

GEOMETRY CHALLENGE

Creative Projects to Reinforce Geometry Concepts

NATE CATTELL graduated from Pennsylvania State University with a BA in graphic design. He later earned his Pennsylvania Elementary Teaching Certification and has taught sixth grade at the State College Area School District since 1971. In 1991, Nate was the Pennsylvania teacher chosen to receive the Presidential Award for Excellence in Mathematics Teaching. He delights in creating new challenge activities for his students. For Interact he wrote CHALLENGE MATH PROJECTS and devised a peace strand for GALAXY.

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Project Editors: Kristen Muir and Sharon Keenan
Graphics Editor: Steve Roberts
Managing Editor: Stacy Yount

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The nationwide movement for high standards has not only determined what students should learn, but also has mandated that students demonstrate what they know. GEOMETRY CHALLENGE addresses numerous mathematics and geometry standards as established by the National Council of Teachers of Mathematics (NCTM), as well as cooperative and applied learning standards. These projects encompass aspects of the NCTM 2000 standards that may not be addressed sufficiently by traditional math texts or assignments.

Mathematics Standards

Number and Operations Standard

- Compute fluently and make reasonable estimates

Geometry Standard

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement Standard

- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements

Problem-Solving Standard

- Build new mathematical knowledge through problem solving
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof Standard

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Monitor and reflect on the process of mathematical problem solving

Communication Standard

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

STANDARDS

Connections Standard

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole

Representation Standard

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems

California Applied Learning Standards

Standard 1: Students will understand how to solve problems through a project design process. Students will design a product, service, or system to meet an identified need.

Standard 2: Students will understand how to solve problems through planning and organization.

Standard 4: Students will understand how to solve problems through meeting client needs. Students will conduct a commissioned project.

Standard 6: Students will understand how to apply communication skills and techniques. Students will demonstrate ability to communicate orally and in writing.

Standard 8: Students will understand the importance of teamwork. Students will work in teams to achieve objectives.

TABLE OF CONTENTS

Purpose	1
Overview	2
Setup Directions	3
Assessment	7
Teaching Directions and Challenges	
Points, Rays, and Segments Teaching Directions	9
POINTS, RAYS, AND SEGMENTS CHALLENGE	10
Points, Rays, and Segments Concepts	11
Points, Rays, and Segments Challenge Key	12
 Planes Teaching Directions	13
PLANES CHALLENGE	14
Planes Concepts	15
Planes Challenge Key	16
 Triangle Teaching Directions	17
TRIANGLE CHALLENGE	18
Triangle Concepts	21
Triangle Challenge Key	22
 Polygon Puzzler Teaching Directions	23
POLYGON PUZZLER CHALLENGE	24
Polygon Puzzler Concepts	25
Polygon Puzzler Challenge Key	26
 Circle Teaching Directions	27
CIRCLE CHALLENGE	29
Circle Concepts	30
Circle Challenge Key	31
 Triangle Thinking Teaching Directions	32
TRIANGLE THINKING CHALLENGE	33
 Math Thinking—Triangles/Angles Teaching Directions	34
MATH THINKING—TRIANGLES/ANGLES CHALLENGE	35
Math Thinking—Triangles/Angles Challenge Key	36
 Block Letter Y Teaching Directions	37
BLOCK LETTER Y CHALLENGE	38
Block Letter Y Concepts	39
Block Letter Y Challenge Key	40

STENTS
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TABLE OF CONTENTS

STUDENT CONTENTS

Four Squares Teaching Directions	41
FOUR SQUARES CHALLENGE	42
Four Squares Concepts.	44
Four Squares Challenge Key	45
Angle Measure Teaching Directions	46
Angle Measure Diagram	47
ANGLE MEASURE CHALLENGE	48
Angle Measure Concepts	50
Angle Measure Challenge Key.	51
Mystery Path Teaching Directions	52
MYSTERY PATH CHALLENGE	54
Mystery Path Concepts	56
Mystery Path Challenge Key	57
Triangle Trail Teaching Directions	58
TRIANGLE TRAIL CHALLENGE	59
Triangle Trail Challenge Key	60
Jewelry Box Teaching Directions	61
JEWELRY BOX CHALLENGE	62
Jewelry Box Concepts.	63
Jewelry Box Challenge Key	64
Two Pyramids Teaching Directions	65
TWO PYRAMIDS CHALLENGE	66
Two Pyramids Concepts.	67
Two Pyramids Challenge Key	68
Rectangular Fold-up Teaching Directions	69
RECTANGULAR FOLD-UP CHALLENGE	70
Rectangular Fold-up Challenge Key.	71
Student-created Angle Teaching Directions	72
STUDENT-CREATED ANGLE CHALLENGE	73
Sample Student-created Angle Challenge	74
Sample Student-created Angle Challenge Key.	75

Students appreciate a good challenge. They gain self-esteem and confidence as they tackle difficult tasks and succeed. GEOMETRY CHALLENGE projects encourage students to employ logical thinking and common sense as they solve the varied and difficult problems. Each activity is designed to extend the student's problem-solving ability.

Completion of a GEOMETRY CHALLENGE task demonstrates thorough knowledge of specific geometry terms and concepts. Students apply basic geometry concepts to create a final product, either a model, a diagram, or a drawing. Mirror images, reversed drawings, and creative folding solutions emphasize clearly that many math problems have multiple solutions/interpretations.

GEOMETRY CHALLENGE is an excellent supplement to any math program. Students will strengthen their geometry understanding while increasing skill, confidence, and creativity. Specifically your students will gain the following:

Knowledge

- Understand that some problem may have more than one solution
- Apply geometry concepts in a broader context
- Solve related types of problems
- Devise creative and innovative two- and three-dimensional diagrams and models

Skills

- Apply visualization, measurement, and estimation to construct a model
- Attack a wide variety of projects while applying geometry knowledge
- Create challenging materials for use by others
- Learn to apply concepts to new problems
- Tackle a problem in a variety of ways
- Work cooperatively with other students to develop problem-solving strategies

Attitudes

- Confidence to use his/her own approach to create a solution
- Appreciation of the value of collaborative working relationships
- Willingness to take risks based on knowledge, estimation, math concepts, and visualization
- Gain an appreciation for alternate math solutions
- Delight in seeing how others have solved a problem
- Increased knowledge and confidence as they share insights and methods with others

ESOPURPOSE

GEOMETRY CHALLENGE is not a traditional simulation or interaction unit. Each project is discrete and independent. As students progress in their geometry understanding the teacher can select more difficult tasks that extend or reinforce concepts learned.

Students work alone, with a partner, and, on occasion, in teams. They use basic materials for all of the challenges and they finish the projects outside of class. The activities are designed to stretch your students' problem-solving skills. Many of the challenges have more than one correct solution. Solutions will differ because of each individual's interpretations as well as his/her approach to solving the problem. Students thus gain an appreciation for alternate math solutions and realize that there are many ways to approach and solve a problem.

The format of each challenge forces students to read directions and information carefully and then apply those facts appropriately to create a model or diagram. The students solve two-dimensional challenges on rays, polygons, triangles, angles, rectangles, and circles by making line constructions. They also solve three-dimensional challenges on planes, the letter "Y," a rectangular prism, and a double pyramid.

Differentiation

Like all Interact units, GEOMETRY CHALLENGE provides differentiated instruction through its various learning opportunities. Students learn and experience the knowledge, skills, and attitudes through all domains of language (reading, writing, speaking, and listening). Adjust the level of difficulty as best fits your students. Assist special needs students in selecting activities that utilize their strengths and allow them to succeed. Work together with the Resource Specialist teacher, Gifted and Talented teacher, or other specialist to coordinate instruction. Every activity causes students to use higher level thinking skills. Knowledge, application, analysis, evaluation, and synthesis are all needed to work through these challenges. The unit incorporates Howard Gardner's Multiple Intelligences:

Visual-Spatial — drawing and creating verbal/physical imagery

Bodily-Kinesthetic — hands-on creations

Interpersonal — interacting and planning with others

Intrapersonal — setting own goals, independent study, introspection, and use of creative materials

Linguistic — using words effectively in writing instructions and evaluations

Logical-Mathematical — reasoning, calculating, thinking conceptually and abstractly, and seeing and exploring patterns and relationships

1. Before you Begin

Read this entire Teacher Guide. Decide how you will use GEOMETRY CHALLENGE in your classroom. (See **Setup Directions #4, Lesson Planning Guide** for ideas and an explanation of the Challenge procedures.) Throughout the Teacher Guide, Interact employs certain editorial conventions to identify materials.

- In preparing materials, *Class set* means *one per student*.
- All transparency masters and student handouts are listed by name using ALL CAPITAL LETTERS.
- Teacher reference pages are named in **Bold**.

2. Timing Options

Decide how you will use the projects—either weekly, throughout the term, throughout the school year, or after lessons on specific geometric concepts.

- The time required for each challenge varies from a normal *class period* (45 minutes to one hour) up to *two hours*.
- Time suggested is listed on each challenge.

3. Grouping Students

Decide ahead of time if the challenge will be an individual, partner, or team task.

- Some challenges indicate the type of student grouping; you decide the grouping for unmarked challenges.
- Partner projects allow for student interaction, but each student is responsible for his/her own finished project.

4. Lesson Planning Guide

GEOMETRY CHALLENGE is useful in a variety of classroom situations. Some challenges include extension activities.

- Experience the activities. Work through each challenge yourself to be better able to respond to any questions your students may ask.
- Select the activities most appropriate for your students.
- Modify any activity to meet the needs of your students.

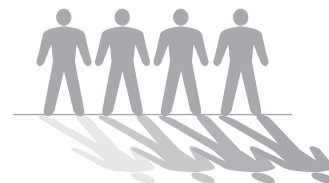
Teaching Directions Pages

The *Teaching Directions* pages include the following:

- Suggestions for student grouping
- Step-by-step procedures for introducing and managing the Challenge
- Information needed to answer many student questions that might arise
- Areas where students might experience trouble



45 minutes–2 hours
per challenge



Individual, Partner, or
Cooperative Groups

SETUP DIRECTIONS



Students must understand concepts before beginning a project.

There may be fewer Concepts pages as you work through the Teacher Guide because once students learn specific geometric concepts it is not necessary to repeat those concepts.



Students must ask questions. Conduct an open class discussion of specific challenge requirements before allowing students to work on any challenge.

- e. Some challenges include *Extensions*. You may assign the Extension after the class completes the challenge. On occasion you may assign the Extension to an individual who has finished a task much more quickly than the others in the class.

Student Pages

Student pages are titled as specific challenges.

- a. Each student page lists the geometric concepts addressed by that challenge.
- b. Each student page includes a description of the challenge.
- c. Each challenge lists all required materials (simple, easy to obtain).

Concepts Pages

- a. A separate **Concepts** page helps the teacher aid the students in succeeding with each difficult challenge by defining the concepts addressed.
- b. You may choose whether students receive the **Concepts** page.
- c. Teach a lesson covering the relevant concepts before asking the students to work on a challenge.

Additional Teacher Resources

- a. Keys are provided as *one example* of a correct solution to each challenge.
- b. Use the key as a resource for instruction or assessment when checking student diagrams/models.
- c. Keys are included for extension activities.

5. Lesson Procedures

Start each activity by having the students ask questions related to the challenge. This gives all students the same information before they begin.

- a. The class discussion will help you understand what the students know and where they might have trouble once they start working.
- b. Class discussions will generate a number of interesting questions that will help students gather the information they need to begin.
- c. You might introduce the lesson by saying, “Please read the instructions. There may be statements that you will not understand. Ask questions that will help everyone have a clearer picture of the nature of the assignment.”

6. Duplication

For each challenge you use, duplicate a master page of instructions for each student or student pair. These pages are listed in ALL CAPITAL LETTERS (see the Table of Contents) and follow the teaching directions.

- Duplicate the **Concepts** page if you choose to use it for the challenge.

7. Materials

Required materials are simple and easy to provide. For each challenge you will need some of the following supplies in the quantity indicated in *Italics*.

- *Calculators — *class set (optional)*
- Centimeter rulers — *class set*
- Compass — *class set*
- Glue — *class set*
- Graph paper — *class set*
- Heavyweight paper (8.5" x 11" or 12" x 18") — *class set (minimum)*
- Protractor — *class set*
- Scissors — *class set*
- Tape (transparent) — *class set*
- Tracing paper — *class set*

— OR —

clean transparency sheets and transparency pens — *class set*

- White paper (lined or unlined) — *class set*

*When appropriate, allow calculators to be a part of the solution.

8. Mock-ups and Models

On the fold-up projects many students will need to work through their ideas using scrap paper (white lined or unlined paper will do) before laying out their final design, cutting, and folding.

- Some students work best using quarter-inch graph paper that helps them maintain parallel lines and draw rectangles easily.
- Students who have trouble figuring out a fold-up pattern should draw and cut out each piece separately, tape them all together, and make a mock-up that way. Then they cut apart the mock-up, lay it flat, and trace around the shapes thereby creating a flat pattern. Do not use this as a method for all students.



Use heavyweight paper the thickness of a file folder (oaktag).

Some students work best using quarter-inch graph paper that helps them maintain parallel lines and draw rectangles easily.

SETUP DIRECTIONS



Suggest that students balance the time spent at home to roughly equal the amount of time spent in class. Although some students will spend more time at home because they want to finish their project, do not expect those who cannot devote that much time to always have a finished project.

After working together during class, by the end of the class period, each student should have an idea of how to proceed with the challenge when he/she gets home.

9. Student Time Commitment

Time spent on each activity varies according to the ability of your students. On occasion, only a few students may discover a solution to the challenge. Your class will include some students who cannot satisfactorily complete each of these tasks. Encourage all students to experiment fully by having a time safeguard.

- a. Agree that students may have a parent sign a note that says, “_____ (name) worked on this challenge at home for _____ (amount of time).”
- b. The student turns in the note and all the work from the experimentation stages of the challenge.
- c. Grade the model pieces fairly and the student will not hesitate to attack the next challenge.
- d. Sometimes allow the class to determine how much time should be spent working on the model at home. Generally, this time should be balanced—work at home not to exceed the amount of time devoted to the challenge at school.
- e. Be aware that some students will spend additional time at home just for the satisfaction of completing the challenge.

10. Cooperative Learning

Always allow discussion while the students are working on a challenge.

1. NCTM Principle Regarding Assessment

Assessment should support the learning of important mathematics concepts and furnish useful information to both the teacher and the students. Assessment should enhance students' learning.

- a. Assessment is a valuable tool for making instructional decisions.
- b. To maximize the instructional value of assessment, teachers need to move beyond a superficial "right or wrong" analysis of tasks and focus on how students are thinking about the tasks. (See **Assessment #3, Writing About the Experience.**)
- c. Scoring guides, or rubrics, help teachers analyze and describe students' responses to complex tasks and determine students' levels of proficiency.

2. Objective Assessment Criteria

GEOMETRY CHALLENGE teaches problem solving in geometry in a way that allows for more than one solution to a challenge.

Activities provide opportunities for students to complete written analysis, develop specific concepts, experience an appreciation for various solutions through reviewing others' work, interact with partner(s), and participate in teacher led class discussions.

- a. Students experiment and apply previous concepts while they gain more skill and confidence related to geometry.
- b. Completing the challenge generates satisfaction, permits you to recognize student accomplishments, and helps students realize that problem solving is a valuable math tool.
- c. You may evaluate the projects using the following criteria:
 - Finished products
 - Measure certain aspects of the finished project for accuracy
 - Grade projects on creative solutions as well as accuracy
 - Accept projects that come close to the final solution
 - Accept one mistake that causes the final project to be incorrect
 - Grade math journals or reflections



When well done, assessment helps teachers make decisions about the content or form of instruction (often called formative assessment) and also helps judge students' attainment (summative assessment).

Activities include varied assessments. Students develop an appreciation for neatness and accuracy.

Students have numerous and varied experiences with problem solving as a method of inquiry and application.

Students model situations using oral, written, concrete, pictorial, graphical, and algebraic methods.

Students visualize and represent geometric figures with special attention to developing spatial sense.

Students estimate and use measurement.

Reproduce and distribute these assessment criteria or post them in your classroom for your students prior to introducing the Challenges.

- Challenge Project Evaluating
 - Is the project complete?
 - Are all aspects of the challenge met?
 - Are the measurements correct?
 - Are diagrams labeled as directed in the instructions?
 - Is the project neat?
 - Are the fold-up models solid? (No holes where you can see inside)
 - Does the project possess any unique qualities?
 - Does the project meet all criteria but present a completely different interpretation?
- Use a rubric for evaluating math journals/reflections

3. **Writing About the Experience**

Your program probably has writing as one assessment tool.

