibilities.

2 **Nearly There**—You *sometimes* help your classmates to achieve their goals and help them understand the lesson.

1 **Must Do Better**—You *do very little* to help your classmates to achieve their goals or understand the lesson.

If your evaluation is less than expected, try to use your cooperating skills more consistently.

Cooperative Group Work Rubric

	4 Exceeds Expectations	3 Meets Expectations	2 Nearly There	1 Must Do Better
Contributing	I consistently contribute to the group by sharing my opinions and ideas.	I usually contribute to the group by sharing my opinions and ideas.	I sometimes contribute to the group by sharing my opinions and ideas.	I rarely contribute to the group by sharing my opinions and ideas.
Listening	I actively listen to and support other people's opinions, ideas, and efforts.	I usually listen to and support other people's opinions, ideas, and efforts.	I sometimes listen to and support other people's opinions, ideas, and efforts.	I rarely listen to and support other people's opinions, ideas, and efforts.
Teamwork	I actively encourage all members to participate and work together.	I often encourage all members to participate and work together.	I occasionally encourage all members to participate and work together.	I rarely encourage all members to participate and work together.
Problem solving	I consistently help my team work through problems by actively seeking and suggesting solutions.	I often help my team work through problems by seeking and suggesting solutions.	I sometimes help my team work through problems by seeking and suggesting solutions.	I do not try to help my team to work through problems or to any suggest solutions.
Staying on-task	I consistently stay on the task and complete the work required.	I usually stay on the task and complete the work required.	I stay on the task some of the time and complete some of the work required.	I am often off-task and hardly ever complete the work required .

Concept Content Rubric for Math Problems

4 **Exemplary**—You demonstrated a *clear* understanding of the math problem(s). You *accurately* and *completely* described/drew it and wrote your answer in detail using *correct labels*. You communicated your understanding clearly with few, if any, spelling or grammatical errors.

3 **Expected**—You demonstrated a *good understanding* of the math problem(s). You *accurately* described/drew it and wrote your answer using *some detail* and *correct labels*. You communicated your understanding clearly with few, if any, spelling or grammatical errors.

2 **Almost There**—You demonstrated *some* understanding of the math problem(s). However, you did not described/draw it accurately and your answer was not complete and/or had some incorrect label. Or you did not communicate your understanding clearly because of spelling or grammatical errors. You may need to meet with your team or teacher to learn the concept more completely, or you need to redo your work correcting the errors.

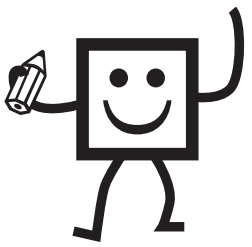
1 **Incomplete**—You demonstrated *little or no understanding* of the math problem(s), so you could not describe/draw it/solve it. You need to meet with your team or teacher to relearn the material.

If your evaluation is 2 or 1, strive to correct your work to at least level 3.

Content/Skills Checksheet **Student names**																									
Square One Concepts—*Students will:*																									
Understand that fractions are numbers that can be written as $\frac{a}{b}$ ($b \neq$ zero).																									
Understand that the top number of a fraction is called the numerator and the bottom number is the denominator.																									
Understand that the number "1" can be written as a fraction where the numerator and denominator are the same. $\frac{n}{n}$																									
Create equivalent fractions by creating a table.																									
Create equivalent fractions by multiplying by "1" written as a fraction.																									
Create equivalent fractions by dividing by "1" written as a fraction.																									
Determine if two fractions are equivalent fractions by cross-multiplying and comparing the cross-products.																									
Use cross-products to create an equivalent fraction when only three of the four terms are known.																									
Square Two Concepts—*Students will:*																									
Understand that ratios are descriptions comparing two items or measures.																									
Understand that a ratio can be written three ways: using "to," using a colon, or writing as a ratio fraction.																									
Understand that the order of a ratio is important and ratios must be written as a sentence or with labels so that the description is clear. *Example: The ratio of red to blue is 4 to 3.*																									
Understand that a proportion is an equation stating that two ratios are equal.																									
Write ratios from a given statement.																									
Write proportions from a given statement.																									
Simplify ratios.																									
Determine the "whole."																									
Determine the part and part to whole ratios.																									
Solve problems involving ratio and proportion by applying skills learned with equivalent fractions.																									

Square Three Concepts*—Students will:*																									
Understand that all rates are ratios, but not all ratios are rates.																									
Understand that a rate always has 2 different labels. ex mph, price per pound, cost per hour, etc.																									
Understand that a rate often contain the word "per" and implies per1, as in mph means miles per 1 hour.																									
Recognize rates in word problems.																									
Convert rates of speed to ratios in word problems.																									
Solve rate problems using proportions.																									
Determine unit prices.																									
Square Four Concepts*—Students will:*																									
Understand that rates of speed involve time and distance.																									
Understand that rates of speed are generally expressed as mph or miles per 1 hour or 60 minutes).																									
Understand that rates can be translated into smaller time units, such per hour can be per 60 minutes, per day can be per 24 hours, etc.																									
Recognize rates of speed in word problems.																									
Solve word problems involving time, rate, and distance by using proportions.																									
Golden Square Challenge																									
Use ratios and proportions to solve a Golden Square Challenge involving similar triangles.																									
Use ratios and proportions to solve a Golden Square Challenge involving scale.																									
Use ratios and proportion to solve complicated word problems.																									

Name: _____ Team: _____



Stop/Think/Draw/Write

(Be sure your labels are spelled correctly)

Name: _____ Team: _____

How to Create Word Problems

Numbers in the problem: You should have created a proportion *before* writing the problem so that you know your problem can be solved.

Example: $\frac{10}{12} = \frac{25}{\textcircled{30}}$ Then decide which term will be the unknown (N). $\frac{25}{N}$

Create a Context: This establishes where the word problem takes place.

Examples: *At the store Molly bought...*, *While driving his car from...*, or *In the garden there was....*

Put the numbers together with the context in the problem:

Example: *At the store Molly bought 10 pens for $12.*

Ask the Question: The question should be answered by whatever you decided will be the unknown (N).

Example: *How much will Molly spend if she wants to buy 25 pens?*

Copy your word problem neatly and make an **answer key** that you will use to correct your classmates' papers. ***N= $30***, but you can use the 4-3-2-1 Content Concept rubric to grade their papers.

Write your proportion: _____ = _____ What term will be N? _____

Context:

Put numbers and context together:

Write the question:

Make an answer key with a label:

Name: _____

Name: _____ Team: _____

Equivalent Fractions—Part 1

Square One

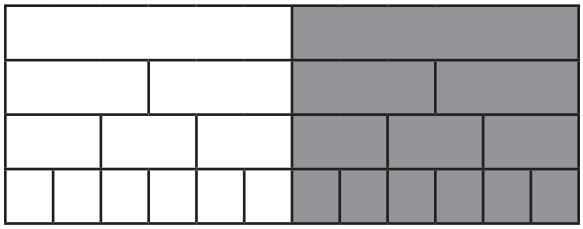
Let's review what you know about equivalent fractions. First consider what you have already learned about **all fractions**.

- $\frac{N}{D}$ You know that fractions have numerators (top number) and denominators (bottom number.)
- You know that a whole can be divided into fractional parts.

$\frac{4}{8}$ 

- You know that when you divide a whole, the denominator tells how many parts in the whole and the numerator tells how many you have identified by shading.

Now consider **equivalent fractions**:



You have discovered that sometimes different fractions describe the same value. Look at the diagram at the right. The fractions listed below the diagram describe each shaded area. The shaded areas cover an equal space so the fractions must be equivalent.

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{6}{12}$

One way to check equivalency is to cross-multiply.

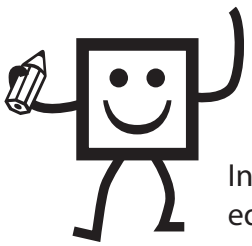
$\frac{1}{2} = \frac{2}{4}$ $1 \times 4 = \mathbf{4}$ and $2 \times 2 = \mathbf{4}$ Both cross-products = **4**

Remember this rule: **If the products of cross-multiplication are equal, then the fractions are equivalent.**

See which of the following are equivalent. Draw the circles around the numbers and show the cross-products. If the cross products are UNequal, then the fractions are *not* equivalent. Draw a line through the equal sign: ≠

$\frac{1}{2} = \frac{6}{12}$ $\frac{5}{20} = \frac{1}{4}$ $\frac{1}{4} = \frac{3}{12}$ $\frac{3}{4} = \frac{3}{6}$ $\frac{5}{30} = \frac{2}{12}$

Name: _____ Team: _____



Stop/Think/Draw/Write 1

(Be sure your labels are spelled correctly)

In the following space, tell someone what it means that two fractions are equivalent. Use drawings and/or examples to make your point.

Practice: Show the **cross-products** to prove which of the following are truly equivalent fractions:

$$\frac{12}{32} = \frac{27}{72} \qquad \frac{12}{15} = \frac{38}{45}$$

$$\frac{72}{90} = \frac{27}{45} \qquad \frac{6}{39} = \frac{8}{52}$$

Name: _____ Team: _____

Equivalent Fractions—Part 2

Square One

How do you create equivalent fractions?

1—Make a table:

Write multiples of the numerator (in this case count by ones.) Then write multiples of the denominator (in this case count by threes.) If you did this correctly, the last fraction will be equivalent to the first. Use cross-multiplying to check the fractions in your table.

Numerator	**1**	2	3	4	5	6	**7**
Denominator	**3**	6	9	12	15	18	**21**

$\frac{1}{3} = \frac{7}{21}$ (The cross-products 21 and 21, so the fractions must be equivalent.)

Numerator	**3**						
Denominator	**4**						

Write a new fraction and a table of equivalent fractions. Check your last fraction against your first fraction by cross multiplying to make sure the cross products are equal.

Numerator							
Denominator							

2—Multiply or divide by one:

You know that any number times 1 is that same number. For example, $(6 \times 1) = 6$ or $(297 \times 1) = 297$. You also know that any number divided by one is that same number. $(6 \div 1) = 6$ or $(297 \div 1) = 297$. Multiplying and dividing by one does not change the value of a number. However, it can change what it looks like. How? Think about this...

One bar is divided into five parts and all five are shaded. 5 of the 5 parts or the whole is shaded.

$\frac{5}{5}$ = one whole 

From this example you should realize that **1** can be written as $\frac{5}{5}$ or $\frac{10}{10}$ or $\frac{24}{24}$. ***All fractions where numerator and denominator are the same are equal to 1.***

Multiplying by one:

Think about it. Have you ever heard someone say, "If you multiply the numerator by 3, then you must multiply the denominator by 3"? That is true, but what you are actually doing is multiplying the fraction by $\frac{3}{3}$ or 1.