Students could write in their math journals:

- a. Comparing or contrasting their final project to another
- b. Describing the difficulties encountered while working on the project
- c. Detailing the steps used to complete a portion of the project
- d. Developing new writing prompts
- e. Writing challenge instructions for others
- f. Writing about the thinking that enabled them to complete the task

4. **Peer Evaluation**

Before you evaluate a finished project as a class, allow the students to compare results with a partner. This gives students time to evaluate and appreciate the work of others.

- a. Pair the students with someone from the other side of the room, not with a planning partner or table partner.
- b. Students can measure parts of the finished projects for accuracy, and study them for good design qualities or creative solutions.
- c. After grading the fold-up models, allow students to carefully cut along the taped edges and lay them flat and study the original fold-up pattern looking for commonalities and ingenious solutions. Discussion follows.

POINTS, RAYS, AND SEGMENTS

TEACHING DIRECTIONS

POINTS, RAYS, AND SEGMENTS

Materials

- POINTS, RAYS, AND SEGMENTS CHALLENGE — *class set*
- Centimeter rulers — *class set*
- White paper (8.5" x 11") — *class set (turned horizontally)*

Procedure

1. Distribute the POINTS, RAYS, AND SEGMENTS CHALLENGE and conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.
2. Diagram information
 - Construct line XY horizontally in the middle of the paper.
 - Ray XZ can be drawn pointing either “up” or “down.”
 - Segment RS has no specific length, but may need to be made longer in order to construct segment AR.
 - The length of the lines CD and AK have no specific measure.
 - The length of segment XM will indicate where point M is on ray XZ.
 - Recognize that $\overline{MX} \cong \overline{AR} \cong \overline{BY}$. (This information helps students locate point R and also point M.)
 - Ray YR should be drawn at least twice as long as segment YR. The ray YR lays on top of the segment YR.
 - Line HJ intersects segment XP and that intersection creates point E.
3. Based on individual student interpretations and approaches to problem solving, allow these variations in diagrams:
 - a. Diagram may be a mirror image if points X and Y are reversed on the first line drawn.
 - b. The diagram may end up flipped horizontally if ray XZ points “down” instead of “up.”
 - c. The diagram may be flipped vertically if the students label segment RS with “S” at the bottom instead of at the top.
4. Make a transparency of **Points, Rays, and Segments Challenge Key** to use in checking each diagram.



Up to 1 hour



Individuals



Copy and distribute the Concepts page as appropriate. Note: to reduce duplication more than one set of concepts are included on the page.

Some students work best using quarter-inch graph paper that helps them maintain parallel lines.



POINTS, RAYS, AND SEGMENTS CHALLENGE

Read the **Instructions** and use the **Challenge Information** to create a neat well-labeled diagram.

Concepts: point, parallel lines, midpoint, ray, segment, perpendicular, congruent, and intersection



Up to 1 hour

Materials

- Centimeter rulers
- White paper (8.5" x 11")

Instructions

1. Label all points.
2. Use a ruler to make all straight lines.
3. Include arrows on the rays and lines.

Challenge Information

1. Label point P as the midpoint on line segment \overline{XY} that is 14 cm long.
2. Ray \overrightarrow{XZ} is perpendicular to \overline{XY} .
3. Point B is the midpoint of \overline{PY} .
4. Point A is the midpoint of \overline{PB} .
5. \overline{RS} is parallel to \overrightarrow{XZ} intersecting \overline{XY} at A. (Make point S the top of the segment.)
6. \overrightarrow{AK} intersects \overrightarrow{XZ} at M.
7. $\overline{MX} \cong \overline{BY}$.
8. Draw line \overleftrightarrow{CD} parallel to \overrightarrow{AK} intersecting \overline{XY} at X.
9. $\overline{MX} \cong \overline{AR}$.
10. \overrightarrow{YR} intersects \overline{RS} at R.
11. \overline{YO} is twice the length of \overline{XP} . Point O is located on \overrightarrow{YR} .

Answer these questions on your diagram:

1. \overleftrightarrow{HJ} is parallel to \overline{RS} , intersecting \overrightarrow{YR} at O. Does this create a \overline{XE} smaller than \overline{AB} .
2. If so, how much smaller is \overline{XE} than \overline{AB} ?

POINTS, RAYS, AND SEGMENTS CONCEPTS



POINTS, RAYS, AND SEGMENTS CONCEPTS

You must understand these concepts in order to solve the challenge:

- All geometric figures are made up of points.
- We create a point by making a dot and labeling it.
- Two points determine exactly one line.
- Lines extend forever in both directions. \leftrightarrow
- A segment is a part of a line. It has two endpoints. —
- A ray has a starting point (endpoint), but goes on forever in only one direction. \rightarrow
- Lines in a plane that do not intersect are called parallel lines. $=$
- Intersecting lines cross at one point. $+$
- A midpoint is the middle of a segment.
- Same size segments are congruent (equal). \cong
- A line segment may be contained in a ray.
- Two lines or planes that intersect and form 90 degree angles are perpendicular. \perp

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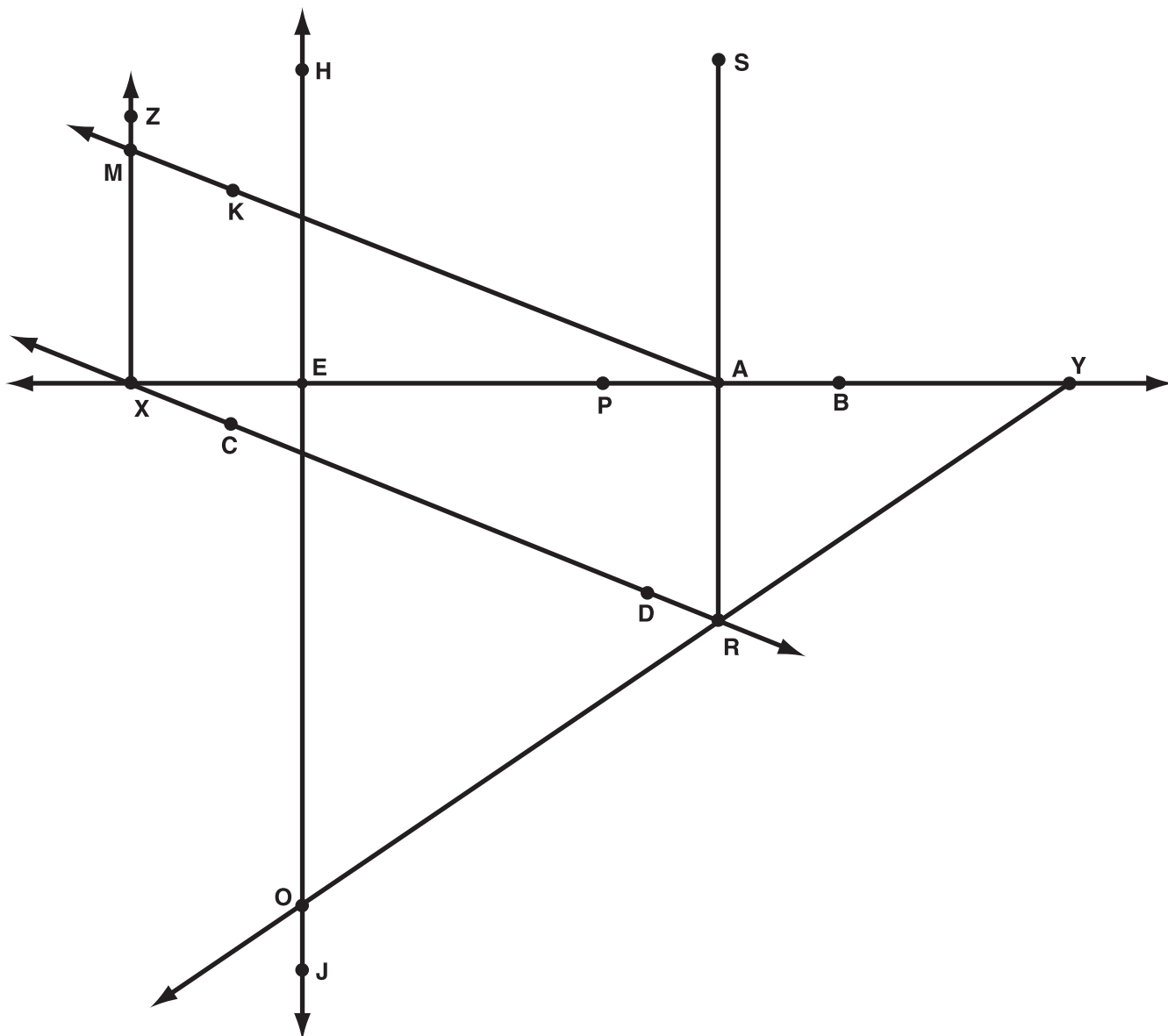
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POINTS, RAYS, AND SEGMENTS CHALLENGE KEY



PLANES

Materials

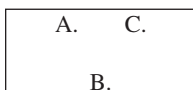
- PLANES CHALLENGE — *class set or one per student pair*
- Centimeter rulers — *class set*
- Heavyweight white paper (8.5" x 11") — *class set (minimum)*
- Scissors — *class set (minimum)*

Procedure

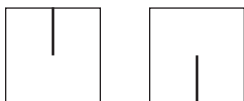
1. Distribute the PLANES CHALLENGE and conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.

2. Construction information

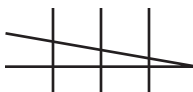
- Each plane is labeled with only three points (e.g., A, B, and C).



- The model should be structurally sound. It can hold its form when picked up and dropped.
- “No fasteners” means don’t use staples, brass fasteners, glue, tape, etc.
- Paper may be folded or used as a fastening device.
- Slits may be cut in the paper.



- The top rated models are the ones that are rigid and maintain parallel planes when they are dropped.
- Example of a set of planes (top view) that creates a rigid model.



3. **Planes Challenge Extension**

Create the whole model from just one piece of paper—only by cutting and folding.



Less than 1 hour



Student Pairs



Copy and distribute the Concepts page as appropriate. Note: to reduce duplication more than one set of concepts are included on the page.



PLANES CHALLENGE

Read the **Instructions** and use the **Challenge Information** to create a paper model of four intersecting planes.

Concepts: parallel planes, planes go on forever, perpendicular planes, three points indicate a plane, acute angles



Less than 1 hour

Materials

- Centimeter ruler
- Heavyweight paper
- Scissors

Instructions

1. Label all planes correctly.
2. You may not use tape.
3. You may not use glue.
4. You may not use any fasteners.
5. Model should be rigid enough to be dropped without having any change in its shape. The planes will remain parallel and retain the original angles.

Challenge Information

1. Plane ABC is parallel to plane DEF.
2. Plane RST is parallel to plane DEF.
3. Plane GHI is perpendicular to plane RST.
4. Plane XYZ cuts (intersects) plane GHI at less than a 45° angle.

PLANES CONCEPTS



PLANES CONCEPTS

You must understand these concepts in order to solve the challenge:

- Any three points in space (not all on the same line) identify a plane.
- A plane extends without limits in all directions.
- Planes that don't intersect are parallel planes.
- Planes that meet at a ninety degree angle are perpendicular planes.
- An acute angle is less than ninety degrees.
- Intersecting planes create a line where one plane cuts through another.

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- A plane extends without limits in all directions.
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- An acute angle is less than ninety degrees.
- Intersecting planes create a line where one plane cuts through another.

PLANES CONCEPTS

You must understand these concepts in order to solve the challenge:

- Any three points in space (not all on the same line) identify a plane.
- A plane extends without limits in all directions.
- Planes that don't intersect are parallel planes.
- Planes that meet at a ninety degree angle are perpendicular planes.
- An acute angle is less than ninety degrees.
- Intersecting planes create a line where one plane cuts through another.

PLANES CONCEPTS

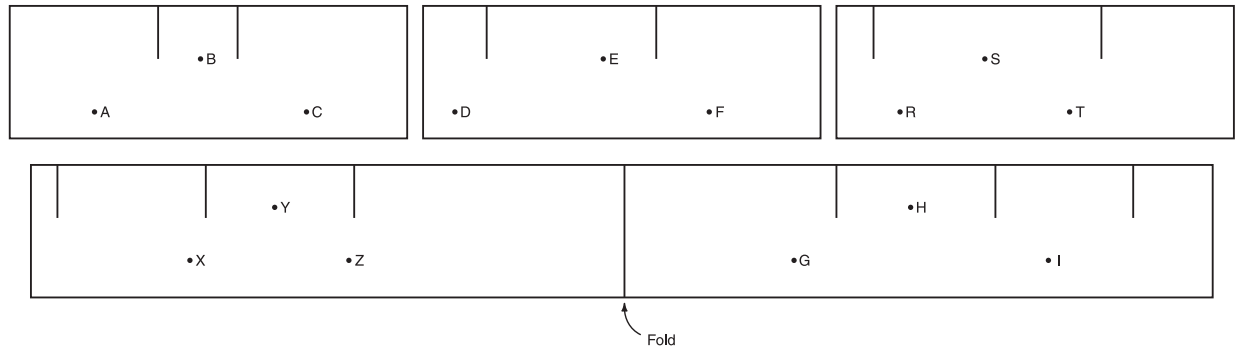
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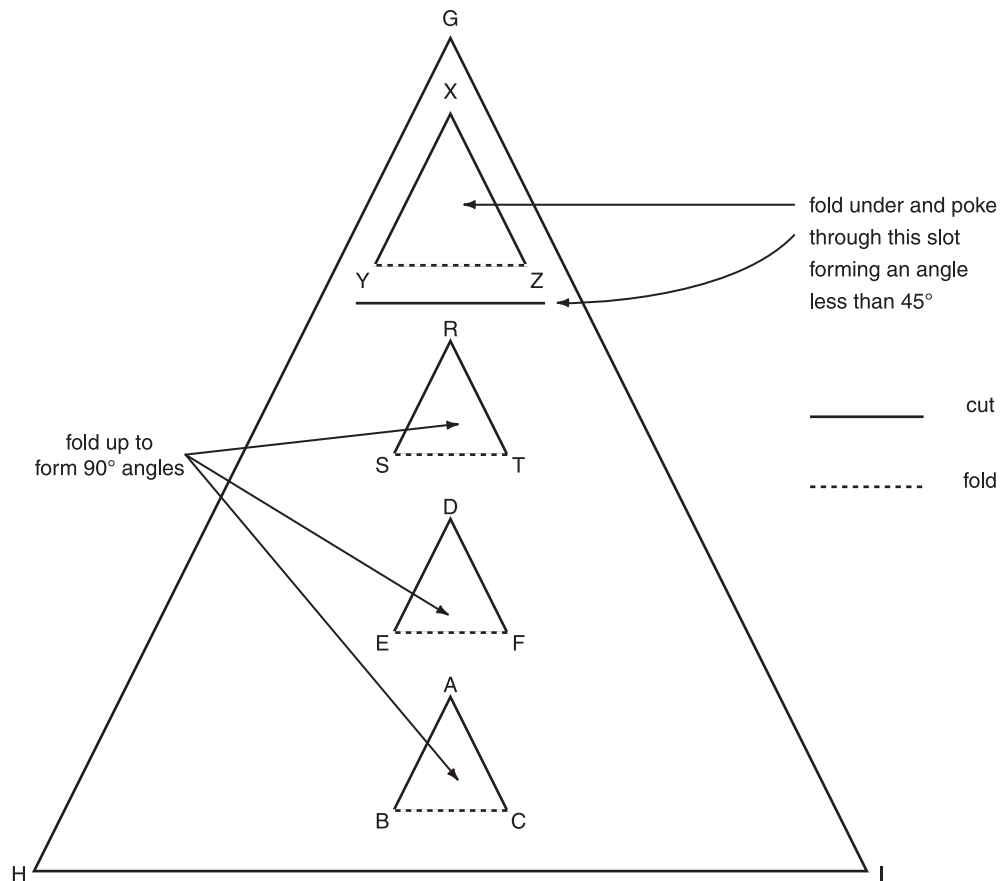


PLANES CHALLENGE KEY

One possible solution for the Planes Challenge.



One possible solution for the Planes Challenge Extension.



TRIANGLE TEACHING DIRECTIONS

TRIANGLE

Materials

- TRIANGLE CHALLENGE 1 or 2 or 3 — *class set*
- Centimeter rulers — *class set*
- White paper (8.5" x 11") — *class set (turned horizontally)*

Challenge Information

1. There are three variations of this challenge.
TRIANGLE CHALLENGE 1 is the easiest.
TRIANGLE CHALLENGE 3 is the most difficult.
2. Decide which version of the puzzle you will give your class.

Procedure

1. Distribute the TRIANGLE CHALLENGE 1 or 2 or 3. Conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.
2. Diagram information
 - Outer edges of the shape are often one side of a given triangle.
 - The largest side of the largest triangle can only fit in one place (the bottom).
 - Every triangle has at least one side that matches the size of another triangle's side. (example: ΔA and ΔF each have one side that measures 6.3 cm).
3. Students may use a variety of strategies to complete their diagrams.
 - a. Students could create the triangles from paper, match up the sides, and place them in the puzzle area.
 - b. Students could construct all of the triangles, cut them out, place them inside the outline, and then trace around the shapes.
 - c. It's all right if some students glue the triangles inside the outline.
4. Make a transparency of **Triangle Challenge Key** to aid your class discussion after the students have completed the challenge.

Additional Notes

Some students will not solve this completely!



2 hours or more



Student Pairs



TRIANGLE CHALLENGE 3 always provides a major challenge for the author's students. Two years in a row they voted to have next year's class attack the puzzle with at least one interior line indicated. The author sometimes gives students the chance to choose which Challenge to tackle and has them work together on their selected version in different parts of the room. (No peeking!)

Copy and distribute the Concepts page as appropriate. Note: to reduce duplication more than one set of concepts are included on the page.

As the class period progresses, encourage students to continue working by indicating to individuals the triangles that are in the correct position.



TRIANGLE CHALLENGE 1

Read the **Instructions** and use the **Challenge Information** to create a neat well-labeled diagram.

Concepts: angles in a triangle

Materials

- Centimeter ruler
- Protractor



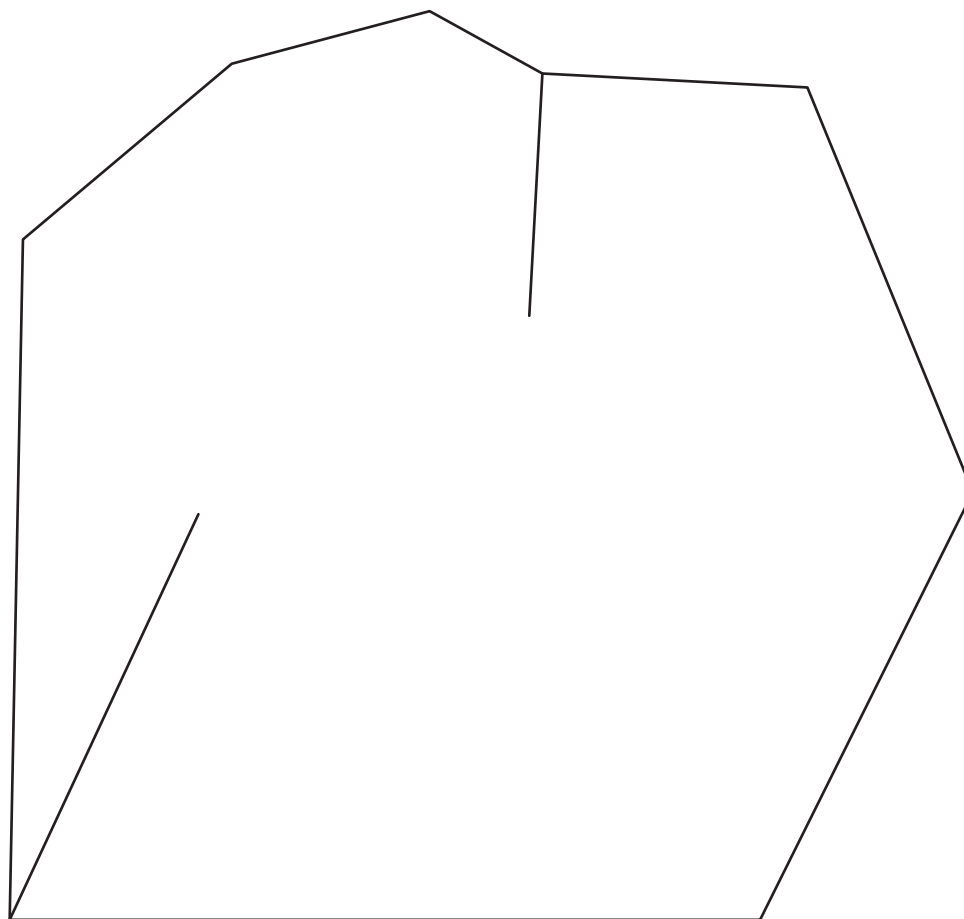
2 hours or more

Instructions

Draw 12 triangles inside the shape.

Challenge Information

ΔA :	angles —	24°	117°	39°	segment lengths —	9.9 cm	13.9 cm	6.3 cm
ΔB :	angles —	33°	24°	123°	segment lengths —	9.0 cm	5.9 cm	4.3 cm
ΔC :	angles —	17°	28°	135°	segment lengths —	2.7 cm	1.7 cm	4.1 cm
ΔD :	angles —	92°	57°	31°	segment lengths —	4.1 cm	4.9 cm	2.5 cm
ΔE :	angles —	34°	21°	125°	segment lengths —	7.4 cm	5.1 cm	3.2 cm
ΔF :	angles —	72°	62°	46°	segment lengths —	6.3 cm	6.8 cm	5.1 cm
ΔG :	angles —	63°	62°	55°	segment lengths —	5.1 cm	4.7 cm	5.1 cm
ΔH :	angles —	61°	45°	74°	segment lengths —	4.7 cm	6.3 cm	5.8 cm
ΔI :	is a right triangle				segment lengths —	3.5 cm	3.2 cm	4.7 cm
ΔJ :	angles —	83°	41°	$___\circ$	segment lengths —	7.1 cm	5.9 cm	4.7 cm
ΔK :	angles —	45°	98°	$___\circ$	segment lengths —	6.0 cm	3.6 cm	4.3 cm
ΔL :	angles —	33°	24°	123°	segment lengths —	3.5 cm	4.9 cm	7.4 cm



TRIANGLE CHALLENGE 2



Read the **Instructions** and use the **Challenge Information** to create a neat well-labeled diagram.

Concepts: angles in a triangle

Materials

- Centimeter ruler
- Protractor



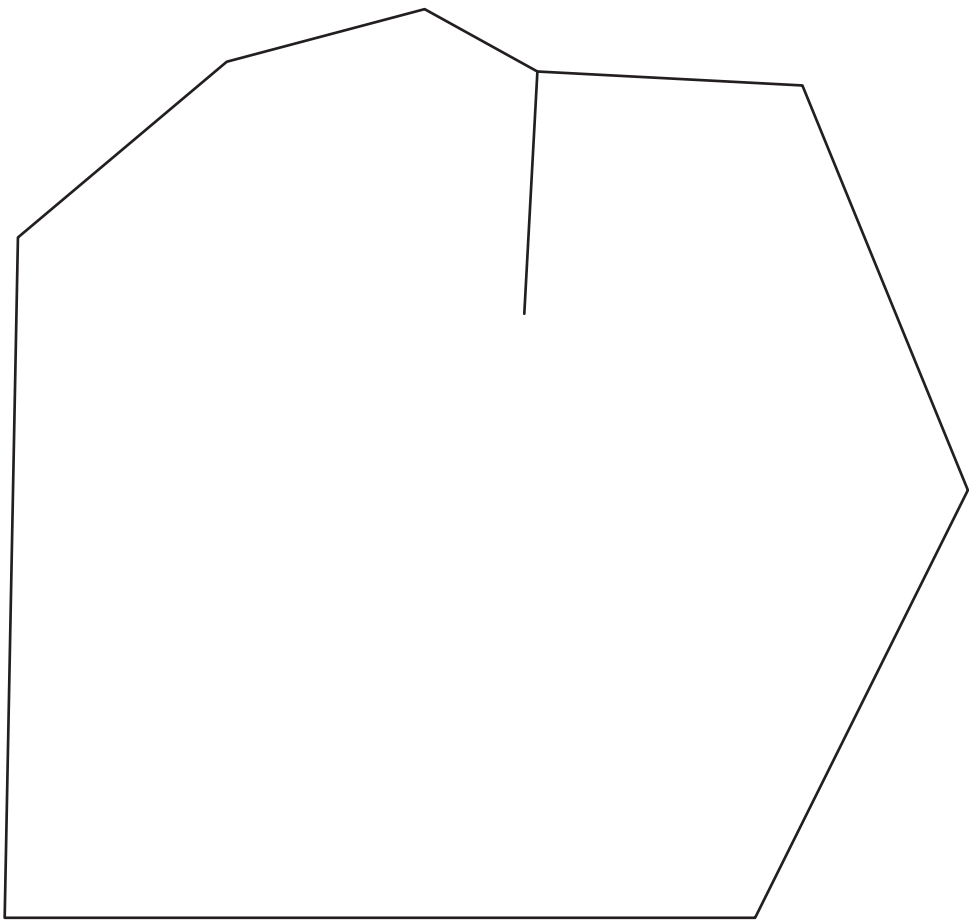
2 hours or more

Instructions

Draw 12 triangles inside the shape.

Challenge Information

ΔA :	angles —	24°	117°	39°	segment lengths —	9.9 cm	13.9 cm	6.3 cm
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ΔK :	angles —	45°	98°	___°	segment lengths —	6.0 cm	3.6 cm	4.3 cm
ΔL :	angles —	33°	24°	123°	segment lengths —	3.5 cm	4.9 cm	7.4 cm





TRIANGLE CHALLENGE 3

Read the **Instructions** and use the **Challenge Information** to create a neat well-labeled diagram.

Concepts: angles in a triangle

Materials

- Centimeter ruler
- Protractor



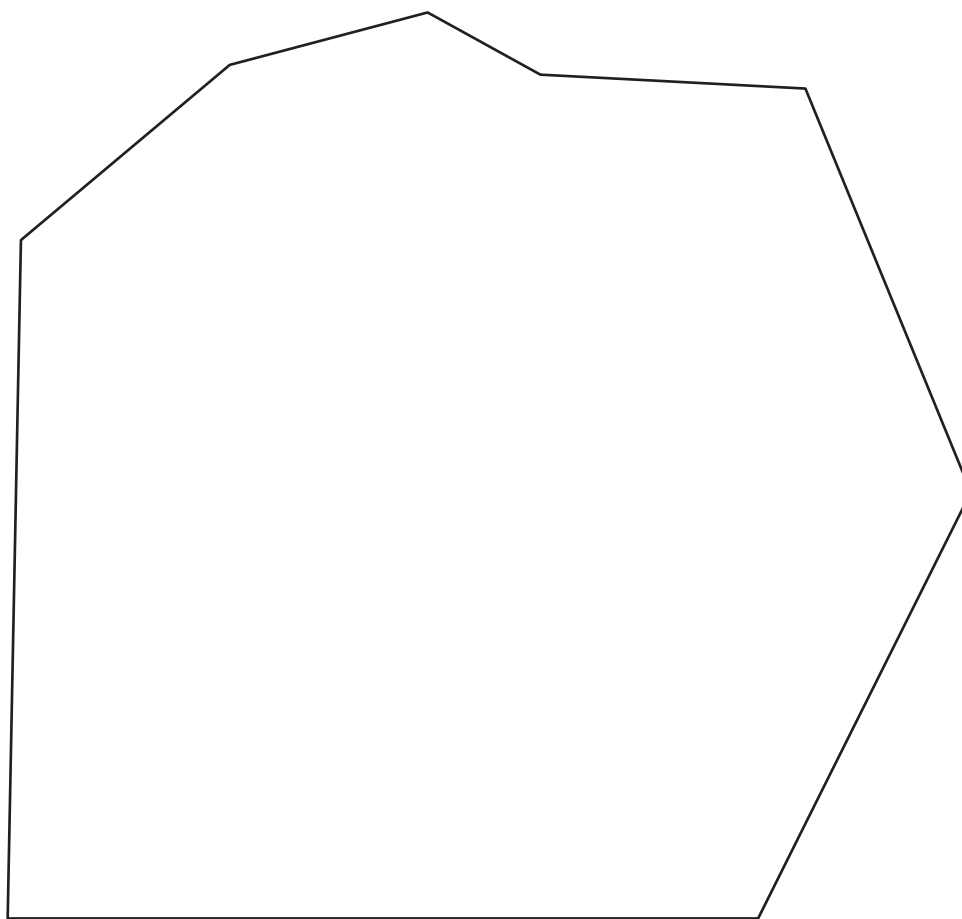
2 hours or more

Instructions

Draw 12 triangles inside the shape.

Challenge Information

ΔA :	angles —	24°	117°	39°	segment lengths —	9.9 cm	13.9 cm	6.3 cm
ΔB :	angles —	33°	24°	123°	segment lengths —	9.0 cm	5.9 cm	4.3 cm
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ΔJ :	angles —	83°	41°	$\underline{\hspace{1cm}}^\circ$	segment lengths —	7.1 cm	5.9 cm	4.7 cm
ΔK :	angles —	45°	98°	$\underline{\hspace{1cm}}^\circ$	segment lengths —	6.0 cm	3.6 cm	4.3 cm
ΔL :	angles —	33°	24°	123°	segment lengths —	3.5 cm	4.9 cm	7.4 cm



TRIANGLE CONCEPTS



TRIANGLE CONCEPTS

You must understand these concepts in order to solve the challenge:

- The angle measure of all triangles equals 180° .
- If you know the angle measure of two angles of a triangle you can figure out the third angle by adding the two and then subtracting that total from 180° .
- Different triangles may share the same size sides.