$\frac{5}{6}$ times $\frac{3}{3}$ means $\frac{5 \times 3}{6 \times 3} = \frac{15}{18}$

Cross multiply to check: $\frac{5}{6} = \frac{15}{18}$

$6 \times 15 = 90$

$5 \times 18 = 90$

Multiply by "**1**" to create equivalent fractions for the following. Show what fraction you chose to equal one, and check using cross-multiplication. (There is more than one correct answer.)

$\frac{2}{5}$ $\frac{6}{15}$ $\frac{10}{11}$

Dividing by one:

Use the same reasoning to create equivalent fractions with division. If you divide the numerator by 5, then you must divide the denominator by 5. This means you are dividing the fraction by $\frac{5}{5}$ or 1.

$\frac{35}{45}$ divided by $\frac{5}{5}$ means $\frac{35 \div 5}{45 \div 5} = \frac{7}{9}$

Cross multiply to check: $\frac{35}{48} = \frac{7}{9}$

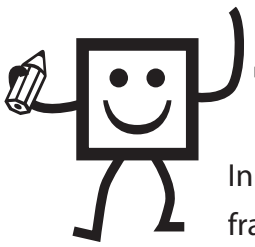
$9 \times 35 = 315$

$7 \times 45 = 315$

Divide by "**1**" to create equivalent fractions for the following. Show what fraction you chose to equal one, and check using cross-multiplication. (There is more than one correct answer.)

$\frac{20}{40}$ $\frac{12}{24}$ $\frac{200}{600}$

Name: _____ Team: _____



Stop/Think/Draw/Write 2

(Be sure your labels are spelled correctly)

In the following space, tell someone how to generate two equivalent fractions for $\frac{5}{12}$ by *multiplying* by "1." Show all your steps.

Stop/Think/Draw/Write 3

Now tell someone how to generate two equivalent fractions for $\frac{18}{36}$ by *dividing* by "1". Show all your steps.

Name: _____ Team: _____

Equivalent Fractions—Part 3

Square One

What do you do when the first strategies don't work?

This strategy works when the other two strategies don't. For example, what if you wanted to know an equivalent fraction and you already knew three of the four numbers in the two fractions. $\frac{8}{12} = \frac{10}{n}$. You can't make a table because you won't get to 10 when you count by 8s. You can't use the strategy of multiplying or dividing by $\frac{n}{n}$ or 1 because no whole number times 8 equals 10.

But you can use cross multiplication to find the fourth number to make an equivalent fraction. Follow the procedure step-by-step:

Step one: Write the problem $\frac{8}{12} = \frac{10}{n}$

Step two: Draw the circles to indicate what you will be multiplying $\frac{8}{12} = \frac{10}{n}$

Step three: Write what you will cross-multiply as an equation $(8 \times n) = (12 \times 10)$.

Step four: Solve the equation:

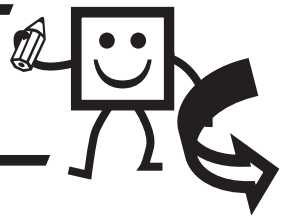
$8 \times n = 120$ $n = 120 \div 8$ $n = 15$ Therefore, $\frac{8}{12} = \frac{10}{15}$

Try a few more. Follow *all* four steps:

$\frac{2}{10} = \frac{9}{n}$	$\frac{12}{16} = \frac{n}{12}$	$\frac{12}{21} = \frac{n}{28}$

Stop/Think/Draw/Write 4

$\frac{6}{21} = \frac{8}{n}$ On the back of this paper, tell someone how to use cross multiplication to find the equivalent fraction in this problem. Write your name and team.

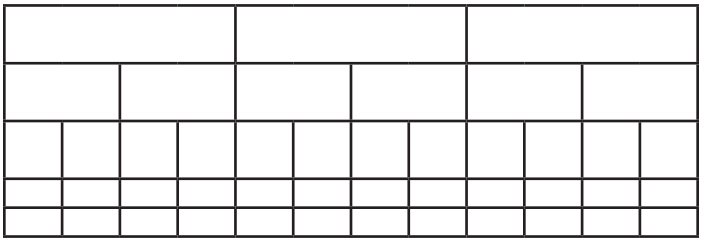


Name: _____ Team: _____

Worksheet to Reinforce Square One Concepts

1. Color this square to prove that

$$\frac{2}{3}=\frac{4}{6}=\frac{8}{12}=\frac{16}{24}$$



2. Which of following fractions are equivalent? Draw circles around the numbers and *show* the cross products. If cross products are unequal, draw a line through the equal sign. ≠

a. $\frac{5}{10}=\frac{3}{6}$ b. $\frac{6}{8}=\frac{9}{12}$ c. $\frac{6}{20}=\frac{12}{30}$ d. $\frac{12}{28}=\frac{6}{21}$

3. Make a chart to find equivalent fractions

Numerator	**3**				
Denominator	**5**				

$$\frac{n}{n}=1$$

4. Multiply by "1" to make equivalent fractions:

a. $\frac{7}{8}\times\frac{2}{2}=\frac{14}{\square}$ b. $\frac{5}{6}\times\frac{6}{6}=\frac{\square}{36}$ c. $\frac{2}{3}\times\frac{\square}{\square}=\frac{20}{30}$ d. $\frac{5}{7}\times\frac{\square}{\square}=\frac{20}{28}$

5. Divide by "1" to make equivalent fractions:

a. $\frac{10}{20}\div\frac{2}{2}=\frac{\square}{10}$ b. $\frac{6}{36}\div\frac{6}{6}=\frac{1}{\square}$ c. $\frac{20}{25}\div\frac{\square}{\square}=\frac{4}{5}$ d. $\frac{25}{100}\div\frac{\square}{\square}=\frac{1}{4}$

6. Use cross- multiplication to find N.

a. $\frac{8}{18}=\frac{12}{N}$	b. $\frac{12}{30}=\frac{N}{25}$	c. $\frac{16}{24}=\frac{14}{N}$	d. $\frac{9}{24}=\frac{N}{40}$

Name: _____ Team: _____

Quick Team Quiz One

Use calculators!

1. Divide the following 3 shapes and color to show that $\frac{3}{5} = \frac{6}{10} = \frac{9}{15}$.

2. Make a table to show four more equivalent fractions for $\frac{10}{25}$.

10				
25				

3. Multiply by **"1"** to make three more equivalent fractions. *Hint: "1" can equal $\frac{3}{3}$, $\frac{5}{5}$, or any $\frac{n}{n}$.*

$\frac{7}{8}$ $\frac{7}{8}$ $\frac{7}{8}$

4. Divide by **"1"** to get three more equivalent fractions:

$\frac{12}{36}$ $\frac{12}{36}$ $\frac{12}{36}$

5. Find the fractions that are ≠. Show the cross-products:

$\frac{9}{12} = \frac{15}{20}$ $\frac{10}{20} = \frac{30}{40}$ $\frac{12}{7} = \frac{14}{6}$

6. Solve for **n**.

$\frac{15}{35} = \frac{n}{21}$ $\frac{27}{33} = \frac{18}{n}$

Name: _____ Team: _____

Square One Test

Use calculators!

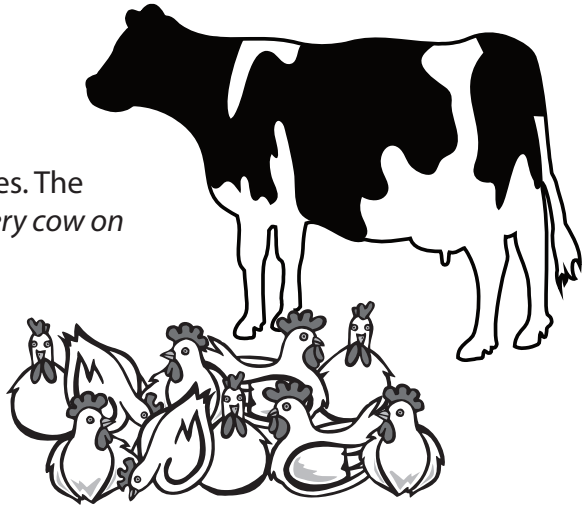
Choose the correct answer and write the letter(s) in the space to the left. Sometimes there is *more than one answer*.

1.	What fraction of this shape is shaded? List *all* correct answers. 	A. $\frac{8}{12}$ B. $\frac{3}{12}$ C. $\frac{4}{6}$ D. $\frac{12}{16}$
2.	Which of the following equals **one**? List *all* correct answers.	A. $\frac{3}{1}$ B. $\frac{3}{3}$ C. $\frac{1}{1}$ D. $\frac{1}{3}$
3.	What number must we multiply the *numerator* by in order to make an equivalent fraction? $\frac{3}{7} \begin{matrix} \times \mathbf{?} \\ \times 5 \end{matrix}$	A. 7 B. 3 C. 5 D. 24
4.	What number must we divide the denominator by in order to make an equivalent fraction? $\frac{27}{\mathbf{36}} \begin{matrix} \div \\ \div \end{matrix} \frac{\mathbf{9}}{\mathbf{?}}$	A. 3 B. 18 C. 27 D. 9
5.	What are the cross-products of these following equivalent fractions? $\frac{5}{6} = \frac{10}{12}$	A. 50 and 72 B. 2 and 2 C. 60 and 60 D. 30 and 120
6.	What is the missing numerator? Write your answer in the space to the right. → $\frac{20}{30} = \frac{n}{18}$	
7.	What is the missing denominator? Write your answer in the space to the right. → $\frac{80}{100} = \frac{36}{n}$	

Name: _____ Team: _____

Ratios and Proportions

Square Two



1. Writing ratios

Ratios are descriptions comparing two items or measures. The following example is a ratio: *There are 10 chickens for every cow on the small farm.*

This ratio can be written 3 ways.

With words → The ratio of chickens to cows is **10 to 1**

With a colon → The ratio of chickens to cows is **10 : 1**

or as a **ratio fraction** → $\frac{10 \text{ chickens}}{1 \text{ cow}}$

Note that there are always **two different labels** in ratios. You must always include the labels after the ratios or state the ratio in a sentence so that the reader understands what you are describing. For example, "***The ratio of chickens to cows is 10 to 1***." The order of items in the ratio is *very* important. (Picture the difference on the farm if there were 10 cows for every chicken! You'd need a bigger farm!)

Complete the following ratios using the <u>three</u> different ways:

- There are 2 left-handed people for every 15 right-handed people.

  Therefore the ratio of left-handers to right-handers is _____ **to** _____.

- The recipe called for 2 cups of fruit for every 4 cups of juice.

  Therefore the ratio of cups of fruit to cups of juice is _____ **:** _____.

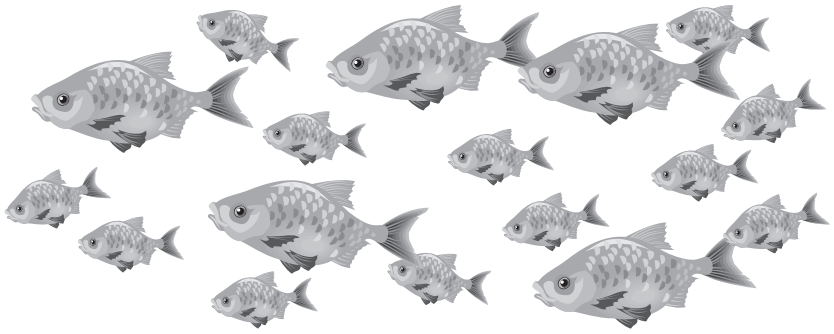
- In the flock of birds there were 14 sparrows for every 3 robins.

  Therefore, the ratio in fraction form is $\frac{\square \text{ sparrows}}{\square \text{ robins}}$

Make a drawing to show how the minute hand moves in comparison to the hour hand on a clock.

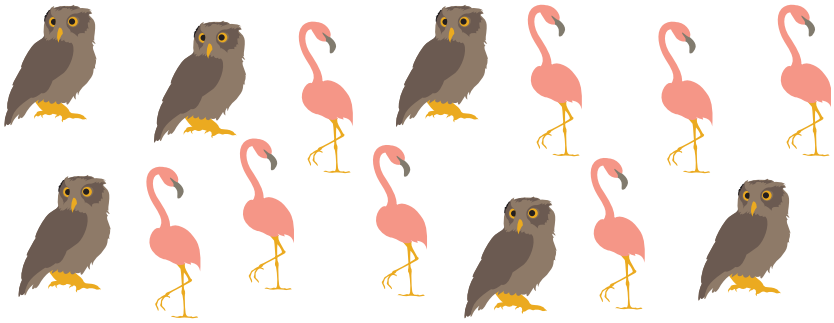
Fill in the blanks:
The ratio of minutes to hours is ___ to ____

Writing Ratios



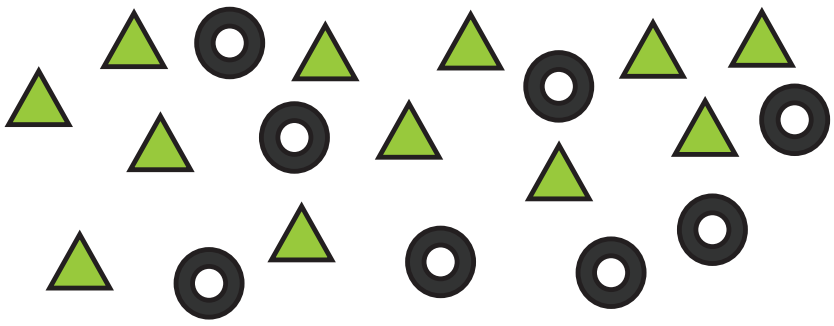
a. The ratio of big fish to small fish is

_____ to _____.



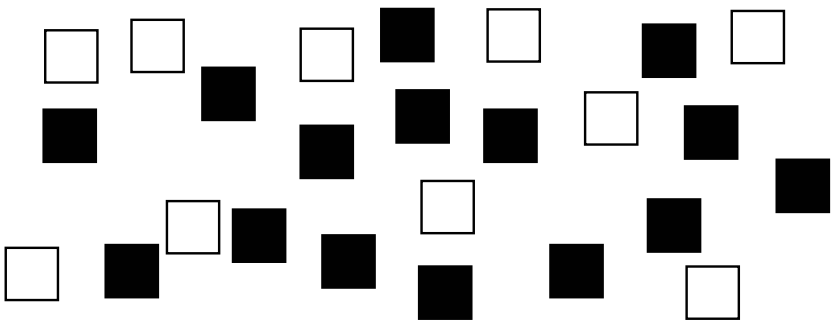
b. The ratio of flamingos to owls is

_____ : _____.



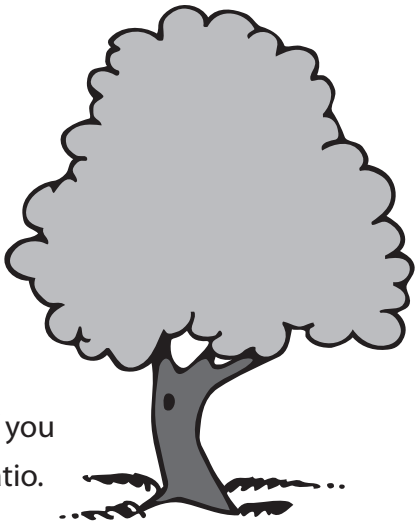
c. The ratio of rings to triangles is

_____ to _____.



d. ☐ white squares
☐ black squares

2. Simplifying ratios



Ratio fractions can be simplified just as you would simplify any other fraction. For example, if the forestry service counted trees in a certain area of the forest, they might find that there are 20 elm trees for every 25 oak trees in an area. **The ratio of elms to oaks is 20 to 25.** Therefore, it could be written three ways:

20 : 25 (elms to oaks) or 20 to 25 (elms to oaks) or $\frac{20 \text{ elms}}{25 \text{ oaks}}$.

From what you know about simplifying fractions, you know that if you divide this ratio fraction by "1" written as $\frac{5}{5}$, you can simplify this ratio.

$\frac{20 \div 5}{25 \div 5} = \frac{4}{5}$ or $\frac{4 \text{ elms}}{5 \text{ oaks}}$. This means, "The ratio of elms to oaks is 4 to 5."

Practice

a. 25 to 50 12 to 16 12 : 18

$\frac{25}{50}$

b. Write an original ratio statement that uses the ratio of 16 to 20.

The ratio of _____ to _____ is 16 to 20.

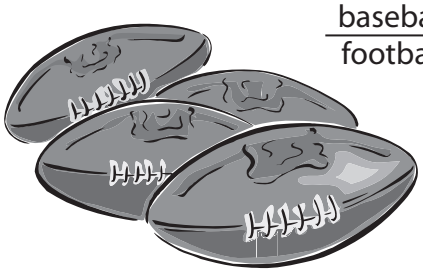
c. Simplify your ratio above.

The ratio of _____ to _____ is _____ to _____.

3. What's a proportion?

An equivalent ratio is a ratio that is equal to another ratio. If you write an equation stating that two ratios are equivalent, then you have written **a proportion**. When you write a proportion, you will notice it looks like you are writing equivalent fractions. In fact, you can use all that you know about equivalent fractions when working with equivalent ratios and proportions. Look at this example.

If the proportion of baseballs to footballs in the closet was 3 to 5, you could write a proportion to show how many footballs there would be if there were 9 baseballs.



$\frac{\text{baseballs } 3}{\text{footballs } 5} = \frac{9}{15}$ If there were 25 footballs, then the proportion would be → $\frac{\text{baseballs } 3}{\text{footballs } 5} = \frac{15}{25}$

You can check that these are proportions because they have equal cross-products.

4. Solving problems with equivalent ratios and proportions

You should feel confident solving the following problems because you already know how to find equivalent fractions. What method you choose depends on the problem.

a. For some, you can make a quick table using the two labels: minutes and $:

For every 20 minutes he walks the dog, John earns $4. How much will he earn for 1 hour and 20 minutes of dog walking?

The ratio of minutes to dollars is 20 to 4. Make a table using the two labels.

minutes	20	40	60 (1 hourr)	80
$	4			

Answer: John will earn _____.

b. Sometimes you can multiply or divide by "1" with a fraction that is $\frac{n}{n}$.

There are 7 blue marbles for every 5 red marbles in each sack. If you had 30 red marbles, how many blue marbles do you need to fill more sacks at the same ratio?

Step one: Set up the ratio with labels (red and blue)

The ratio of blue to red is 7 to 5, or $\frac{\text{blue } 7}{\text{red } 5}$.

Step two: Write a proportion $\frac{\text{blue } 7}{\text{red } 5} = \frac{n}{30}$. *Think*. What $\times 5 = 30$?

Step three: Solve the problem

Multiply by $\frac{6}{6}$. $\frac{\text{blue } 7 \times 6}{\text{red } 5 \times 6} = \frac{n}{30}$ Answer: _____ blue marbles.

c. Sometimes you must cross multiply:

The recipe says *"To make sweet tea, add 8 cups of tea to 1 cup of sugar. How much tea should you add for 3 cups of sugar?*

Step one: Write a ratio with labels. $\frac{\text{T8}}{\text{S1}}$

Step two: Write the proportion. $\frac{\text{T8}}{\text{S1}} = \frac{n}{3}$

Hint: You can abbreviate your labels to initial letters.

(Note that you must keep track of labels. In this problem, tea is in the numerator position and sugar is in the denominator position.)

Step three: Solve by cross multiplying or multiplying by $\frac{3}{3}$.

$\frac{\text{tea } 8}{\text{sugar } 1} = \frac{n}{3}$ $n = 8 \times 3 = 24$ Answer: _____ cups of tea.

d. ***Practice***

Sam's stamp collection had 16 American stamps for every 14 Canadian stamps. He had 104 American stamps. How many Canadian stamps did he have?

Step one: Write the ratio fraction.

Step two: Write the proportion.

Step three: Solve the problem.

$\frac{\text{Am } 16}{\text{Can } 14} = \frac{104}{n}$

Cross multiply $16 \times n = 14 \times 104$

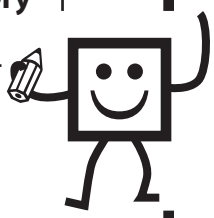
$16 \times n = 1456$

$16\overline{)1456}$ Answer: _____ Canadian stamps.

Stop/Think/Draw/Write 5

Sandra tried to solve this problem, but her answer was wrong. Find her mistake, explain what she did wrong, and solve the problem correctly.

Problem: The recipe for cinnamon sugar is 3 tablespoons cinnamon to every 4 cups of sugar. How many tablespoons of cinnamon for 12 cups of sugar?



Sandra's work:

$\frac{3}{4} = \frac{12}{N}$ $3 \times n = 4 \times 12$ $3 \times n = 48$ $n = 16$

Was there anything Sandra could have done to avoid her mistake?

5. Finding the ratio in a given statement

You should always read a problem carefully and think about what it means. For example, suppose you read an ad that says, "*4 out of 5 dentists prefer Sparkling Toothpaste*." What does this mean?

If you guessed that the ratio here was 4 to 5, you would be wrong!

"4 out of 5 dentists prefer Sparkling Toothpaste," describes a part of collection (dentists). This is actually a fraction, not a ratio. The denominator describes *all* dentists and the numerator describes the part representing those dentists who prefer Sparkle.

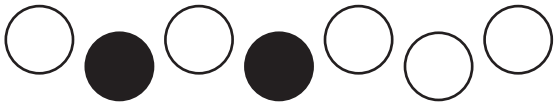
$\frac{4}{5}$ dentists prefer Sparkling Toothpaste.

If you were comparing the dentists who prefer Sparkling Toothpaste to those dentists who do not, then you would have a ratio. Note, however, that it's not 4 to 5 but 4 to 1! Look at the picture to see the comparison. *Think* about this.