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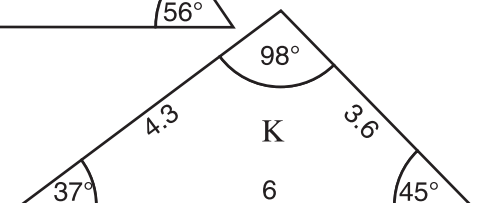
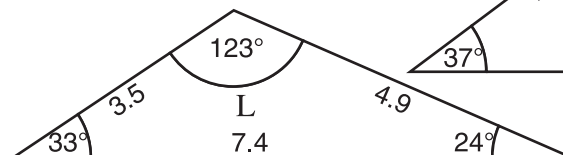
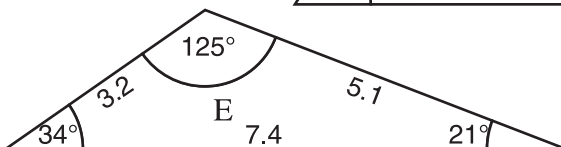
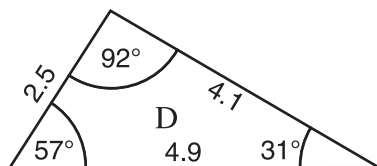
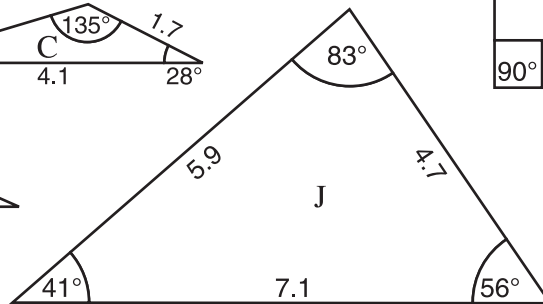
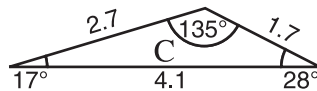
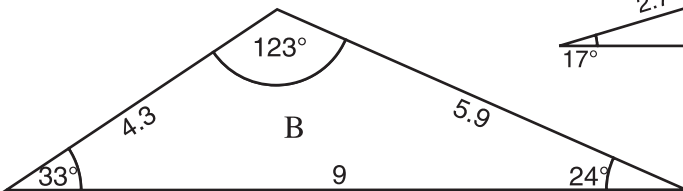
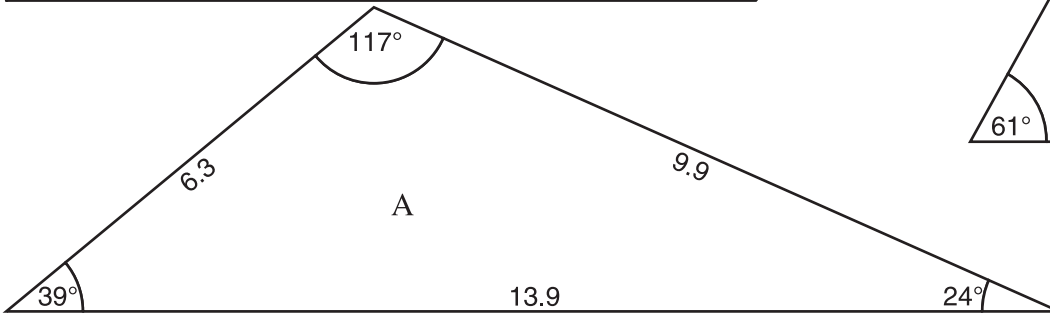
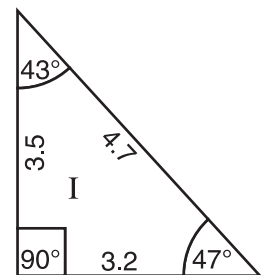
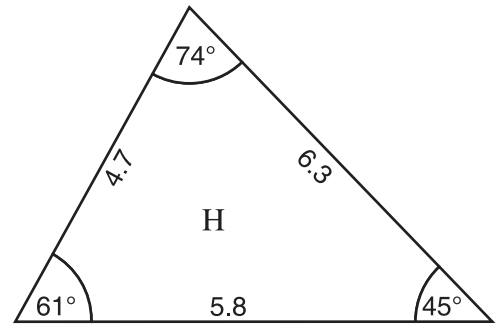
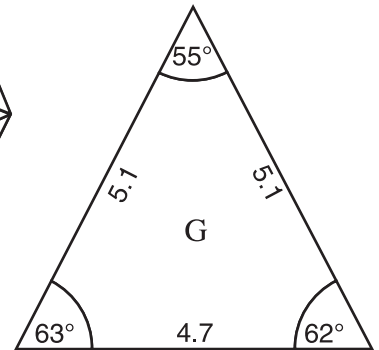
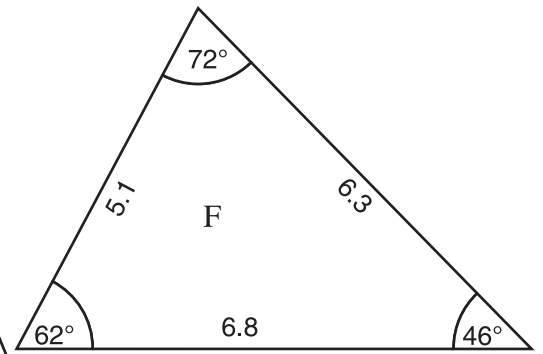
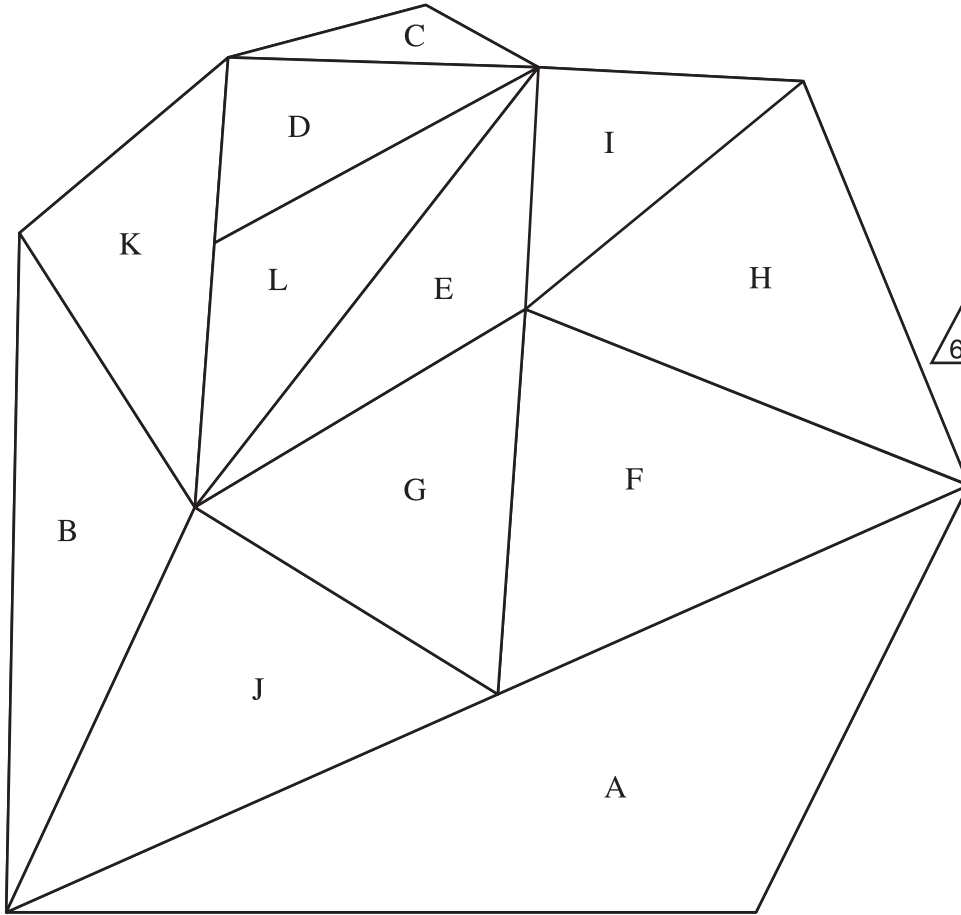
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- Different triangles may share the same size sides.



TRIANGLE CHALLENGE KEY



POLYGON PUZZLER

TEACHING DIRECTIONS

POLYGON PUZZLER

Materials

- POLYGON PUZZLER CHALLENGE — *class set*
- Centimeter rulers — *class set*
- Protractor — *class set*

Procedure

1. Distribute the POLYGON PUZZLER CHALLENGE and conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.
2. Diagram information
 - Depending on how they start diagonal segment AB, their finished construction may be flatter or taller than the construction sample provided. (See **Polygon Puzzler Challenge Key**).
 - Use the top of the letter “A” as the starting point.
 - Students need to know that shapes share the same segments.
 - Students need to know definitions of various geometric shapes.
 - Students most often reverse right angle LKM. The project can still be completed even if they make that mistake. Final assessment of the challenge is the measuring of the three angles in triangle AOB.
 - To correctly answer #11 on the POLYGON PUZZLER CHALLENGE, students need to have the three angles equal 180° .
 - When assessing, use a protractor to measure the acute and obtuse angles of triangle AOB. Students may have different angle measures for triangle AOB, but will have completed the work and should get full credit if the three angles equal 180° .
3. Make a transparency of **Polygon Puzzler Challenge Key** to aid in your class discussion following students’ completion of the challenge.



1 hour



Individuals



Copy and distribute the Concepts page as appropriate. Note: to reduce duplication more than one set of concepts are included on the page.



POLYGON PUZZLER CHALLENGE

Read the **Instructions** and use the **Challenge Information** to create a neat and well-labeled diagram.

Concepts: angle measure, shapes can share segments, shapes have special names, proper labeling of constructions



1 hour

Materials

- Centimeter ruler
- Protractor

Instructions

1. Label all constructions.
2. Use a ruler.
3. Answer #11 on this paper.

Challenge Information

1. Construct parallelogram ABCD starting at "A." \overline{AB} is a diagonal line that slants up to the right and is 2.2 cm long. \overline{BC} is 2.4 cm long and runs parallel to the bottom of the paper. $\angle ABC = 150^\circ$.
2. Construct trapezoid DCEF with \overline{DF} parallel to the bottom of the paper. $CE \cong BC$. \overline{DF} is 5 cm long and runs parallel to the bottom of the paper.
3. Construct rhombus EFGH. $\angle HEF = 70^\circ$.
4. Construct square EHIJ.
5. Construct rectangle JILK. \overline{LI} is twice as long as \overline{JI} .
6. $\triangle LKM$ is a right triangle. $\overline{MK} \cong \overline{BC}$.
7. Construct rectangle MKON. $\overline{OK} \cong \overline{LI}$.
8. Construct square NOQP.
9. Draw \overline{OB} . Draw \overline{OA} .
10. Measure all the angles of $\triangle OBA$.
11. $\angle OBA =$ $\angle BAO =$ $\angle AOB =$

•
A



POLYGON PUZZLER CONCEPTS

You must understand these concepts in order to solve the challenge:

- Shapes can share sides.
- Shapes have special names.
- Parallelogram—opposite sides parallel and congruent
- Trapezoid—two sides only are parallel
- Rhombus—all sides congruent (example: diamond)
- Label shapes using letters in the consecutive order printed on the information sheet.

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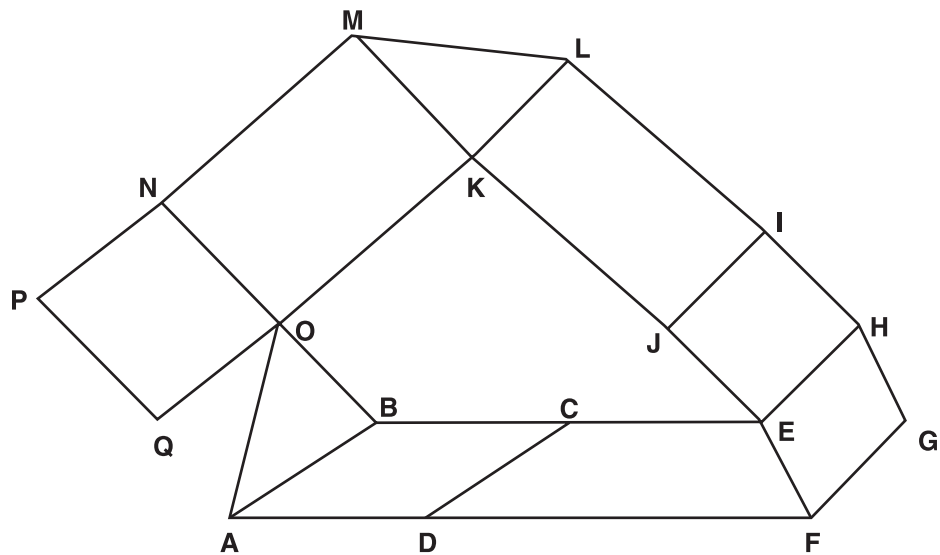
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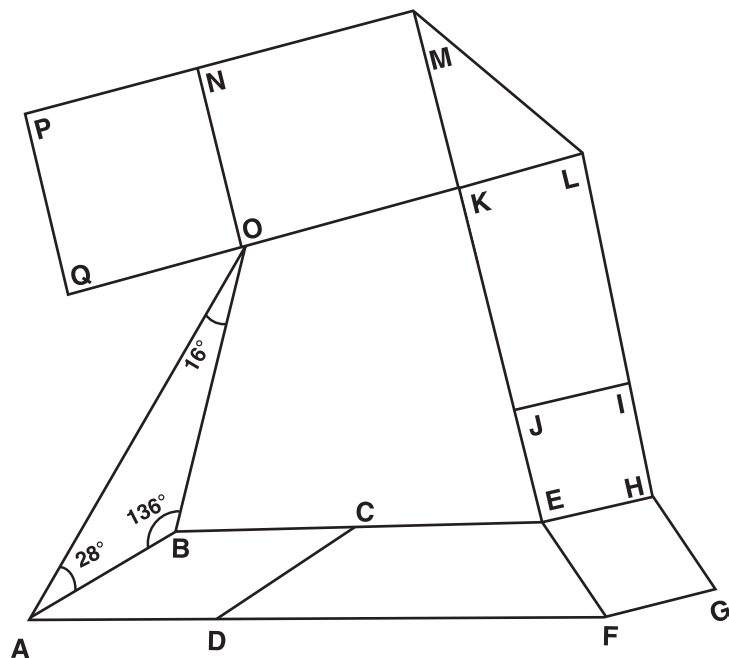
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POLYGON PUZZLER CHALLENGE KEY



Sample of student work. This shows how a finished diagram may not match the key, but is still correct if $\triangle AOB$ angles are measured correctly.



CIRCLE

Materials

- CIRCLE CHALLENGE — *class set*
- Centimeter rulers — *class set*
- Compass — *class set*
- White paper (8.5" x 11") — *class set (turned horizontally)*

Procedure

1. Teach the parts of a circle first: center point, diameter, radius, chord, and circumference.
2. Discuss inscribing shapes in circles: demonstrate if necessary.
3. Distribute CIRCLE CHALLENGE and conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.
4. Diagram information
 - Point "A" may be near either end of line AB (right side or left side of the paper).
 - Line AB should be drawn in the middle of the paper orientated horizontally.
 - All circles intersect at point "A."
 - On scrap paper inscribe a triangle in circle "O" by first constructing triangle DEF. You then must move the compass point (and change the size of the circle being drawn around the triangle) until the circle touches all three vertices of the triangle. Construct circle "O" around triangle DEF.
 - To make circle "O" intersect point "A" just put the scrap paper (with the triangle inscribed inside the circle) under the white paper and trace them so that circle "O" goes through point "A" on line AB. Circle "O's" center is also on line AB.
 - The right angle drawn in circle "Q" just touches the center of the circle and the rays from that angle may be oriented in any direction.
 - Construct a 6 cm x 6 cm square on the scrap paper and inscribe it in circle "R." Then, without changing the opening of the compass, draw circle "R" through point "A."
5. Make a transparency of **Circles Challenge Key** to aid your class discussion after they have completed the challenge.



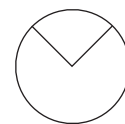
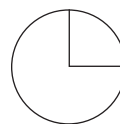
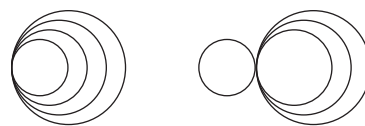
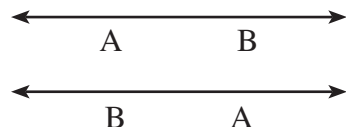
1 hour



Individuals



Copy and distribute the Concepts page as appropriate. Note: to reduce duplication more than one set of concepts are included on the page.



CIRCLE TEACHING DIRECTIONS

Additional Notes

1. Most students will want to construct the square inside circle “R” on the diagram, but the instructions never state that they have to, so no points should be lost if the finished diagram does not contain a square.
2. You may choose to skip giving students the concept sheet and make them use a dictionary or glossary to gather information before they start the project.
3. If a mistake is made on one of the early circles the larger circles will be disproportionate even if the student has done the rest of the work correctly. You will need to adjust your assessment to give credit for circles that are correctly generated based on the incorrect measurements they get from the previous circles. It is easy to see which diagrams have this problem. Usually, if the fifth circle (S) ends up being too large or small it’s a result of an earlier mistake where another circle is the incorrect size.

CIRCLE CHALLENGE



Read the **Instructions** and use the **Challenge Information** to create a neat well-labeled diagram.

Concepts: circles have centers, inscribing shapes in a circle, radius (radii), diameter, circumference, intersection, chord, and vertex



1 hour

Materials

- Centimeter ruler
- Compass
- White paper (8.5" x 11")

Instructions

1. Turn paper horizontally.
2. Construct line AB in the middle of the paper and parallel to the bottom of the page.
3. Label the centers of the circles from the smallest to the largest—"O, P, Q, R, S."
4. Write the answer to #7 on the diagram paper.

Challenge Information

1. You must construct five circles.
All circles intersect at point "A."
Each circle's center is located on line AB.
2. Triangle DEF is inscribed in the smallest circle (O).
 \overline{DE} is 3.5 cm
 \overline{EF} is 3.3 cm
 \overline{FD} is 2 cm
3. The second circle (P) has a radius .7 times the diameter of the smallest circle (O).
4. The third circle (Q) has a radius equal to the chord that is created by two radii of the second circle that form a right angle.
5. The fourth circle (R) has a circumference that allows a 6 cm square to be perfectly inscribed.
6. The fifth circle's center is located as far away from "A" as one half of the total measure of the radii of each of the first four circles. Label the fifth circle (S).
7. What is the length of \overline{PR} ?



CIRCLE CONCEPTS

CIRCLE CONCEPTS

You must understand these concepts in order to solve the challenge.

- The center of any circle may be labeled at that point by a letter.
- A shape is inscribed in a circle if all of its vertices touch the circumference of the circle.
- A diameter is a line segment from any point on the circle to any other point on the circle that passes through the center point of the circle.
- A radius is a segment from the center of a circle to a point on the circle.
- Radii is the plural of radius.
- A circumference is the measure around the edge of the circle.
- Intersection is where lines meet.
- A chord is a straight line touching the edge of the circle at two points, but not running through the center of the circle.
- A circle can always be constructed (drawn) to touch the three vertices (angles) of any triangle.
- A vertex is the angle created by two rays originating from one point. (the plural of vertex is vertices)

center



diameter



radius



radii



chord



CIRCLE CONCEPTS

You must understand these concepts in order to solve the challenge.