The ratio of those who prefer to those who do not prefer is **4 to 1**.

Look at the box below: *think*. How many total marbles? Answer:7. Your fraction denominator should be 7. Look at the ratio and add the two numbers in the ratio. The sum of the ratio numbers should match the denominator (2 + 5) = 7.

Marbles



Write a ratio:

The ratio of black to white marbles

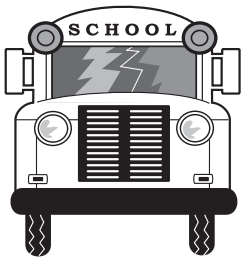
is _____ to _____.

Write the ratio fractions:

$\frac{\text{W marbles } 5}{\text{marbles } 7}$

$\frac{\text{BLK marbles } \square}{\text{marbles } \square}$

6. Finding the "whole"



Most problems begin with a statement.

a. Example 1—Read this statement and think about what it is telling you:

5 out of 6 hockey players rode the bus back to school.

Discuss what "out of" means. Be certain students understand that if 5 out of 6 players rode the bus, that means that 1 out of 6 players got back to school some other way.

Compare the two groups to make a ratio: The ratio of hockey players who rode the bus to hockey players who got home some other way is _____ **to 1**.

Note—If you add the numbers of the ratio, they should equal the "whole." This does not mean that there were only 6 hockey players, but it allows you to set up a proportion to solve this problem. ***If 5 out of 6 players rode the bus back to school and there were 48 players, how many rode the bus?***

$$\frac{\text{rode bus } 5}{\text{all players } 6} = \frac{n}{48}$$

b. Example 2

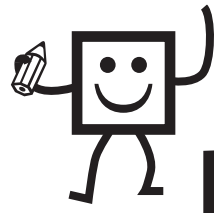
For every 3 diners who ate hotdogs, 2 diners did not.

The ratio of hotdog eaters to those who did not eat hotdogs is _____ **to** _____.

What is the "whole?" _____ (Hint: Add the numbers in the ratio.) Again, this does *not* mean there were only a few diners, but it allows you to make a proportion to solve this problem. ***If the ratio of hotdog eaters to those who didn't eat hotdogs was 3 to 2, and 63 people ordered hotdogs, how many diners were there?***

$$\frac{\text{HD } 3}{\text{diners } 5} = \frac{63}{n}$$

Stop/Think/Draw/Write 6



In the space below, use a drawing to explain how you figured out the ratio and the "whole" described in this statement:

8 out of 10 concert-goers bought tickets online before the concert. The rest bought their tickets at the door.

The ratio of concert-goers who bought tickets online to those who bought them at the door **is** _____ **:** _____.

to

What is the "whole?"_____ How did you figure that out?

Solve this problem using the ratio above: *If 700 people attended the concert, how many bought their tickets online?* Don't forget labels!

_____ = _____

7. Keeping track of the labels in the question

a. Look at this first problem:

Mike collected foreign stamps. For every 8 French stamps he had 28 Spanish stamps. How many French stamps does he have if he has 42 Spanish stamps.

It begins with a statement and ends with a question that asks about French stamps and Spanish stamps. Here's the thing to remember: **Start with the question because the question helps you know what labels you need to solve the problem!!!!**

Follow the steps, show your work, and don't forget *labels*:

Step one: On the first line below, write the labels mentioned in the question.

Step two: Write the numbers from the statement and the question on the lines to make the proportion:

_____ = _____

Step three: Solve the problem.

Answer: _____ French stamps.

b. Look at this problem, which is a little different.

Mike collected foreign stamps. For every 2 French stamps he had 7 Spanish stamps. How many Spanish stamps does he have if he has a collection of 54 stamps in all?

Think: You need two labels to make a proportion, and the question tells you the two labels: all stamps and Spanish stamps.

To help you count the number of all stamps, *draw* the ratio of French stamps to Spanish stamps in the box.

Look at your ratio drawing and ask yourself to look for the "whole" or how many total stamps. _____

to

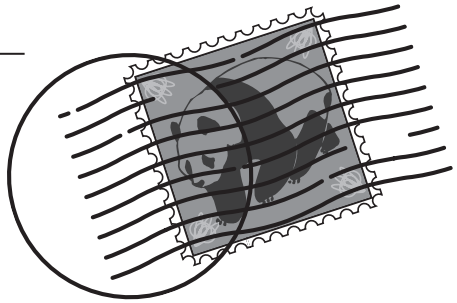
Step one: On the first line below, write the labels of the *question*.

Step two: Write the numbers from the statement and question to make the proportion:

_____ = _____

Step three: Solve the problem.

Answer: _____ Spanish stamps.



Name: _____ Team: _____

Worksheet to Reinforce Square Two Concepts

1. C C A C C A A C A C C
   a. The ratio of Cs to As is _____ to _____.
   b. The ratio of As to Cs is _____ to _____.
2. Simplify the following
   a. 8 : 10 = _____ : _____ b. 14 to 35 = _____ to _____ c. $\frac{18}{81} = \frac{\square}{\square}$
3. The biologists counted 125 frogs and 25 toads.

   a. The ratio of frogs to toads is _____ to _____.
   b. Simplify the ratio: The ratio of frogs to toads is _____ to _____.
4. The library had 4,200 books. There were 2,400 fiction books and 1,800 non-fiction books.
   a. The ratio of fiction to non-fiction is _____ to _____.
   b. Simplify the ratio: The ratio of fiction to non-fiction is _____ to _____.
5. Make a table to solve this proportion. The recipe for light green paint called for 8 cups of yellow to every 3 cups of blue paint. How much blue for 32 cups of yellow?

yellow				
blue				

Solve the following proportion problems using any the strategy that works for you. However, you must show your work so do your problems on a separate piece of paper. *Remember that the questions will tell you the labels for your proportion.*

6. The ratio of cats to dogs is 9 to 8. How many cats for 48 dogs? _____ cats
7. The ratio of spoons to forks is 7 : 3. How many spoons for 24 forks? _____ spoons
8. The ratio of horses to cows is $\frac{\text{horses } 48}{\text{cows } 12}$. How many cows for 8 horses? _____ cows
9. The ratio of geese to ducks is 7 to 12. How many ducks for 14 geese? _____ ducks
10. 5 out of 7 cats were long-haired. What is the ratio of long-haired to short-haired cats? _____ to _____. If there were 84 cats, how many cats had long hair? _____
11. The ratio of red ants to black ants was 2 to 20. If you counted 132 ants, how many were red ants? _____

Name: _____ Team: _____

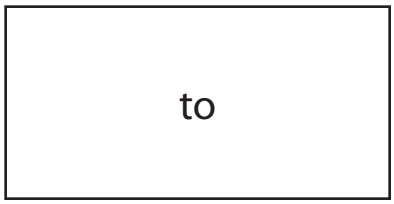
Practice

1. The ratio of high schoolers to middle schoolers who attended the Homecoming game was 9 to 5. How many high schoolers were at the game if 560 students went to the game?

   a. What two labels are in the question? _____ and _____.

   b. You need to know the total number of students. The ratio can help you. Using the black and white circles, draw the ratio of HS to MS in the box: Now *add* the ratio numbers to find the total or "whole."

   The total number is _____.

   c. Follow the steps to solve the problem:

   Step one: In front of the lines below, write the labels of the *question*.

   Step two: Write the numbers from the statement and question to make the proportion.

   _____ = _____

   Step three: Solve the problem.

2. Solve the following problems. **Remember** to look at the question to find the labels you need for the proportions:

   a. In an election between Jon and Ted, Ted won by a ratio of 7 to 3. If Ted earned 1015 votes, how many voted for Jon? (Show your work.)

   Follow the steps to solve the problem:

   Step one: On the first line below, write the labels of the *question*.

   Step two: Write the numbers from the statement and question to make the proportion.

   _____ = _____

   Step three: Solve the problem.

   b. In another election Alice beat Marcia by a ratio of 9 to 2. If Marcia got 612 votes, how many people voted in the election? Show your work. There are different ways to solve this, but use a proportion or equivalent fractions for this problem. Hint: Use the ratio to find the total or "whole."

   _____ = _____

Name: _____ Team: _____

More Practice

Solve these problems using proportions or equivalent fractions. Remember, look at the question to find the labels you need.

3. In the orchard, the ratio of apple trees to pear trees is 14 to 2. If there are 588 apple trees, how many pear trees are there?

_____ = _____

4. In another orchard there are 6 cherry trees for every 4 peach trees. If there are 936 cherry trees, how many trees are there in the whole orchard?

_____ = _____

Solve these problems any way you'd like, but show your work!

5. The farmer harvested on average about 140 pounds of cucumbers for every 40 plants. If the farmer harvested a total of 3780 pounds of cucumbers, how many cucumber plants did he have on his farm?

_____ = _____

6. A safety official sat at a busy intersection and watched the cars. He saw that 2 out of every 15 drivers forgot to fasten their seatbelts. If 1530 cars passed through the intersection, how many drivers forgot to fasten their seatbelts?

_____ = _____

Challenge: Write an original word problem using the numbers from one of the problems on this page

Name: _____ Team: _____

Quick Team Quiz Two

Use calculators!

1. Which of the following are equivalent fractions? Show the cross-products.

   a. $\frac{21}{35} = \frac{3}{7}$ b. $\frac{9}{24} = \frac{21}{56}$ c. $\frac{30}{45} = \frac{20}{30}$

2. In the boxes, write a ratio for this sentence three different ways:

   There were 23 photographs to every 5 paintings.

The ratio of photos to paintings is
The ratio of photos to paintings is
Ratio fraction: You need *labels*. _____

3. Simplify these ratios

   a. $\frac{8}{24} =$ b. $\frac{14}{21} =$ c. $\frac{300}{5000} =$

4. Make a table to solve this ratio problem: 3 peanuts for every 11 pieces of popcorn. How many pieces of popcorn for 18 peanuts?

peanuts						
popcorn						

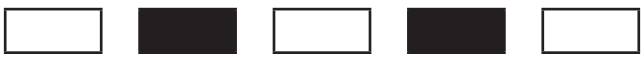
5. Use equivalent ratios to solve this problem. Show your work on the back or on another piece of paper.

   If the ratio of guppies to goldfish in the fish tank was 11 to 6, how many guppies would there be if you counted 48 goldfish? *Label your answer.*

6. Use equivalent ratios to solve this problem. Show your work on the back or on another piece of paper.

   In the parking lot, the ratio of foreign cars to American-built cars was 7 to 5. If you counted 120 American cars, how many foreign cars were there? *Label your answer.*

7. Use equivalent ratios to solve these problems. Show your work on the back or on another piece of paper.



   a. What is the ratio of black to white boxes? _____to_____

   b. If you had 21 white boxes, how many black boxes would you have? _____

   c. If you had 40 boxes in all, how many would be black? _____

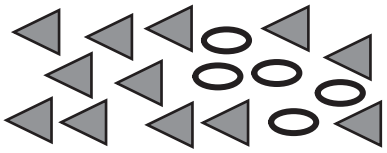
Name: _____ Team: _____

Square Two Test

Use calculators!