- The center of any circle may be labeled at that point by a letter.
- A shape is inscribed in a circle if all of its vertices touch the circumference of the circle.
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center



diameter



radius



radii



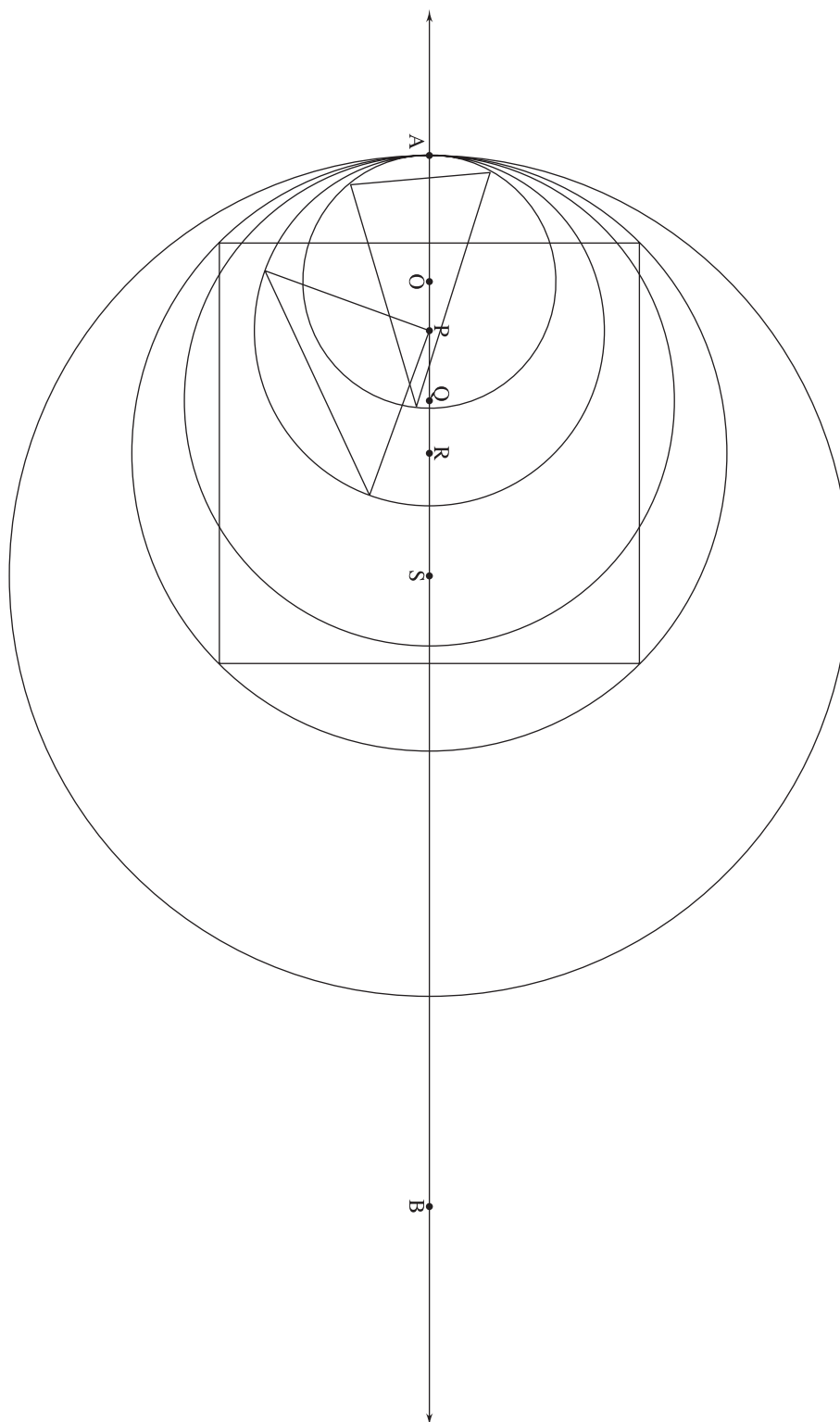
chord



CIRCLE CHALLENGE KEY



Line segment PR equals 1.75 cm.



TRIANGLE THINKING

TEACHING DIRECTIONS



Less than 1 hour



Individuals

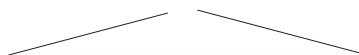
TRIANGLE THINKING

Materials

- TRIANGLE THINKING CHALLENGE — *class set*
- Centimeter rulers — *class set*

Procedure

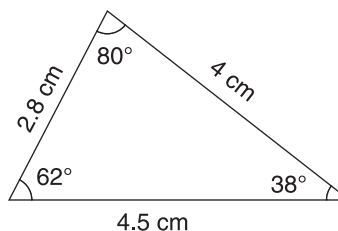
1. Decide ahead of time how you will use this challenge. It can be used in the following ways:
 - a. An introduction to triangles
 - b. A starting point for discussion
 - c. A challenge activity
 - d. An assessment tool
2. Distribute the TRIANGLE THINKING CHALLENGE and conduct a short class discussion of the **Instructions**. Elicit questions and conclusions from the students.
3. Diagram information
 - #1 *cannot* be constructed. The two short sides do not equal the longest side. Students should end up with a triangle shape that is not a closed line.



- #2 *cannot* be constructed. The two short sides equal the longest side. Students should end up with a triangle shape that is not a closed line or a line segment where the two shorter line segments are laying on the long segment.



- Only *one* triangle can be constructed. The shape may be flipped or rotated, but when cut out the triangles are identical, unless a mistake is made in construction.



TRIANGLE THINKING CHALLENGE



Read the **Instructions** to create neat well-labeled diagrams.

Materials

- Centimeter ruler
- Protractor



Less than 1 hour

Instructions

1. Construct a triangle using the information given.
Write statements showing what new math understandings you have gained.
Triangle ABC:
 $\overline{AB} = 12 \text{ cm}$
 $\overline{AC} = 5.5 \text{ cm}$
 $\overline{BC} = 5.9 \text{ cm}$
2. Construct a triangle using these three segments.
3 cm, 7.3 cm, 4.3 cm
Write statements showing what new math understandings you have gained.
3. Construct all the triangles you can, using the angles and segments given.
Use segments measuring 4 cm, 2.8 cm, 4.5 cm.
Use angles measuring 80° , 62° , and 38° .
Write statements showing what new math understandings you have gained.

MATH THINKING—TRIANGLES/ANGLES

TEACHING DIRECTIONS



About 1 hour



Individuals



The author didn't let his students construct a rectangle or a square unless they could prove to him that they had constructed perfect 90° angles (corner of paper folded or corner of their ruler traced).

MATH THINKING—TRIANGLES/ANGLES

Materials

- MATH THINKING CHALLENGE — *class set*
- Centimeter rulers — *class set*

Procedure

1. Distribute the MATH THINKING CHALLENGE and conduct a short class discussion of the **Instructions**. Elicit questions and conclusions from the students.
2. Diagram information
 - This challenge is designed to help students think creatively.
 - Students need to know: angle measure, rays, angle sum.
 - This is a simple challenge activity that allows math thinkers to come up with brilliant solutions to common problems. We take it for granted that we use a protractor or a compass to help us construct specific angles. Some common angles may be constructed using the most basic tools.
3. Make a transparency of **Math Thinking Challenge Key** to aid your class discussion after they have completed the challenge.
4. **Math Thinking Challenge Extension**

Photocopy some interesting student diagrams and give them to your class. Instruct students to study the solutions and write reflections describing the thought process of the student who solved that specific challenge. This follow-up activity is also well suited to partner or small group discussions.

MATH THINKING—TRIANGLES/ANGLES CHALLENGE



Read the **Instructions** to create neat well-labeled diagrams.

Materials

- Centimeter ruler



About 1 hour

Instructions

1. Construct the angles listed below.
You may not use a compass or a protractor.
Use only this paper, a ruler, a pencil, and your math sense!
2. **Label all constructions!**
3. **Include arrows on all rays.**

Construct a 180° angle.

Construct a 90° angle.

Construct a 45° angle.

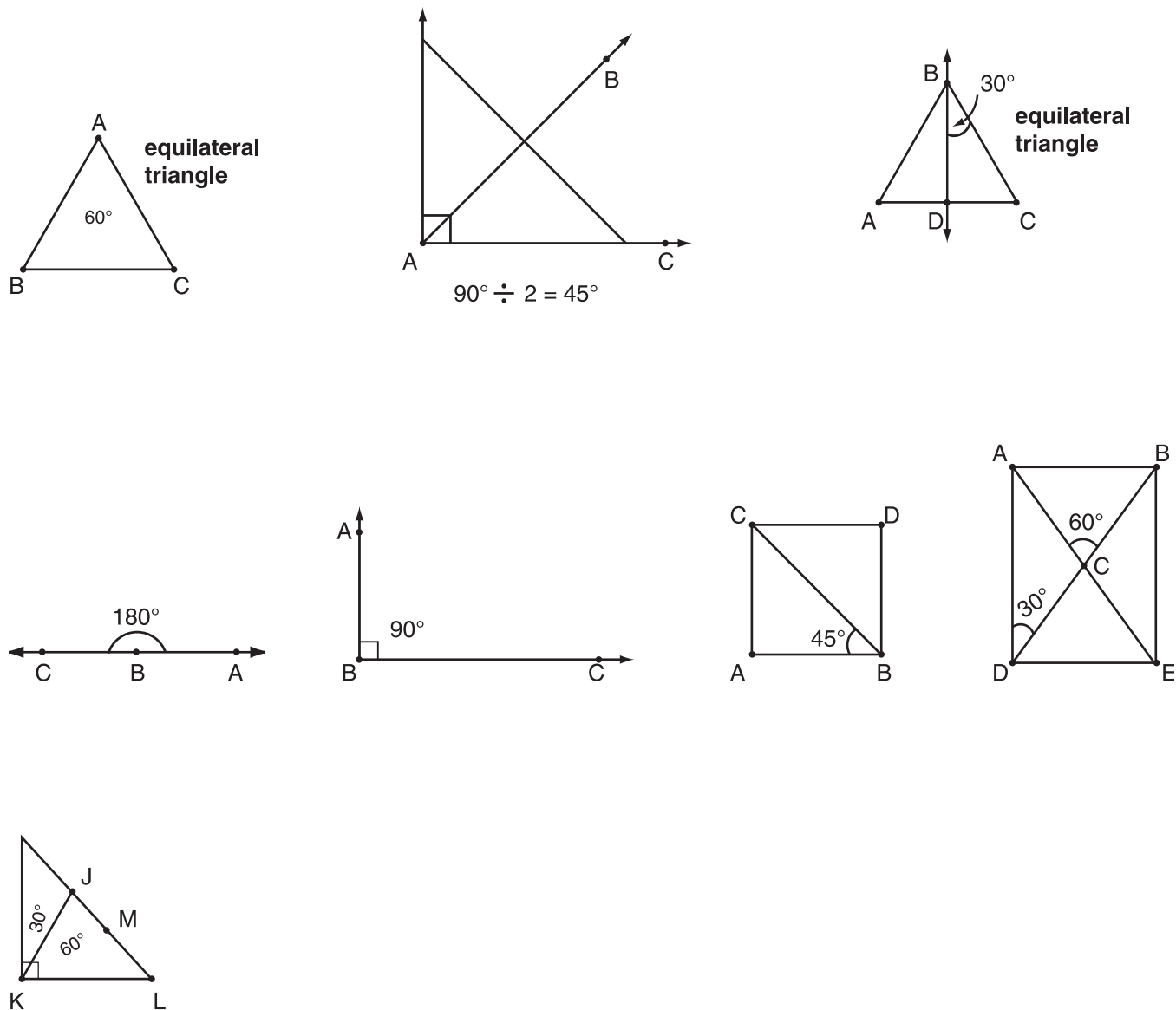
Construct a 60° angle.

Construct a 30° angle.



MATH THINKING—TRIANGLES/ANGLES CHALLENGE KEY

Samples of student solutions.



BLOCK LETTER Y

Materials

- BLOCK LETTER Y CHALLENGE — *class set or as needed*
- Centimeter rulers — *class set*
- Heavyweight paper (12" x 18") — *class set*
- Protractor — *class set*
- Scissors — *class set*
- Tape (transparent) — *class set*
- White paper — *class set*

Procedure

1. Distribute the BLOCK LETTER Y CHALLENGE and conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.
2. Construction information
 - This is a fold-up project of medium difficulty. Some students will instantly realize that they need a reverse image of the letter in the fold-up pattern. The real difficult part is finding a way to design the pattern so that there is adequate material available to fold in for the thickness (edges) of the letter.
 - Allow students to bring in partially completed models and a note from home that states how much time they spent on the challenge.
 - Students may need a larger sheet of paper if the initial design is not well thought out. Sometimes students will need to tape a piece of paper on the edge of their pattern because their design runs off the paper. Allow for this solution.
 - It makes a lot of sense for them to make a rough prototype of their idea using scrap paper to keep from wasting the heavyweight paper.
 - The major challenge of this project—the top branches of the “Y” where paper strips are needed to close in the sides.
3. Make a transparency of **Block Letter Y Challenge Key** to aid your class discussion after they have completed the challenge.
4. **Block Letter Y Challenge Extension**
Assign each student a different letter and then have him/her write instructions for a partner to use to develop a fold-up pattern for that specific letter.



60–90 minutes

Use this challenge after students learn to construct acute and obtuse angles.



Some students work best using quarter-inch graph paper that helps them maintain parallel lines and draw rectangles easily.

Copy and distribute the Concepts page as appropriate. Note: to reduce duplication more than one set of concepts are included on the page.



BLOCK LETTER Y CHALLENGE

Read the **Instructions** and use the **Challenge Information** to create a flat fold-up pattern for the letter “Y,” that when folded and taped will create a solid three-dimensional letter.



60–90 minutes

Concepts: angle measure, reflection, symmetry

Materials

- Centimeter ruler
- Heavyweight paper (12" x 18")
- Protractor
- Scissors
- Tape
- White paper

Instructions

1. Cut and fold the letter from one piece of paper with no additional pieces taped on.
2. When finished, the letter should appear solid (no openings).
3. Make a rough prototype of your idea using scrap white paper before you begin work on the final model.

Challenge Information

1. The straight part of the “Y” is 2 cm wide.
2. The letter is 1.5 cm thick.
3. The bottom part branches to form the upper part of the “Y” at the 6 cm mark.
4. The angle of the branching part is 143° .
5. The inner angle of the top part of the “Y” is 75° .
6. The branching parts of the “Y” are 6 cm long when measured along the outer edge (starting from the vertex of the 143° angle).
7. The letter is symmetrical.

BLOCK LETTER Y CONCEPTS



BLOCK LETTER Y CONCEPTS

- Angle measure—the units (in degrees) used to represent an angle (90° = a right angle)
- Reflection—the mirror image of a line, shape, or geometric construction
- Symmetry—Having two identical halves (a shape, line, or geometric construction that has two similar sides when cut in half)

BLOCK LETTER Y CONCEPTS

- Angle measure—the units (in degrees) used to represent an angle (90° = a right angle)
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BLOCK LETTER Y CONCEPTS

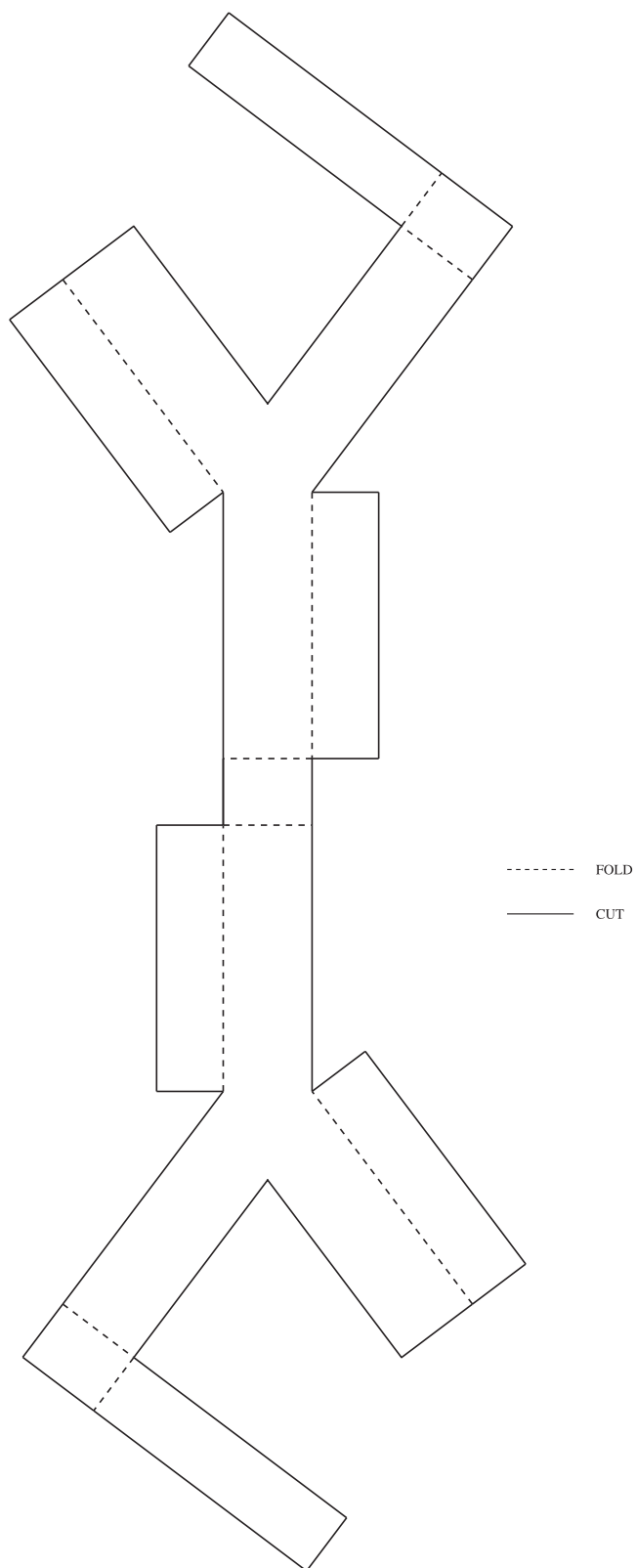
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- Symmetry—Having two identical halves (a shape, line, or geometric construction that has two similar sides when cut in half)



BLOCK LETTER Y CHALLENGE KEY

One possible fold-up pattern:

Not full size



FOUR SQUARES TEACHING DIRECTIONS

FOUR SQUARES

Materials

- FOUR SQUARES CHALLENGE — *class set or one per pair or team*
- Tracing paper — *class set*

— OR —

clean transparency sheets and transparency pens — *class set*

Procedure

1. Distribute the FOUR SQUARES CHALLENGE and conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.
2. Diagram information
 - This challenge is more about logical thinking than geometric concepts.
 - This activity is designed to be a break from the regular challenges that require use of concepts to find the solution.
 - It is possible that a student could solve this very quickly.
 - If students need help, give a hint by indicating where the largest rectangle is located.
 - Tracing may be an easier approach than moving around the shapes.
 - Consider giving credit if students can get the overlapping rectangles to form even one of the $\frac{1}{4}$ inch squares.
 - The rectangles can be matched in some fashion by focusing on the measurements of each (the .25 inch and the 1.125 inch measurements will show where some of the shapes share edges).
3. Make a transparency of **Four Squares Challenge Key** to aid your class discussion after they have completed the challenge.



30 minutes



Student Pairs
or Teams



Copy and distribute the Concepts page as appropriate. Note: to reduce duplication more than one set of concepts are included on the page.



FOUR SQUARES CHALLENGE (1)

Read the **Instructions** and use the **Challenge Information** to create a neat and well-labeled diagram.

Concepts: rectangles, squares, overlapping



30 minutes

Materials

- Tracing paper or clean transparency sheets and transparency pens

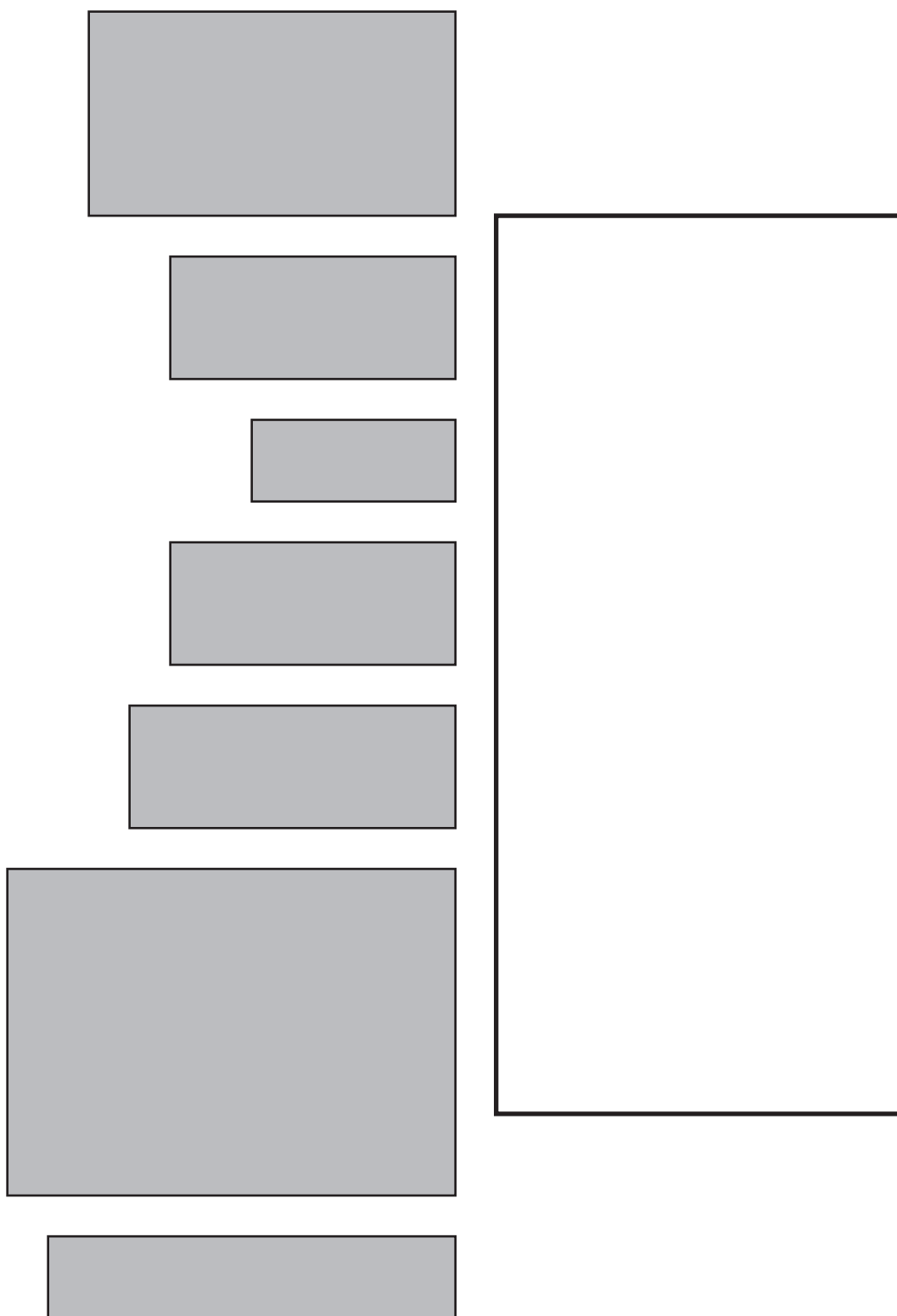
Instructions

1. Examine FOUR SQUARES CHALLENGE (page 2) and use as a template.
2. Trace the large rectangle onto tracing paper or a transparency.
3. Trace around and arrange the rectangles so that they fit in the large rectangle. Some rectangles will overlap at corners or in other ways and create *four* perfect $1/4$ inch squares.
4. If you use a transparency, draw the large rectangle on one side then flip it over. Trace all of the smaller rectangles on the opposite side so you can easily wipe them off without ruining your main rectangle.
5. Carefully draw or trace the shapes in the large rectangle.
6. When finished shade the rectangles except where they overlap, indicating the location of the four white squares.

Challenge Information

1. The four overlapped squares will each measure $1/4$ inch on a side.
2. The large rectangle will be covered completely by the shapes you have traced.

FOUR SQUARES CHALLENGE (2)





FOUR SQUARES CONCEPTS

FOUR SQUARES CONCEPTS

- Rectangle—a parallelogram with all right angles (opposite sides congruent)
- Square—a rectangle with congruent sides and all right angles
- Overlapping—lines that form one shape extend over the lines that form another shape

FOUR SQUARES CONCEPTS

- Rectangle—a parallelogram with all right angles (opposite sides congruent)
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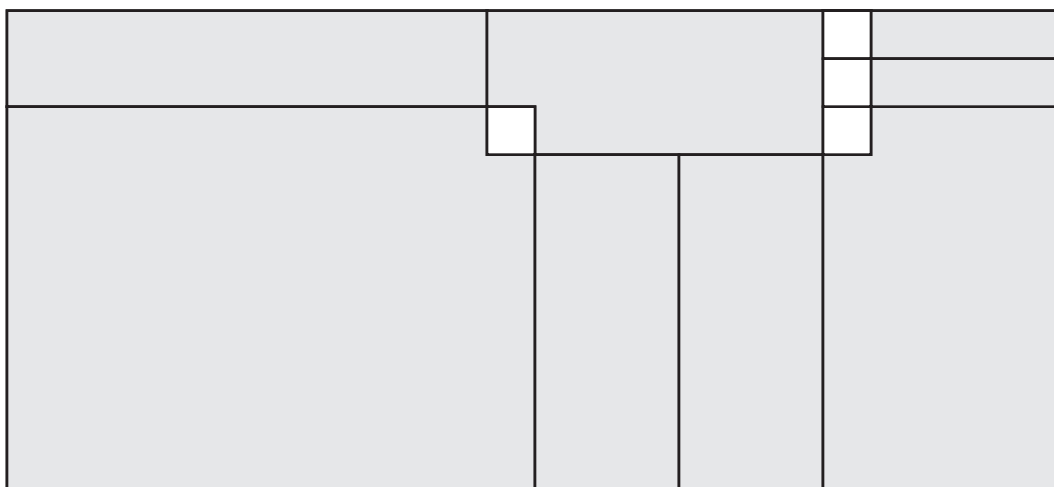
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FOUR SQUARES CHALLENGE KEY



ANGLE MEASURE

TEACHING DIRECTIONS



15–45 minutes



Student Pairs



Copy and distribute the Concepts page as appropriate. Note: to reduce duplication more than one set of concepts are included on the page.

ANGLE MEASURE

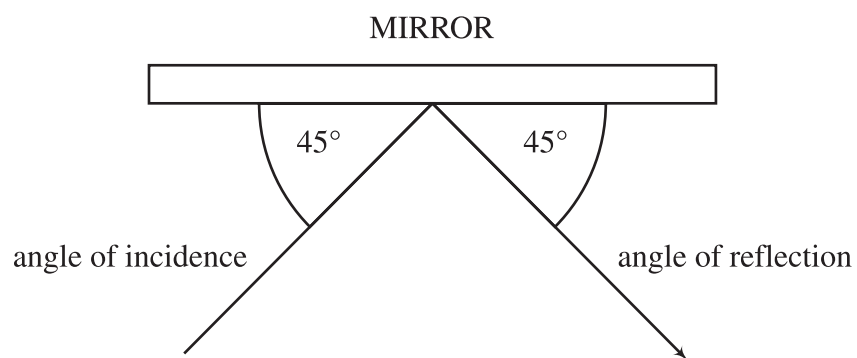
Materials

- ANGLE MEASURE CHALLENGE — *class set*
- Centimeter rulers — *class set*
- Protractor — *class set*

Procedure

1. Teach about *the angle of incidence equals the angle of reflection*. Make a transparency of **Angle Measure Diagram** on page 47.
2. Review the use of a protractor and the measuring of acute and obtuse angles. (This challenge only has acute angle measures of incidence and reflection).
3. Distribute the ANGLE MEASURE CHALLENGE and conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.
4. Diagram information
 - If students are not accurate, their lines of reflection will miss the next mirror.
 - Allow them to lengthen the mirror's surface so that the ray will hit a mirror.
 - If students cannot get the ray to strike the target when it reflects off of the last mirror allow the students to draw an additional mirror so that the ray *will* strike the target.
 - When finished, have students work in pairs to measure each other's angles for accuracy.
5. Make a transparency of **Angle Measure Challenge Key** to aid your class discussion after they have completed the challenge.
6. **Angle Measure Challenge Extension:**
Students who finish quickly should be directed to create their own challenge “angle of incidence/angle of reflection” activity for others to try. If well done, use copies of those new challenge papers for other classes or for next year's group, giving credit to the designer.

ANGLE MEASURE DIAGRAM





ANGLE MEASURE CHALLENGE (1)

Concepts: angle of incidence is equal to the angle of reflection, angle measure

Materials

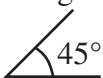
- Centimeter ruler
- Protractor



15–45 minutes

Instructions

1. Examine ANGLE MEASURE CHALLENGE (page 2).
2. The lines on the page represent mirrors.
3. Use the “mirrors” on the paper to help you reflect the laser light from the start to the end of the maze.
4. When finished, measure each angle of incidence and reflection and write the angle measure near each angle (see diagram).



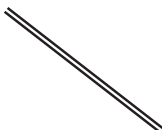
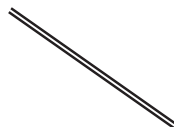
Challenge Information

1. Use each mirror only once.
2. There is only one solution.

ANGLE MEASURE CHALLENGE (2)



START



• END



ANGLE MEASURE CONCEPTS

ANGLE MEASURE CONCEPTS

- Angle measure—the units (in degrees) used to represent an angle.
(90° = a right angle)
- Angle of incidence—the angle created by a light ray as it strikes a smooth surface.
- Angle of reflection—the angle created by a light ray as it is reflected away from a smooth surface.

Note The angle of incidence and the angle of reflection concepts are experienced in everyday life. A ball bouncing off of a smooth wall is one example. Another common experience is glancing at a mirror and seeing objects that may be in the next room.

ANGLE MEASURE CONCEPTS

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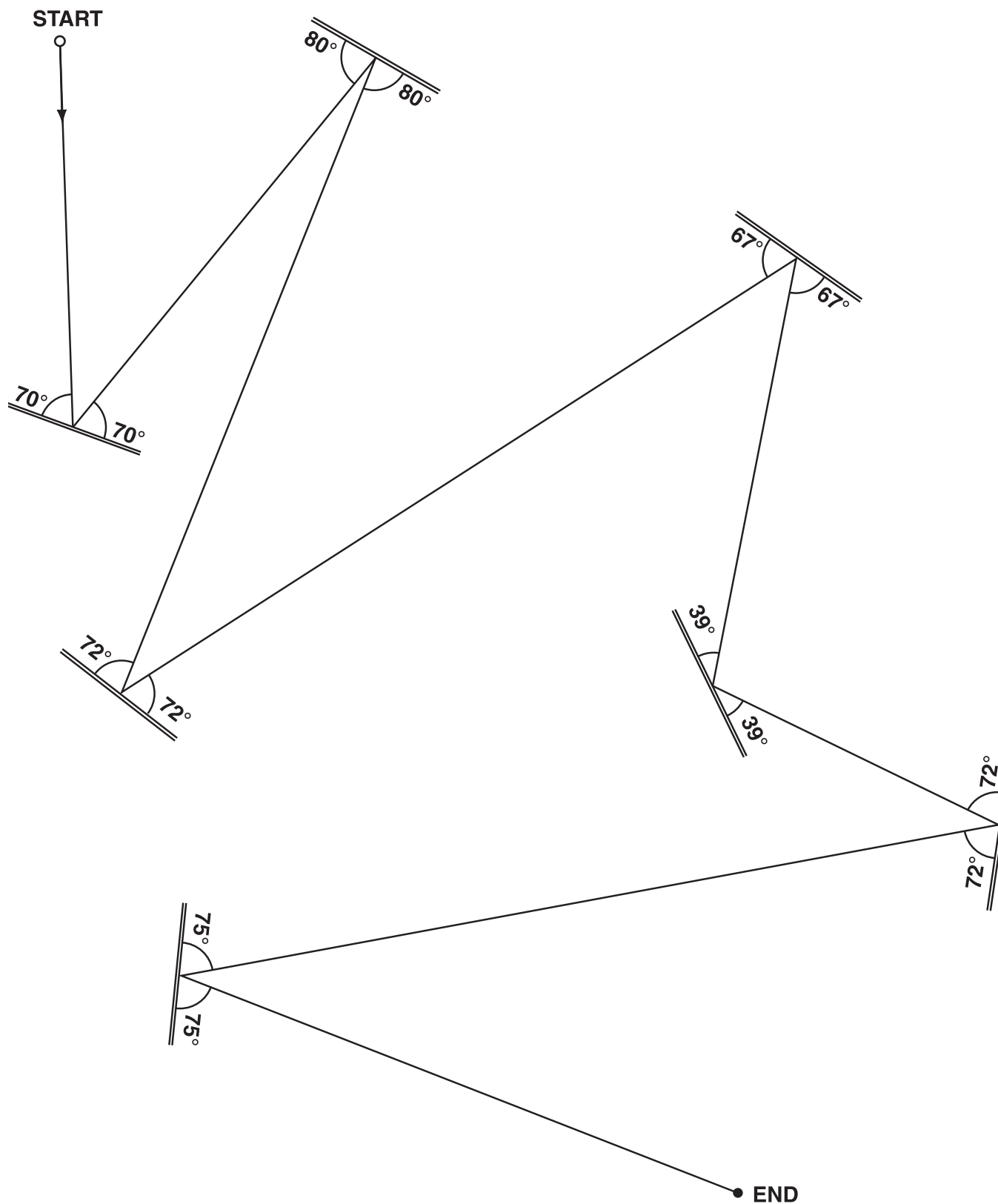
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ANGLE MEASURE CHALLENGE KEY



MYSTERY PATH

TEACHING DIRECTIONS



1–2 hours



Individuals or
Student Pairs



Copy and distribute the Concepts page as appropriate. Note: to reduce duplication more than one set of concepts are included on the page.