1. What are the cross products?

$$\frac{20}{25} = \frac{24}{30}$$

2. 

The ratio of ◀ to ⬭ is _____ to _____.

3. Write this ratio in two other ways.

The ratio of squares to rectangles is **7 to 9**.

4. What is the ratio of hours in a day to a day?

The ratio of hours in a day to a day is _____ to _____.

5. Simplify the ratio.

The ratio of math books to science books is 18 to 6:

The ratio of math books to science books is _____ to _____:

6. Students read 4 biographies for every 14 books of fiction. How many biographies if kids read 42 books of fiction?

Solve with a table for this ratio.

biographies				
fiction				

Answer: _____

7. The company sells 32 orders every 12 days. How many orders in 27 days?

Solve this word problem using equivalent ratios. Show your work here:

_____ = _____

Answer: _____

8. In the yard there were red and white rose bushes. 2 out of 3 rose bushes were red. For questions b and c, show work on back of paper.

Draw the bushes at the bottom *before* you start:

What is the ratio of red rose bushes to white roses? _____ to _____

If there were 42 red rose bushes, how many white rose bushes?

If there were 96 rose bushes in all, how many would be red?

Name: _____ Team: _____

Rates

Square Three

1. Rates and ratios:

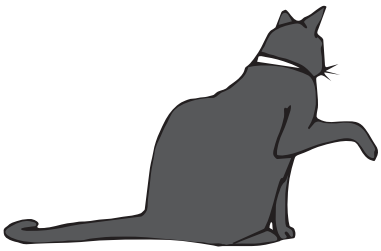
Rates are special kinds of ratios. When a ratio compares two quantities using *different* kinds of units, it is called a **rate**. You can often recognize a rate because you will see the word **per**. If the problem says **per** hour, **per** case, **per** month, it means per 1 hour, per 1 case, or per 1 month.

Look at the examples and note the two different units in the rate.

Ratio	***Rate***
The ratio of red marbles to blue marbles is 3 to 4. (unit = marbles)	3 cans of soup for $4.00. (units = cans and $$$) Her pay was $8 per hour. (units = $$$ and hour)

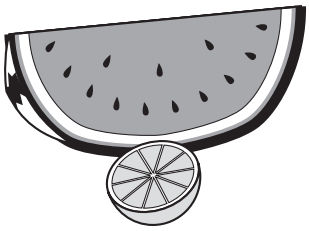
a. You work with rates all the time in your daily life. Note the different units in each example and fill in the correct numbers in the rates.

- Your washing machine uses 20 gallons of water for every load of wash: $\frac{\text{gallons } \square}{\text{wash } \square}$
- The average cat eats 7 oz of cat food per day: $\frac{\text{oz } \square}{\text{day } \square}$
- Your hair grows 1 inch in two months: $\frac{\text{inch } \square}{\text{months } \square}$



b. If you think about a grocery store, you will find all sorts of rates. Write rates like the ones above for each example.

- Soda is often sold at a rate of 4 cans for $6.00. _____
- Soup is sold 3 cans for $7.00. _____
- Fresh fruits and veggies are either sold by the pound or sold by the number of items.



Lemons are 3 for $2.00. _____

Watermelon is $.50 per pound. _____

2. Solving rate problems:

a. One way to solve rate problems is to set up proportions and solve for the missing N. (You can do these without ratios, but this format keeps all the numbers straight, and you will never mix up when you should multiply or when you should divide.)

Fish cost $2.99 per pound. How many pounds of fish can you buy for $20.93?

Set up the ratio, solve it, and write the answer with a label.

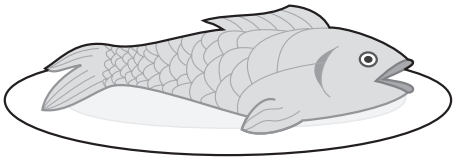
$$\frac{\text{price } \$2.99}{\text{pound } 1} = \frac{\$20.93}{N}$$

$\$2.99 \times N = 1 \times \$20.93$ $\quad$ $2.99\overline{)20.93}$ $\quad$ $N = 7$ pounds of fish

b. You can also solve this problem:

Fish cost $2.99 per pound. How much fish did 8 pounds cost?

Set up the ratio, solve it, and write the answer with a label.



$$\frac{\text{price } \$2.99}{\text{pound } 1} = \frac{N}{8}$$

$1 \times N = \$2.99 \times 8$ $\quad$ $N = \$23.92$

3. Practice:

Find the answers to these rates. Show your work, write labels, and be sure your answers are reasonable.

a. Ice Cream $6.50 per pint. How much will 10 pints cost? _____

b. Apple juice is $3.10 per quart. How many quarts can you buy for $18.60? _____

c. Buy 2 cans of soup for $3.50. How many cans of soup can you buy for $14.00? _____

d. Frozen pizzas are 3 for $16.50. How many pizzas can you buy for $44? _____

e. Sunglasses cost $39.96 for 3. How many sunglasses can you buy for $67.00? _____

4. Finding unit rates, or the cost of one item

a. You run into situations involving unit rates often in your everyday life, especially in stores. **A unit rate describes the cost *per* one item.**

Again the word "per" is signaling a rate. With problems as simple as Example 1, there is no need to use equivalent fractions. You can do this problem using mental math.

Example 1: *If you could buy 4 apples for $1, what's the unit price?*

$5\overline{)1.00}$ $\frac{.25}{}$ or 25¢ per apple.

b. Sometimes you cannot easily do the math in your head.

Example 2: *If bars of imported chocolate are sold for 4 bars for $16.80, how much does each bar cost or what is the unit price?*

If you are not sure what to do, first set up the ratio. $\frac{\text{bars } 4}{\$16.80} = \frac{1}{N}$ (1 unit)

It becomes obvious what to do when you write the proportion.

Hint: If your calculator said 4.2, you must change that into money: $4.20.

$4 \times N = 1 \times 16.80$ $4\overline{)16.80}$ $\frac{4.20}{}$ So the unit price is $4.20.

c. Example 3: Again the ratio helps. *The bookstore sells 8 books for $15. What's the unit price ?*

Solve by cross multiplying. $\frac{\$15}{8 \text{ books}} = \frac{N}{1}$ (1 unit)

Round your answer to the nearest cent. 1 book cost _____.

5. Two ways to finding the better buy

a. Often you are asked to look at two prices and decide, "Which is the better buy?" Sometimes you can just think about it and do the math in your head. For example: *Which is the better buy? 4 limes for 80 cents or 6 limes for $1.10.*

In you head you can figure out that the unit price of 4 for 80 cents= 20¢ for each lime. At this rate, 6 limes (6×20) would cost $1.20. However, the second rate says you can buy 6 limes for only $1.10—so it's a better buy. When the numbers don't allow you to do simple calculations in your head, there are 2 ways to figure out the better buy.

b. The first way is to set up a proportion using the first rate and then compare your answer to the other rate. Look at this problem: *16 pens for $19.20 or 7 pens for $10.50. Which is the better buy?* Set up the first rate and then create a proportion to determine how much 7 pens will cost at this rate. After you solve the proportion, compare your answer to the cost of 7 pens at the second rate.

$\frac{16 \text{ pens}}{\$19.20} = \frac{7 \text{ pens}}{\$n}$ $16 \times n = 7 \times 19.20$ $16n = 134.4$ $(134.40 \div 16)$

n = 8.4 dollars or $8.40

This means that at the rate of 16 pens for $19.20, 7 pens would cost only $8.40. That makes this the better deal because the second offer was 7 pens for $10.50.

c. The second way to solve this problem is to determine the unit price for each offer and then compare the unit prices. *16 pens for $19.20 or 7 pens for $10.50.*

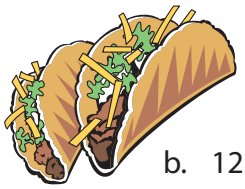
$\frac{16 \text{ pens}}{\$19.20} = 16\overline{)19.20}$ unit price is $1.20 $\frac{\$10.50}{7 \text{ pens}} = 7\overline{)10.50}$ unit price $1.50

Compare the unit prices. What's the better buy? _____

6. Practice finding the better buy:

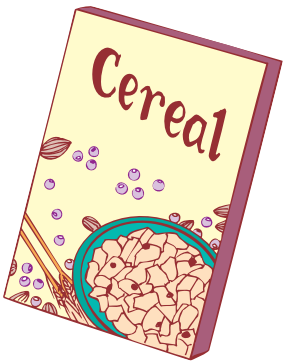
Solve these problems using either way. (See b and c above.) *Show your work.*

a. 8 tacos for $16 or 5 tacos for 11? What's the better buy?



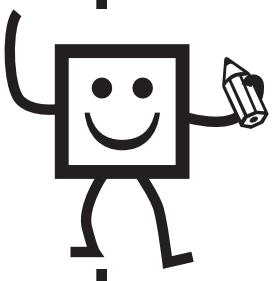
b. 12 oz chips for $2.16 or 16 oz for $2.56. What's the better buy?

c. You could buy 28 oz of cereal for $4.20 or 50 oz of cereal for $7.50. What's the better buy?



Stop/Think/Write/Draw 7

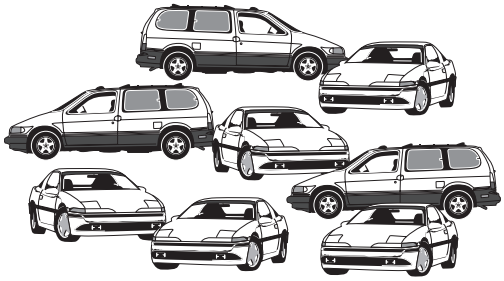
Which method did you use for finding the unit price? Write the steps using the method you preferred so that someone else could follow them



Name: _____ Team: _____

Worksheet to Reinforce Square Three Concepts

1. Write the ratio of minivans to cars in three different ways:



2. In Tinytown, there were **2 cats** to every **10 dogs**. If there were **9 cats**, how many dogs were there?

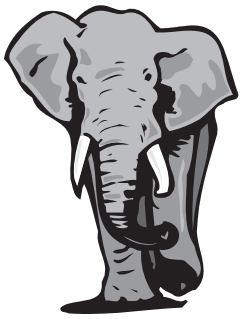
$$\frac{C}{D} \text{ _____} = \text{_____}$$

3. The ratio of goats to sheep is 12 : 21. How many goats for **28 sheep**?

_____ = _____

4. The ratio of **forks to knives** is **12 : 16**. If there were 84 pieces of silverware, how many forks would there be?

_____ = _____



5. Write a rate for the following:
   a. An elephant drinks 40 gallons of water per day. _____
   b. An elephant eats 300 pounds of food every two days. _____
   c. The Earth spins once every 24 hours. _____
   d. The Moon makes one revolution around the Earth every 30 days. _____

Solve the following problems. Show your work and label your answers.

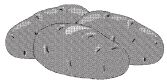
6. If frozen cookie dough costs $2.60 per package, how much will 4 packages cost? _____

7. If 5 lunches cost $67.90, how much would 1 lunch cost? _____

8. If the total cost of 3 video games was $361.89, what is the unit price of each game? _____

9. If the special price of 5 concert tickets is $566, what is the unit price for each ticket? _____ Round your answer to the nearest penny if necessary.

10. What was the better buy? 6 pounds of potatoes for $4.50 or 9 pounds for $6.50? _____



Name: _____ Team: _____

Quick Team Quiz Three

Use calculators!

1. Draw the picture and write a ratio for each of the following:

a. 4 gold coins and 9 silver coins:	b. Red and green grapes. 2 out of 3 grapes were green. The ratio of red to green grapes is:
_____ to _____	_____ to _____

2. Simplify these ratios

$\frac{6}{16}$ $\frac{20}{28}$ $\frac{400}{1200}$

3. Use proportions to solve this problem. **Show your work. Label your answer.**

   If the ratio of pansies to tulips in the garden was 5 to 1, how many pansies would there be if you counted 48 tulips?

   _____ = _____

4. Use the information in #3 and proportions to solve this problem. **Show your work. Label your answer.**

   If there were 72 flowers in the whole garden, how many would be pansies?

   _____ = _____

5. Solve these problems any way you choose. Show your work and label your answers.

   a. 6 lollipops costs $1.56. What is the unit price? _____

   b. Pens sold for $1.39 each. How many pens can you buy for $11.12? _____

   c. Bananas sell at a rate of 5 for $1.39. What is the unit price? _____

6. What's the better buy? Show your work.

   9 muffins for $12 *or* *7 muffins for $10*

Name: _____ Team: _____

Square Three Test

Use calculators!

1.	a. The ratio of minutes in an hour to an hour is _____ to _____. b. The ratio of hours in a day to a day is _____ to _____.		
2.	4 out of 5 players were boys.	The ratio of boys to girls was _____ to _____.	
3.	Simplify. (Put in lowest terms.)	$\frac{12}{36}$	$\frac{400}{1600}$
4.	Solve the problem using ratio and proportion. Show your work and label your answer.	The ratio of hawks to rabbits is 2 to 15. If there are 75 rabbits, how many hawks?	
5.	Solve the problem using ratio and proportion. Show your work and label your answer.	In the pond the ratio of green frogs to bull frogs was 3 to 5. If there were 56 frogs in the whole pond, how many were green frogs?	
6.	Solve this problem using ratio and proportion. Show your work and label your answer.	If 7 bars of soap cost $17, how many bars of soap can you buy for $204?	
7.	What are the unit prices for these? Show your work.	a. 6 boxes for $15	b. 3 meals for $51.60
8.	What's the better buy? Show your work and circle your answer.	12 hats for $53 or 16 hats for $72?	

Name: _____ Team: _____

Rates of Speed—Part 1

Square Four

Rate of Speed

1. Rate of speed is one of the most common kinds of rates we meet in everyday life. When someone says that a car was going 60 miles per hour, they are expressing a rate. As you know, the word *per* is key to recognizing a rate. Therefore, 60 mph can be written as $\frac{\text{miles } 60}{\text{hour } 1}$ or $\frac{\text{hour } 1}{\text{miles } 60}$. You must put a 1 in the rate because 60 mph means 60 miles per one hour.

a. Take a moment to write the *rates* for mph. Complete the chart:

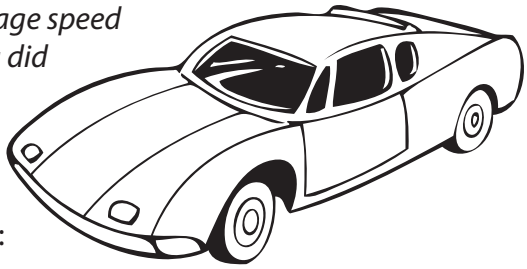
mph	means	written as a *rate* miles per hour	written as a *rate* miles per minute
35 mph	35 miles per hour	$\frac{\text{miles } 35}{\text{hour } 1}$	$\frac{\text{miles } 35}{\text{minutes } 60}$
45 mph			
100 mph		$\frac{\text{miles}}{\text{hour}}$	
	115 miles per hour		$\frac{\text{miles } 115}{\text{minutes } 60}$
		$\frac{\text{miles } 250}{\text{hour } 1}$	

b. Take another moment to visualize what mph means. If a car moves at 40 mph, in one hour's time, it will be 40 miles away from where it started. Complete the chart.

Car starts at place A at one o'clock	1 hour later, how many miles has it traveled?	2 hours later, how many miles has it traveled?	5 hours later, how many miles has it traveled?
30 mph			
50 mph			
60 mph			

2. You have seen many word problems that sound like this:

Ted drove 180 miles to visit his grandparents at an average speed of 45 mph. Assuming he didn't stop for coffee, how long did the trip take?



In order to solve a proportion problem, you need three of the four numbers. (The fourth number will be N.) It looks as though you only have two numbers: 180 and 45. But the third number is hidden in "mph." Mph is a *rate* that means **miles per 1 hour**.

Follow the steps below to solve the problem.

What do you know?	Speed was 45mph. He drove 180 miles.	First write his speed as a rate with labels: $\frac{45 \text{ miles}}{1 \text{ hour}}$
What do you want to know?	How long did it take to drive 180 miles?	Next, write the proportion: $\frac{45 \text{ miles}}{1 \text{ hour}} = \frac{180}{n}$ Note the labels—*Miles* is in the numerator position, *hours* in the denominator position.
Solve the problem using cross multiplication.	$\frac{45 \text{ miles}}{1 \text{ hour}} = \frac{180 \text{ miles}}{n \text{ hours}}$ $\frac{45 \text{ miles}}{1 \text{ hour}} = \frac{180 \text{ miles}}{n \text{ hours}}$	$45 \times n = 180 \times 1$ $45n = 180$ $n = 4$ $45\overline{)180}$ 4
Add the correct label and check if your answer is reasonable.	How long did the trip take? Answer: $n = 4$ hours. The answer is reasonable. (At 50 mph, you can drive 200 miles in 4 hours.)	

3. Here's another problem for you to solve:

Sara rode her bike to raise money for her favorite charity. She rode 91 miles in 7 hours. What was her average speed? (mph)

Hint: *In this case you are trying to find the rate—miles per 1 hour. Not to worry! You follow a similar process.*

What do you know?	She rode 91 miles. It took her 7 hours.	First write his speed as a rate with labels: $\frac{91 \text{ miles}}{7 \text{ hours}}$
What do you want to know?	What was her average speed (mph)? Miles per hour means the distance she can ride in 1 hour.	Next, write the proportion: $\frac{91 \text{ miles}}{7 \text{ hours}} = \frac{n}{1}$ Note the labels—*Miles* is in the numerator position, *hours* in the denominator position.
Write the equivalent fractions and solve the problem using cross multiplication.	$\frac{91 \text{ miles}}{7 \text{ hours}} = \frac{n \text{ miles}}{1 \text{ hour}}$ $\frac{91 \text{ miles}}{7 \text{ hours}} = \frac{n \text{ miles}}{1 \text{ hour}}$	$7 \times n = 91 \times 1$ $7n = 91$ $n = 13$ $7\overline{)91}$ 13
Add the correct label and check if your answer is reasonable.	What was her average speed? Answer: 13 miles per hour. The answer 13 mph is reasonable. (Estimate $7 \times 12 = 84$)	

4. Sometimes the answer is not a whole number. In the following problem, *Sally rode 108 miles in 8 hours. What was her average speed?*

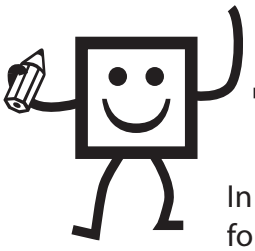
$\frac{8 \text{ hours}}{108 \text{ miles}} = \frac{n \text{ miles}}{1 \text{ hour}}$ $\frac{8 \text{ hours}}{108 \text{ miles}} = \frac{n \text{ miles}}{1 \text{ hour}}$

$8 \times n = 108 \times 1$

$8n = 108$

$n = 13.5$ in one hour, *or 13.5 mph.*

Name: _____ Team: _____



Stop/Think/Draw/Write 8

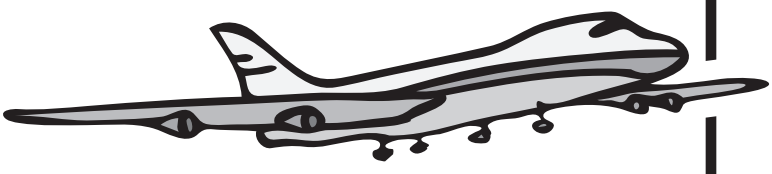
(Be sure your labels are spelled correctly)

In the following space, demonstrate the steps you followed to solve the following problem by using equivalent fractions:

The Shakespeare Club at Pilgrim High flew by jet from New York City to London to attend an international Shakespeare festival. The distance between New York City and London is 3,458 miles and the flight was 7 hours long. What was their jet's average rate of speed in mph? *(Hint: begin with what you know...)*

_____ = _____

Is your answer reasonable?



Name: _____ Team: _____

Rates of Speed—Part 2

Square Four

5. Rates and Time:

a. When talking about **rates of speed**, there is always the element of time—how far you go in how much time. Examples of rates usually include the familiar mph or miles per hour. But sometimes, the time is expressed in minutes, seconds, hours, days, or years. It doesn't matter as long as the units of time within the equation always match.

b. What happens when a problem says that someone traveled a distance (*n)* at 45 mph in less than an hour—say 30 minutes. What can you do?

One way is to change the rate from mph into miles per minute. 45 mph also can be written as 45 miles per 60 minutes:

$$\frac{45 \text{ miles}}{60 \text{ minutes}} = \frac{n \text{ miles}}{30 \text{ minutes}}$$

Or you can change the minutes into a decimal part of an hour. 30 minutes equals 0.5 hours. *If you don't know the decimal equivalent, divide the minutes by 60. (30 ÷ 60 = 0.5)*

$$\frac{45 \text{ miles}}{1 \text{ hour}} = \frac{n \text{ miles}}{0.5 \text{ hour}}$$

c. Solve this problem using equivalent fractions.

Although the burning house was 12 miles away, the fire trucks arrived in 15 minutes. How fast must they have been traveling?

_____ = _____

Is your answer reasonable?

d. Sometimes a word problem reports a time that is more than 1 hour, such as 90 minutes. What would you do to solve those problems? The same as you did before. Change the fraction label from per hour to per 60 minutes. Or change the 90 minutes to be 1.5 hours.