MYSTERY PATH

Materials

- MYSTERY PATH CHALLENGE — *class set*
- Centimeter rulers — *class set*
- Protractor — *class set*

Procedure

1. Distribute the MYSTERY PATH CHALLENGE and conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.
2. Review the use of a protractor and the measuring of acute and obtuse angles.
3. Diagram information
 - This challenge is a practice in using segments and measuring angles in a logical math approach.
 - Not all students will complete the challenge successfully.
 - Students measure the smaller angle for each change in direction.



- Using acute angles at the beginning keeps the angle total from getting too high too fast.
- $56 \text{ cm} / 10 = 5.6 \text{ cm}$ per average segment (students need to use some short segments).
- Nine angles generate ten segments needed to return to the “X.”
- $365^\circ / 9 =$ approximately 40° per angle (some will be larger and some smaller).
- This challenge may frustrate some students. Allow them, in the end, to get as close as they can to the target goals for both total degrees (365°) and total length (56 cm).
- Grade on a scale related to pathway length and angle sum, even if the last segment doesn’t return exactly to the “X” or if the last angle puts the total angle sum over or under the 365° .
- Be on the lookout for identical pathways.

MYSTERY PATH TEACHING DIRECTIONS

4. Following this challenge conduct a discussion focusing on the successful strategies that students used to change the segments and angle measures. Consider the following questions for discussion:
 - a. What worked for you?
 - b. What strategies did you discover?
 - c. Is this task possible?
 - d. Are there really multiple solutions? Why or why not?
5. Make a transparency of **Mystery Path Challenge Key** to aid your class discussion.



MYSTERY PATH CHALLENGE (1)

Read the **Instructions** and use the **Challenge Information** to create a neat and well-labeled diagram.

Concepts: angle sum, acute, obtuse, angle measure, metric measure

Materials

- Centimeter ruler
- Protractor



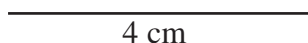
1–2 hours

Instructions

1. Examine MYSTERY PATH CHALLENGE (page 2).
2. Create a pathway, using line segments, which total 365° in nine angle changes.
3. Your line needs to start at “X” and must return to the same “X” without crossing over itself.
4. You are allowed no more than 56 cm of line segments to complete the pathway.
5. Measure the angle every time a line segment indicates a change of direction in the pathway.
6. Write the angle measure in the angle.



7. Write the length of each segment near the middle of the segment.



8. Continually write the degree measures and segment lengths in the proper columns and add them up in the “total” columns as you proceed.

Challenge Information

1. Total number of degrees for the nine angles is 365° .
2. Total length of pathway is 56 cm or less.

Remember: there are many possible solutions to this project, so don’t worry if your solution looks different than everybody else’s.

MYSTERY PATH CHALLENGE (2)



Segments		Angles	
Length	Running Total	Angle	Running Total

X



MYSTERY PATH CONCEPTS

MYSTERY PATH CONCEPTS

- Angle sum—the total when adding the degree measure of more than one angle
- Acute—an angle less than 90°
- Obtuse—an angle whose degree measure is 90° or larger, but no more than 180°
- Angle measure—the units (in degrees) used to represent an angle (90° = a right angle)
- Metric measure—indicating lengths in metric units (mm, cm)

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MYSTERY PATH CONCEPTS

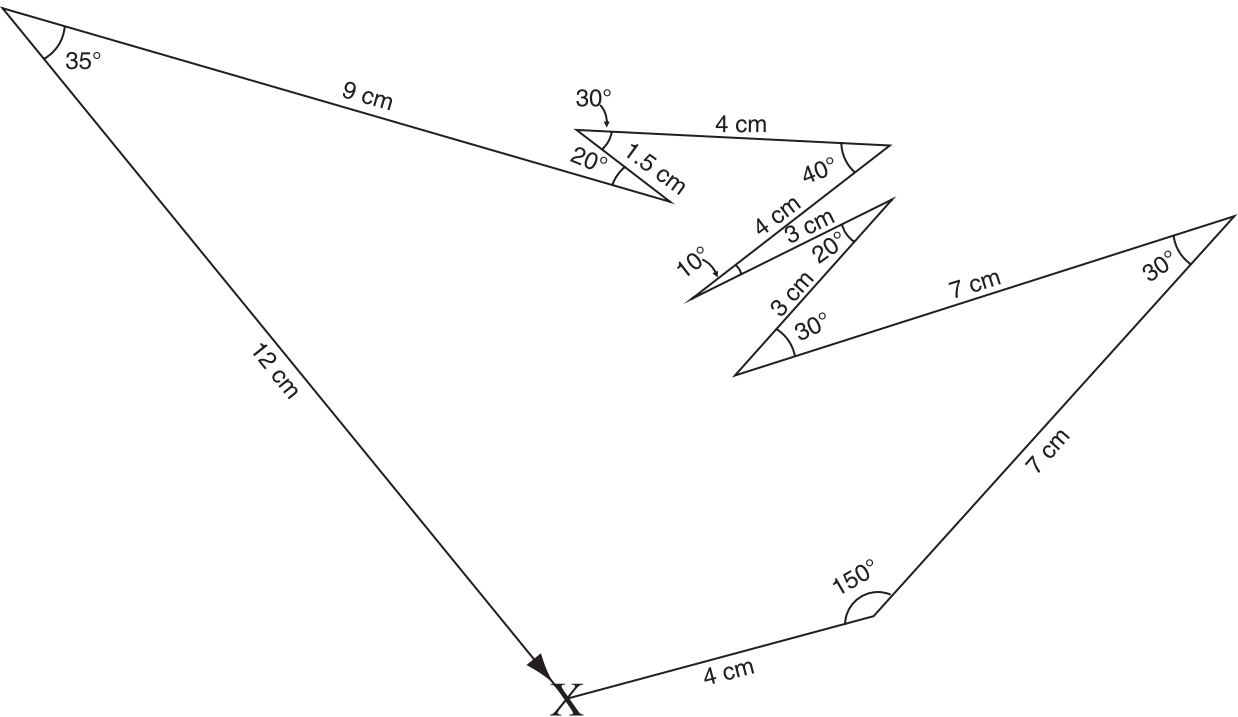
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- Metric measure—indicating lengths in metric units (mm, cm)

MYSTERY PATH CHALLENGE KEY



Segments		Angles	
Length	Running Total	Angle	Running Total
4 cm	4 cm	150°	150°
7 cm	11 cm	30°	180°
7 cm	18 cm	30°	210°
3 cm	21 cm	20°	230°
3 cm	24 cm	10°	240°
4 cm	28 cm	40°	280°
4 cm	32 cm	30°	310°
1.5 cm	33.5 cm	20°	330°
9 cm	42.5 cm	35°	365°
12 cm	54.5 cm		

One possible solution.



TRIANGLE TRAIL

TEACHING DIRECTIONS



1 hour



Student Pairs

TRIANGLE TRAIL

Materials

- TRIANGLE TRAIL CHALLENGE — *class set*
- Centimeter rulers — *class set*
- Glue — *class set*
- Protractor — *class set*

Procedure

1. Distribute the TRIANGLE TRAIL CHALLENGE and conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.
2. Diagram information
 - This is a good activity to assess use of protractor and measuring skills during the creation of the six triangles.
 - Triangles may need to be flipped during the trail building phase.
 - If your students are being frustrated direct them to first match up the sides of the triangles that are not 3.0 cm long.
 - The remaining four 3.0 cm long sides then need to be moved or flipped so that they will make the whole trail settle into the right configuration.
 - If students are struggling, walk around with **Triangle Trail Challenge Key**, giving students reinforcement by indicating that a certain shape or shapes are in the correct location or have the correct orientation.
 - When shapes are glued down in the final solution, congruent sides need to overlap the thickness of a line to make it perfect (since the trail has triangles that are sharing the same line).
 - Finishing the trail is *the challenge* and extra credit or bonus points can be given for those who complete it. But students should be encouraged to glue the shapes down when they get as close as they possibly can.
3. Make a transparency of **Triangle Trail Challenge Key** to aid your class discussion after they have completed the challenge. Final sharing and discussion can reinforce the concepts of this lesson.

TRIANGLE TRAIL CHALLENGE



Read the **Instructions** and use the **Challenge Information** to create a neat and well-labeled diagram.

Concepts: adjacent triangles may share sides, triangles may be constructed with only side-angle-side measures, vertex



1 hour

Materials

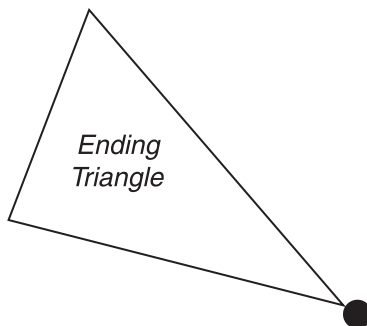
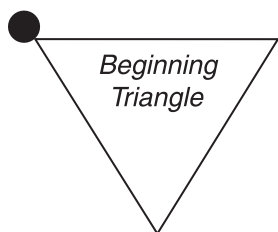
- Centimeter ruler
- Glue
- Protractor

Instructions

1. Use the information provided to create the triangles A, B, C, D, E, and F. Arrange them so that they share sides and create a triangle trail from the *Beginning* triangle to the *Ending* triangle.
2. Some triangles may need to be flipped over to make a perfect trail.
3. Place the triangles so that they share sides (sides touch).
4. Cut the triangles out and match sides to create the trail.
5. Glue them down when you are finished.

Challenge Information

Triangle "A" – legs: 4.3 cm and 3.0 cm	vertex 40°
Triangle "B" – legs: 5.3 cm and 3.0 cm	vertex 35°
Triangle "C" – legs: 3.2 cm and 3.0 cm	vertex 85°
Triangle "D" – legs: 4.4 cm and 4.0 cm	vertex 30°
Triangle "E" – legs: 3.0 cm and 2.2 cm	vertex 95°
Triangle "F" – legs: 4.4 cm and 1.2 cm	vertex 140°

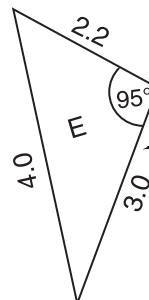
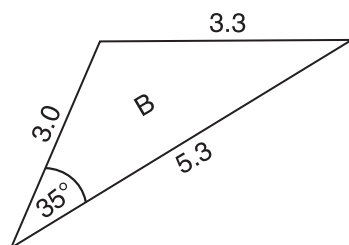
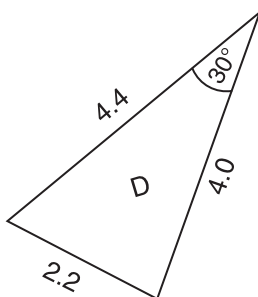
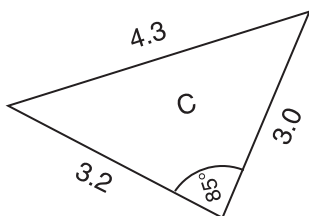
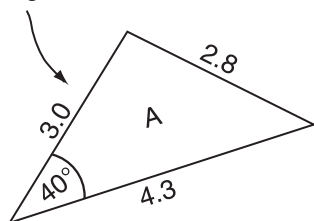




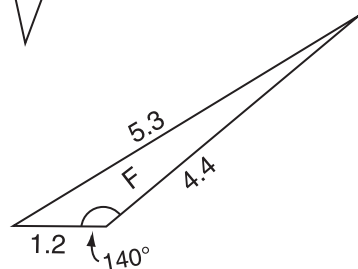
TRIANGLE TRAIL CHALLENGE KEY

The six triangles needed for Triangle Trail Challenge.

matches 3.0 of
top triangle

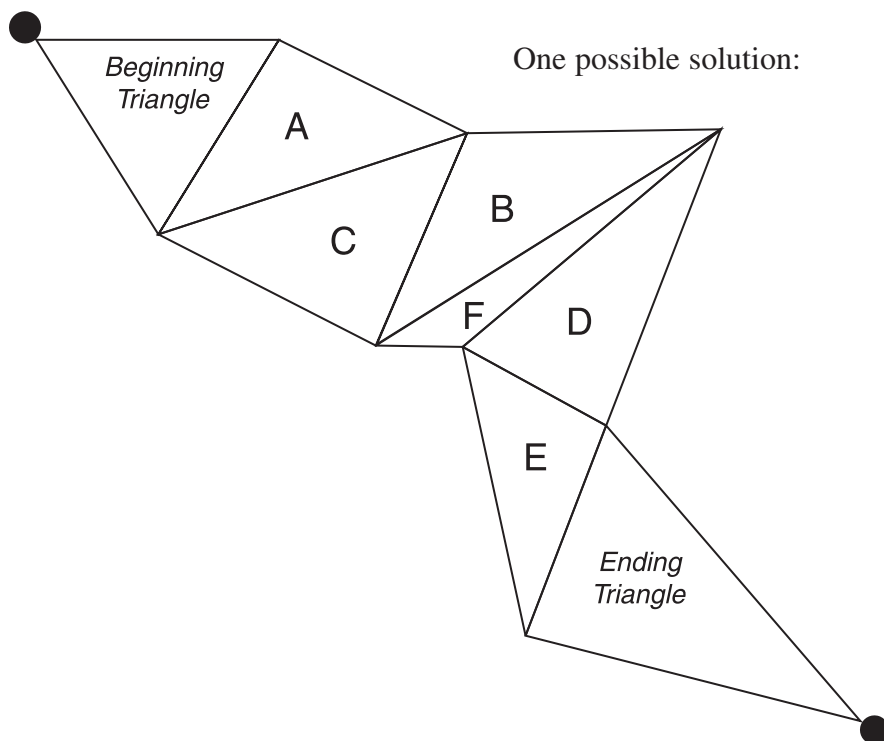


matches 3.0 edge of
bottom triangle



Sides that match:

4.3
4.4
2.2
5.3



One possible solution:

JEWELRY BOX TEACHING DIRECTIONS

JEWELRY BOX

Materials

- JEWELRY BOX CHALLENGE — *class set or one per student pair*
- Centimeter rulers — *class set*
- White paper (8.5" x 11") — *class set*

Procedure

1. Distribute the JEWELRY BOX CHALLENGE and conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.
2. Diagram information
 - Make sure students do *not* interpret the area of 24 sq. cm to mean that it is a “square” shaped box. The box is a rectangle.
 - Students start by saying this isn’t possible, but most will solve the challenge on their own.
 - The jewelry box is a rectangle.
 - There are only a few dimensions possible for the four boxes mentioned in #1 —
1 cm x 18 cm, 2 cm x 9 cm, or 3 cm x 6 cm.
If students get in trouble, direct them to the 3 cm x 6 cm measure for the boxes.
 - Give the following clue at the end of the class period for students who are having difficulty: ***“The area of the jewelry box tray is $(2+3+3+2) \times (12+6)$ or $(2+8) \times (3+3+3+9)$.”***
3. Make a transparency of **Jewelry Box Challenge Key** to aid your class discussion after they have completed the challenge.