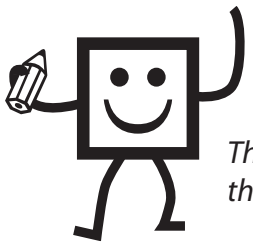
6. Practice

Give the equivalent time with the new time labels. *Example:* $\frac{1}{2}$ *day =* <u>*12 hours*</u>.

$1\frac{1}{2}$ days = _____ hours $\frac{3}{4}$ hour = _____ minutes. 20 days = _____ hours

0.1 hour = _____ minutes $\frac{1}{2}$ year = _____ months

Stop/Think/Draw/Write 9



Show the steps to solve this next problem using equivalent fractions:

The marching band completed the 5 mile parade route in 90 minutes. What was their average marching speed?

_____ = _____

Is your answer reasonable?

Name: _____ Team: _____

Worksheet to Reinforce Square Four Concepts

1. In each candy bag, there were 8 sourballs and 5 lollipops. The ratio of sourballs to lollipops was _____ to _____. Remember *labels*!

   a. If there were 32 sourballs, how many lollipops would there be? _____

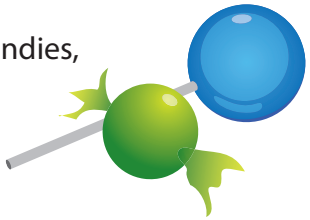
   _____ = _____

   b. If there were 2 bags of candy, how many sourballs would you have? _____

   _____ = _____

   c. If you had emptied all the candy bags and found you had 156 candies, how many would be lollipops? _____

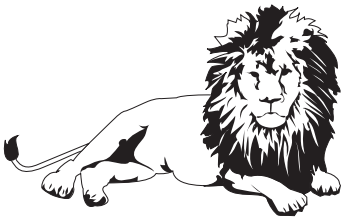
   _____ = _____



   d. If there were 117 pieces of candy with the proper ratio of sourballs to lollipops, how many bags of candy could you put together? _____

   _____ = _____

2. Write a rate for the following.



   a. A lion eats 15 pounds of meat per day. _____

   b. A lioness eats 33 pounds of meat every 3 days. _____

   c. The planet Mercury makes one revolution around the Sun every 88 days. _____

   d. The planet Neptune makes one revolution around the Sun every 165 years. _____

Solve the following problems. Show your work and <u>label</u> your answers.

3. If a dress shirt costs $14.98, how much will 6 dress shirts cost? _____

4. If 6 ties cost $77.88, what is the unit cost? _____

Solve the following problems. Show your work and <u>label</u> your answers.

5. What's the better buy? 4 mp3 players for $178.68 or 5 mp3 players for $217.50?

   _____

6. Experienced hikers can hike at an average speed of 6 mph. How long would it take an experienced hiker to hike 21 miles? _____

   _____ = _____

7. a. The hospital sent a helicopter to pick up the most injured at a car crash. Although the distance from the hospital to the crash site was 45 miles away, the helicopter arrived in 15 minutes. What was the helicopter's average speed in mph? _____

   _____ = _____

   b. The hospital also sent an ambulance to the same crash scene to pick up those who had only minor injuries. The ambulance sped to the scene at an average speed of 60 mph. How long did it take to get there? _____

   _____ = _____

8. a. In 1927, Charles Lindbergh flew solo across the Atlantic Ocean (3,500 miles) in a little over 33 hours. What was his average rate of speed in mph? _____

   _____ = _____

   b. Today most jumbo jets can make the same trip as Lindbergh took but at a speed of 538 mph. How long does it take these jets to fly across the Atlantic? _____

   _____ = _____

Name: _____ Team: _____

Quick Team Quiz Four

Use calculators!

1. Use proportions to solve this problem. **Show your work. Label your answer.**

   If the ratio of lions to giraffes on the African game reserve was 3 to 4, how many lions would there be if you counted 168 giraffes?

   _____ = _____

2. If the ratio of cod to bluefish in the fisherman's net was 2 : 5, how many cod would be in a net filled with 126 fish? *Show your work. Label your answer.*

   _____ = _____

3. Solve these problems any way you choose. **Show your work, round to nearest penny, and label answers.**

   a. 3 digit cameras cost $239.97. What is the unit price? _____

   b. One electric chain saw cost $39.99. How many chain saws can you buy for $325.00? _____

   c. Cantaloupes sell at a rate of 2 for $4.39. What is the unit price? _____

   d. What's the better buy? Show your work.
   3 bags of dog food for $17 or buy one bag for $11 and get one free.

Use proportions to solve these problems. Show your work, round to the nearest tenth, and label your answers.

4. Michael drove 348 miles to the university at an average speed of 58 mph. He stopped for ten minutes to buy gas. How long did the trip take? _____

   _____ = _____

5. The Coast Guard raced to where the ship was sinking 8 miles off shore. It arrived in just 15 minutes, in time to save everyone on board. What was the average speed in mph of the Coast Guard's rescue vessel? _____

   _____ = _____

6. Michelle rode her bike in a 100-mile charity challenge. She completed the ride in 4 hours 30 minutes. What was her average rate of speed? _____

   _____ = _____

Name: _____ Team: _____

Square Four Test

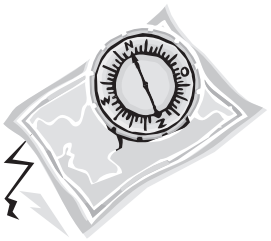
Use calculators!

1.	9 out of 15 athletes played baseball. 6 out of 15 athletes played soccer. a. The ratio of baseball players to soccer players is _____ to _____, *or* b. Simplifying the ratio in part a. _____ to _____	☐ baseball ☐ soccer	
2.	Solve the problem using ratio and proportion. Show your work and label your answer.	The ratio of apples to oranges was 16 to 18. If there were 24 apples, how many oranges? _____ = _____	
3.	Solve the problem using ratio and proportion. Show your work and label your answer.	The ratio of apple trees to pear treas is 3 to 5. If there were 48 trees in the orchard, how many were apple trees? _____ = _____	
4.	Solve this problem using ratio and proportion. Round if necessary to the nearest penny.	If 6 tubes of toothpaste cost $14.34, how much will 8 tubes cost? _____ = _____	
5.	What are the unit prices for these? Round (if necessary) to the nearest penny.	a. 4 candy bars for $3.52.	b. 6 cans of bug spray for $23.68.
6.	What's the better buy? Detemine the unit price and circle your answer.	Cold pills come in two different packages—24 cold pills for $5.28 or 36 pills for $7.20.	
Use <u>proportions</u> to solve the following problems. Show your work and label answers.			
7.	Solve this problem using ratio and proportion. Round if necessary to the nearest tenth.	If a jet flew from Boston to Los Angeles, a distance of 2600 miles, in 5 hours, what was the jet's average speed in mph? _____ = _____	
8.	Solve this problem using ratio and proportion.	Two friends drove 850 miles from Chicago, Illinois to Baton Rouge, Louisiana. Their average speed was 50 mph, but they also stopped for 2 hours and 30 minutes to eat meals and buy gas. How long did the whole trip take? _____ = _____	

Name: _____ Team: _____

Scale Drawings and Similar Triangles

Golden Square



1. Scale:

Scale can be considered a part of the study of ratio, rates, and proportions. We use scale when drawing blueprints, making models, or creating maps. Generally, because of space considerations, we want to take something big and draw it proportionately smaller. How else could we create a world map representing the distance at the Equator of over 25,000 miles on an 8 x 10 inch piece of paper? A formal definition of ***scale*** might be the ratio of a measurement of an actual thing compared to the measurement of a model or drawing.

Map Scale: If you measured a road on the map from City A to City B and it was 3.75 inches, you could use a proportion to figure out the true distance between the cities by using the scale. In this case the scale on a map said that 1 inch is equal to 50 miles:

$$\frac{1 \text{ inch}}{50 \text{ miles}} = \frac{3.75}{n} \qquad n = 50 \times 3.75 \qquad n = \textbf{187.5 miles}$$

Use ratios and proportions to fill in the blanks in this chart. Work on a separate piece of paper and check your answers with the answers under the graph.

distance on a map	scale in inches	actual distance
14 inches	1 inch = 25miles	
	1 inch = 50 miles	325 miles
1.75 inches	1 inch = 2000miles	
3.25 inches		13,000 miles

Answers: 350 miles, 6.5 inches, 3,500 miles, 1 inch = 4,000 miles

Golden Square Challenge 1:

Walk or ask someone to drive from the front door of your school to your house. You can use an existing map for your data gathering, or just the odometer in the car. Drive from the school to your house. **Make a scale map of the route to your house from school to home** with a scale of 1 inch = 0.5 mile. (If you live more than 10 miles away, use the scale of 1 inch = 1 mile. If you live less than one mile away, use the scale of 1 inch = 0.1 mile.) Add landmarks and write street names.

★ Write a paragraph explaining how you completed this challenge and how this challenge relates to ratios, rate, and proportions.

2. Models and scale:

People who build models also use a scale. For example, if the scale was 1 inch (model) = 2 feet (actual), then a model of a 120-foot ship would be 60 inches (5 feet) long. That is a large model!

$\frac{1 \text{ inch}}{2 \text{ feet}} = \frac{n}{120}$ $2n = 120$ $2\overline{)120}$ $\frac{60}{}$ $n = 60$ inches, or convert to feet $12\overline{)60}$ $\frac{5}{}$ $= 5$ feet.

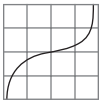
You are going to make a scale drawing of your family car.

Golden Square Challenge 2:

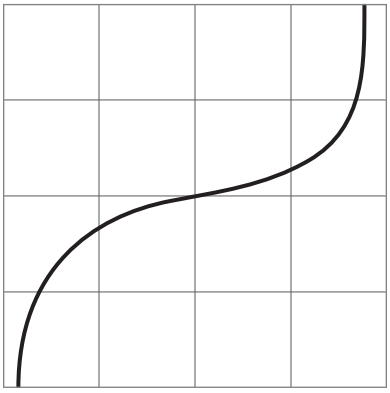
Make a scale drawing of your family car, truck, or SUV. You will need graph paper to complete this activity. If you are using quarter-inch graph paper make a scale of 1 square equal to 6 inches (or 0.5 feet). If using centimeter graph paper, make a scale of 1 square = 10 centimeter. First measure your car from bumper to bumper and then ground to roof. Next, measure the heights of wheel wells and tires. Measure how long the hood is, the trunk, where the doors begin, etc. Use your measurements to draw to make a scale drawing.

★ Write a paragraph explaining how you completed this challenge and how this challenge relates to ratios, rate, and proportions.

3. Size change factors:



We can use scale to scale draw items *bigger* than what they are in reality. If you draw a line on graph paper that has quarter-inch squares and you wanted to make an enlarged scale drawing of that line, you can redraw the line using graph paper that was one-inch square. Your second drawing would be 4 times as large (400% larger) than your original line.



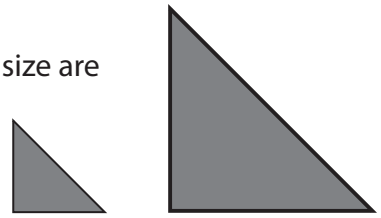
It would have a size change factor of $\frac{4 \text{ drawing}}{1 \text{ actual}}$.

Golden Square Challenge 3:

Make a scale drawing where the drawn object is drawn 400% (4 times) larger than the actual object. Choose something small (a paper clip, pencil, pen, stamp, thumb tack, etc.) Trace it on small graph paper (1 centimeter or $\frac{1}{4}$ inch). If you choose $\frac{1}{4}$ inch graph paper, find one-inch graph paper, or make your own one-inch graph paper. If you choose 1 centimeter graph paper, make your own 4-centimeter-square graph paper. Redraw your object to make a drawing that is 4 times (400%) larger than the original.

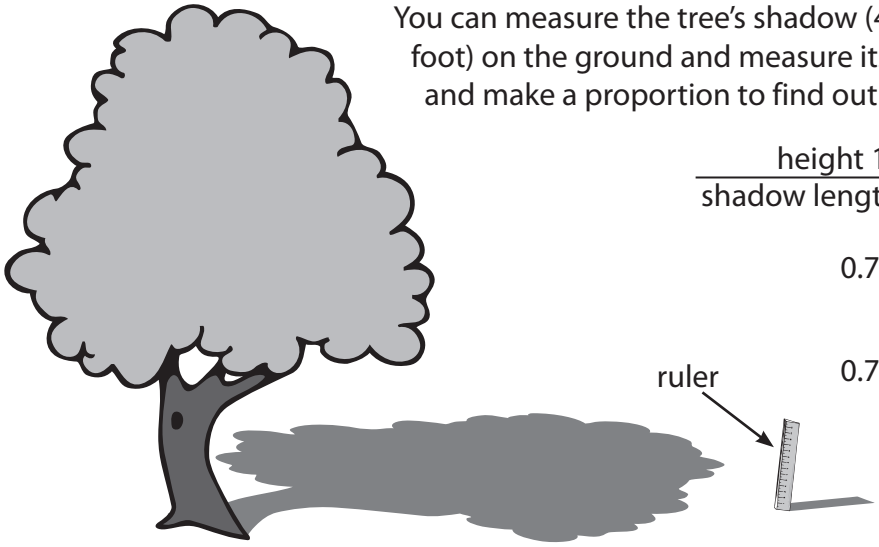
★ Write a paragraph explaining how you completed this challenge and how this challenge relates to ratios, rate, and proportions.

4. Similar figures:



Figures that have exactly the same shape, but not necessarily the same size are called *similar* figures. We can use a proportion to find the length of corresponding sides in similar figures because each side of the enlarged polygon was enlarged by the same size-change factor.

The two right triangles are similar. This means that if you knew the height and the base of the smaller triangle and only the base of the bigger triangle, you could figure out the height of the bigger triangle by creating a proportion. Ancient Greeks figured this out a long time ago, and used it to determine the height of objects they could not reach to measure. Let's see how it works with a tree on a sunny day. The tree and its shadow create two sides of triangle. The shadow of a ruler creates a similar triangle with its shadow on the ground.



You can measure the tree's shadow (40 feet.) You stand a ruler (1 foot) on the ground and measure its shadow 9 inches (0.75 feet) and make a proportion to find out the height of the tree:

$$\frac{\text{height } 1}{\text{shadow length } 0.75} = \frac{N}{40}$$

$$0.75 \times N = 40$$

$$0.75 \overline{)40} \quad 53.3$$

The tree is 53.3 feet tall.

Golden Square Challenge 4:

Finding height by similarity. Choose something that is casting a shadow in your yard or the school yard. (For example, a flag pole, tree, monument, or building.) It must be something that casts a shadow from the base of the object where you can measure it. get through, etc.) Use the procedure described above to find its height. Show your measurements, create your proportion, and solve rounding your answer to the nearest tenth. Repeat your measurements at another time of day. Do you get the same answer?

★ Write a paragraph explaining how you completed this challenge and how this challenge relates to Ratios, Rate, and Proportions.

5. Amazing problem solving:

This last challenge requires you to solve difficult word problems that require several groups of steps. Read the problem carefully, set up ratios as you need them, label answers as you find them, and reread when you finish making sure you have answered the question asked.

Golden Square Challenge 5: Amazing Problem Solving

Solve the two problems below on a separate piece of paper. They require that you organize your work and keep track of what you discover. You may want to make drawings to help you visualize what the problem is saying. Be *certain* that you list all the steps you take and explain your thinking as you work. Remember whenever you use your calculator, you must write the equations. *Label* your final answers.

★ **If you do this outside of the classroom, recopy your work neatly before turning it in. Only problems that show steps with explanations will earn any credit. See Content-Concept Rubric.**

A. The Creature Problem

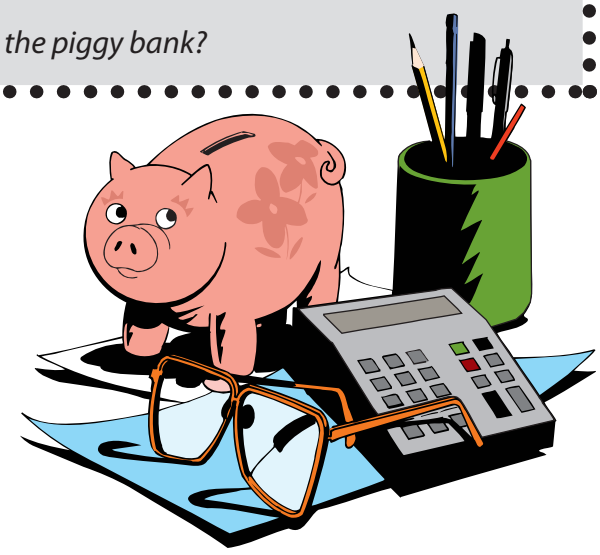
The ratio of creatures with bumpy skin to those with smooth skin is 4 to 1. The ratio of the smooth skin creatures with purple ears to smooth skin creatures with orange ears is 7–3.

If there are 54 creatures with smooth skin and orange ears, how many creatures have bumpy skin?

B. The Piggybank Problem

There were coins and bills in the piggy bank.
The ratio of pennies to nickels is 1 : 5.
The ratio of nickels to dimes is 1 to 3.
The ratio of dimes to quarters is 3 to 4.
The ratio of quarters to dollars is 5 to 2.

If you had 3 pennies, how much money was in the piggy bank?



Special Award

Recently we have been working on a special math unit called *Squared Away*. "Squared away" was originally a nautical term used to announce that the sails of a Square Rigger sailing ship were correctly set. The Navy came to use it to describe sailors who completed a task with competency, as in, "He was right squared away!" We have adopted the term to describe students who demonstrate competency in specific content and skills.

Students can only master the concepts and skills of this *Squared Away* unit by learning a great deal about ratios, rates, and proportions. They must know how to express a ratio, determine a rate, set up a proportion and use these to solve word problems. Throughout this unit they must not only calculate the answers, but they also must explain the steps they took and validate their solutions. Their knowledge of ratios, rates, and proportions will become their special tool to solve many word problems not only in math class, but in situations in the real world.

Congratulations to

_____

For Being Squared Away in
Ratios, Rates, and Proportions

Special Award

Recently we have been working on a special math unit called *Squared Away*. "Squared away" was originally a nautical term used to announce that the sails of a Square Rigger sailing ship were correctly set. The Navy came to use it to describe sailors who completed a task with competency, as in, "He was right squared away!" We have adopted the term to describe students who demonstrate competency in specific content and skills.

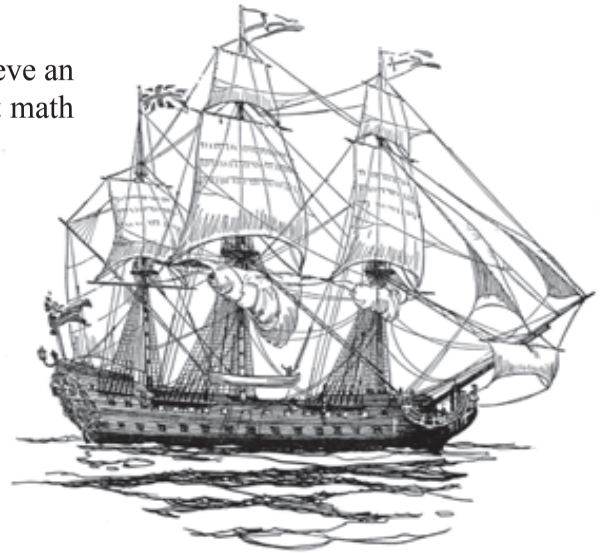
Students can only master the concepts and skills of this *Squared Away* unit by learning a great deal about ratios, rates, and proportions. They must know how to express a ratio, determine a rate, set up a proportion and use these to solve word problems. Throughout this unit they must not only calculate the answers, but they also must explain the steps they took and validate their solutions. Their knowledge of ratios, rates, and proportions will become their special tool to solve many word problems not only in math class, but in situations in the real world.

Only some students also earned a "Golden Square." These students had to achieve an exemplary score on difficult Golden Square Challenges that required excellent math thinking skills and independent work.

Congratulations to

_____

For Being Squared Away in
Ratios, Rates, and Proportions
and Earning a Golden Square!



Teacher Feedback Form

At Interact, we constantly strive to make our units the best they can be. We always appreciate feedback from you—our customer—to facilitate this process. With your input, we can continue to provide high-quality, interactive, and meaningful instructional materials to enhance your curriculum and engage your students. Please take a few moments to complete this feedback form and drop it in the mail. Address it to:

Interact • Attn: Editorial
10200 Jefferson Blvd. • P.O. Box 802
Culver City, CA 90232-0802

or fax it to us at **(800) 944-5432**
or e-mail it to us at **access@teachinteract.com**

We enjoy receiving photos or videos of our units in action!
Please use the release form on the following page.

Your Name: _____

Address: _____

_____

E-mail: _____

Interact Unit: _____

Comments: _____

_____

_____

_____

_____

_____

_____

_____

_____

Release Form for Photographic Images

To Teachers:

To help illustrate to others the experiential activities involved and to promote the use of simulations, we like to get photographs and videos of classes participating in the simulation. Please send photos of students actively engaged so we can publish them in our promotional material. Be aware that we can only use images of students for whom a release form has been submitted.

To Parents:

I give permission for photographs or videos of my child to appear in catalogs of educational materials published by Interact.

Name of Student: _____ (print)

Age of Student: _____ (print)

Parent or Guardian: _____ (print)

Signature: _____ Date: _____

Address:

_____

_____

_____

_____

Phone: _____

Interact
10200 Jefferson Blvd.
Culver City, CA 90232-0802
310-839-2436