*More than 1
hour*



Student Pairs



Some students work best using quarter-inch graph paper that helps them maintain parallel lines and draw rectangles easily.

Copy and distribute the Concepts page as appropriate. Note: to reduce duplication more than one set of concepts are included on the page.

Write this clue on the board and have students copy it down before they leave.



JEWELRY BOX CHALLENGE

Read the **Instructions** and use the **Challenge Information** to create a neat and well-labeled diagram.

Concepts: area, perimeter, math logic



More than 1 hour

Materials

- Centimeter ruler
- White paper

Introduction

You have purchased a fancy jewelry box. To your surprise, when it is delivered the top tray is unassembled. All you receive is a bunch of different size rectangular boxes and some glue. Somehow these rectangular boxes are to fit in that top tray. The directions are limited: “To assemble the top tray, glue the boxes in place.”

Instructions

1. Using the **Challenge Information** “clues,” arrange the boxes and create the top tray of the rectangular jewelry box.
2. Create a neat well-labeled diagram.
3. Use a ruler to make all lines.
4. Write the dimensions in each box on the final diagram.

Challenge Information

These additional instructions came with the boxes.

1. Each one of the four identical boxes has an area of 18 sq. cm and each one touches the sides of two others.
2. The largest box (48 sq. cm) touches the edges of two of the 18 sq. cm rectangles and the narrowest edge of the longest box.
3. A long narrow box (24 sq. cm) is for storing necklaces.
4. Three small boxes, each 6 sq. cm, are arranged in a row for holding earrings.
5. The remaining box is 9 cm long and could hold rings.

Answer these questions on your diagram:

1. How many small boxes make up the top tray of the jewelry box?
2. The outer measurements of the top tray are _____ x _____.

JEWELRY BOX CONCEPTS



JEWELRY BOX CONCEPTS

- Area—the total number of square units in a specific surface (length x width)
- Perimeter—the measure (length) of all the sides of a shape (side + side + side + side for a four-sided shape)
- Math logic—correct math reasoning

JEWELRY BOX CONCEPTS

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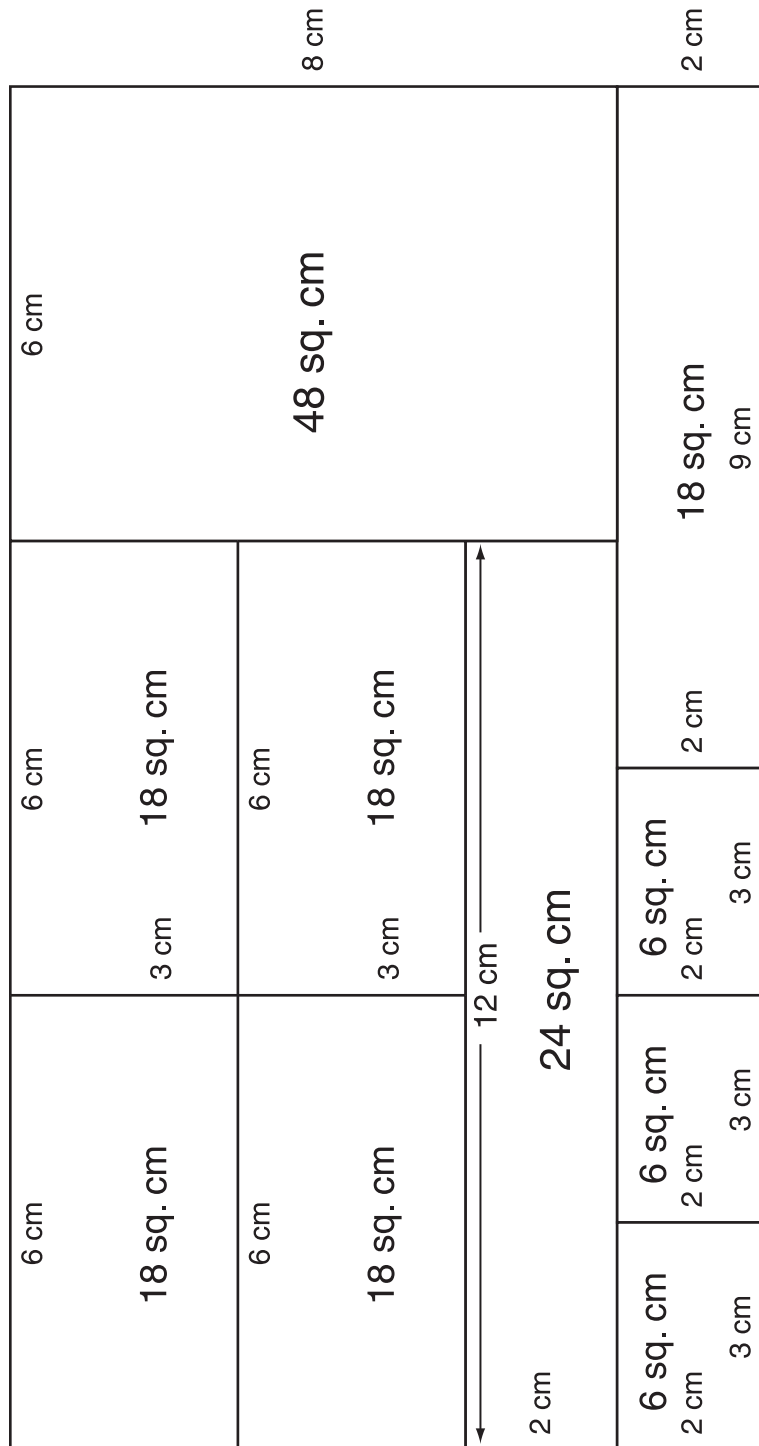
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- Math logic—correct math reasoning



JEWELRY BOX CHALLENGE KEY

One possible solution.



TWO PYRAMIDS

TEACHING DIRECTIONS

TWO PYRAMIDS

Materials

- TWO PYRAMIDS CHALLENGE — *class set or one per student pair*
- Centimeter rulers — *class set*
- Glue — *class set*
- Heavy weight paper — *class set*
- Protractor — *class set*
- Scissors — *class set*
- Tape (transparent) — *class set*
- White paper — *class set*

Procedure

1. Distribute the TWO PYRAMIDS CHALLENGE and conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.
2. Construction information
 - **Note:** it is difficult to fold and tape this project and get it accurate.
 - The base gives the student lots of information needed to construct this pattern.
 - Draw the two bases touching along the 7 cm edge.
 - Use the two 6 cm edges of each base to construct the two known triangles (now you have four triangles drawn).
 - Once you know the base and one side of the pyramid you can construct the whole pyramid.
 - The two sides of the base (excluding the 6 cm side) give you the missing measurements needed to draw the remaining legs of the other triangles (faces of the diamond shape).
 - Fold the pattern and glue the two bases together. **Note:** It is possible to draw the flat pattern having only one base (5 cm x 6 cm x 7 cm). All six faces will be connected to that base which ends up inside the folded shape.
 - The base or both bases end up inside the folded shape.
3. Make a transparency of **Two Pyramids Challenge Key** to aid your class discussion after they have completed the challenge.



1–2 hours

This is a difficult challenge!



Student Pairs



Copy and distribute the Concepts page as appropriate. Note: to reduce duplication more than one set of concepts are included on the page.



TWO PYRAMIDS CHALLENGE

Read the **Instructions** and use the **Challenge Information** to create a flat fold-up pattern for the shape described. When the pattern is folded and taped it will create a solid three-dimensional shape.



1–2 hours

Concepts: triangular pyramids, angles, triangles

Materials

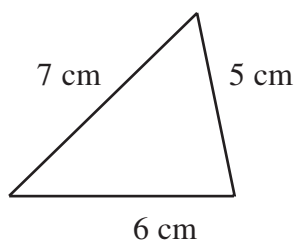
- Centimeter ruler
- Glue
- Heavy weight paper
- Protractor
- Scissors
- Tape
- White paper

Instructions

1. Cut and fold the diamond from one piece of paper with no additional pieces taped on.
2. When finished, the diamond shape should appear solid (no openings).
3. Make a rough prototype of your idea using white paper before you begin to draw the fold-up flat pattern on heavyweight paper.

Challenge Information

1. Two different triangular pyramids have the same size base.



2. When the two triangles have their bases glued together they create a six-sided irregular diamond shape.
3. One side of the first triangular pyramid is 6 cm x 3.2 cm x 4.3 cm.
4. One side of the other triangular pyramid is 6 cm x 4 cm x 4.3 cm.
5. Find the dimensions of *all six sides* of the shape, and then create a flat pattern to cut out and fold that would make the six-sided solid shape.

TWO PYRAMIDS CONCEPTS



TWO PYRAMIDS CONCEPTS

- Triangular pyramid—a polyhedron with a three-sided base.
- Angle—the shape created by two straight lines meeting at a point (measured in degrees).
- Triangle—a shape with three sides (legs).

TWO PYRAMIDS CONCEPTS

- Triangular pyramid—a polyhedron with a three-sided base.
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TWO PYRAMIDS CONCEPTS

- Triangular pyramid—a polyhedron with a three-sided base.
- Angle—the shape created by two straight lines meeting at a point (measured in degrees).
- Triangle—a shape with three sides (legs).



The diagram illustrates a 12-sided polyhedron, likely a dodecahedron, with faces labeled with numbers and the word 'BASE'. The structure is symmetrical about a central vertical axis. The top face is a pentagon with edges labeled 5, 7, 5, 7, 5. The bottom face is a pentagon with edges labeled 5, 5, 5, 5, 5. The side faces are labeled with numbers 3.2, 4, 4.3, 6, 6, 7. The central vertical axis is labeled 7. The word 'BASE' is written on the bottom face.

———— CUT

RECTANGULAR FOLD-UP TEACHING DIRECTIONS

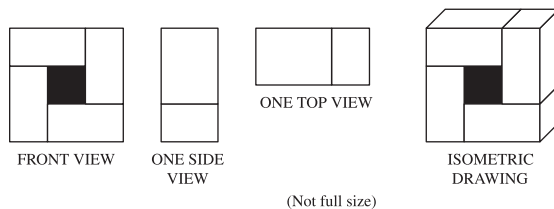
RECTANGULAR FOLD-UP

Materials

- RECTANGULAR FOLD-UP CHALLENGE — *class set or one per student pair*
- Centimeter rulers — *class set*
- Heavy weight paper — *class set*
- Scissors — *class set*
- Tape (transparent) — *class set*
- White paper — *class set*

Procedure

1. Distribute the RECTANGULAR FOLD-UP CHALLENGE and conduct a short class discussion of the **Instructions** and **Challenge Information**. Elicit questions and conclusions from the students.
2. Construction information
 - Accept drawings that show three distinct views (front, top, side) or an isometric version that shows all three faces.



- The shape is a basic box fold-up with the 3 cm x 3 cm “hole” cut in the 7 cm x 7 cm face which is folded into the model to provide paper for the inside wall.
 - The other 7 cm x 7 cm face must have the same size hole cut and folded into the shape to create the opposing inside wall.
 - The 7 cm x 3 cm sides need to have an extra length of paper added to them so each can be folded inside of the model to make the other two inside walls.
3. Make a transparency of **Rectangular Fold-up Challenge Key** to aid your class discussion after they have completed the challenge.



1–2 hours



Student Pairs



Some students work best using quarter-inch graph paper that helps them maintain parallel lines and draw rectangles easily.



RECTANGULAR FOLD-UP CHALLENGE

Read the **Instructions** and use the **Challenge Information** to create a flat fold-up pattern for the rectangle, that when folded and taped will create a solid three-dimensional shape with a 3 cm x 3 cm x 3 cm opening through the center.



1–2 hours

Concepts: rectangular prism

Materials

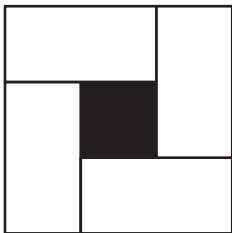
- Centimeter ruler
- Heavy weight paper
- Scissors
- Tape
- White paper

Instructions

1. Cut and fold the shape from one piece of paper with no additional pieces taped on.
2. Draw the shape showing the top, the front, and the side view and get it approved by your teacher.
3. Create the fold-up pattern for the shape you have drawn.
4. Make a rough prototype of your idea on white paper before you begin work on the final model.

Challenge Information

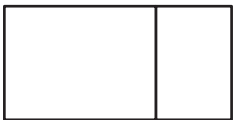
1. Imagine four solid rectangular wood blocks (2 cm x 3 cm x 5 cm) are glued together to form a 7 cm x 7 cm x 3 cm shape with a 3 cm x 3 cm x 3 cm opening through the center.
2. When finished, the rectangle will appear solid with a square opening through the 7 cm x 7 cm sides.



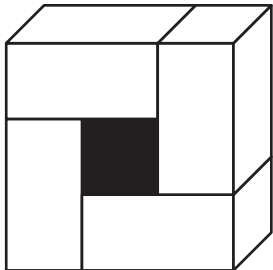
FRONT VIEW



ONE SIDE
VIEW

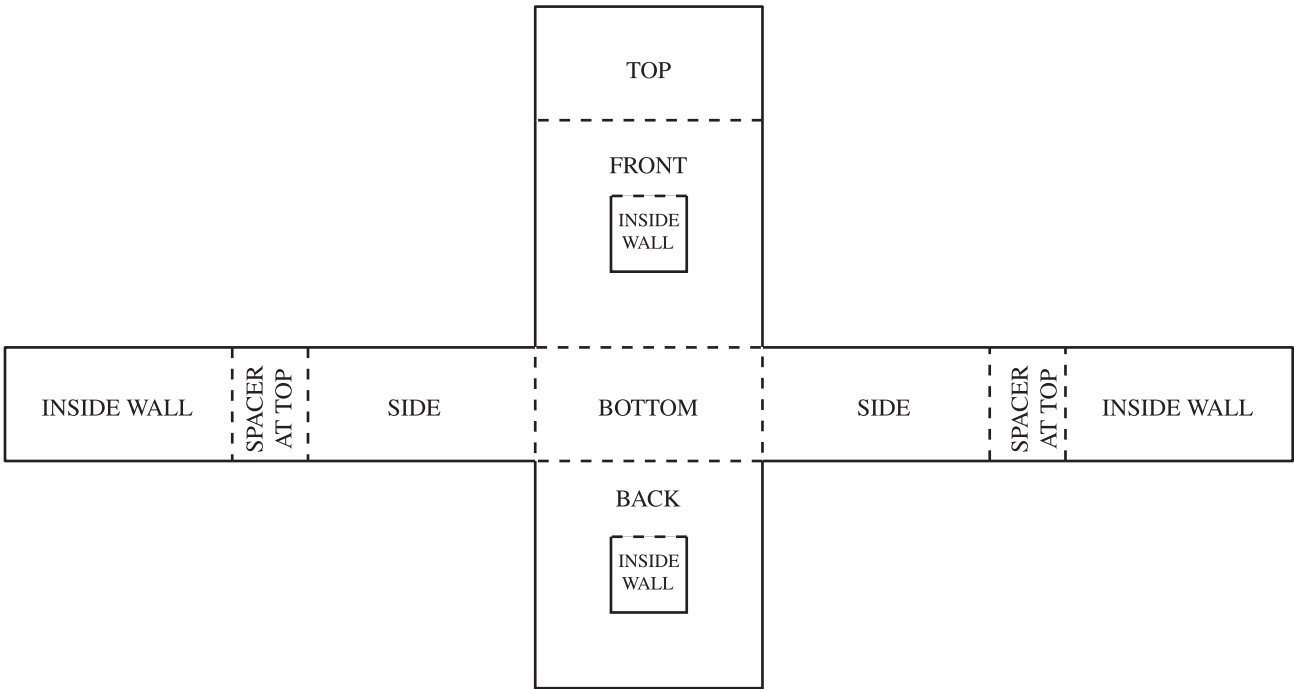


ONE TOP VIEW



ISOMETRIC
DRAWING

(Not full size)



----- FOLD
————— CUT

STUDENT-CREATED ANGLE CHALLENGE

TEACHING DIRECTIONS



*Variable time
commitment*



*Individuals and
Student Pairs*

*This challenge
should take about
two hours. Some
students will take
a lot of time as
they create more
difficult
challenges for
others.*

*Students work
individually to
create their own
Challenge.
Student pairs
exchange
Challenges.*

STUDENT-CREATED ANGLE CHALLENGE

Materials

- STUDENT-CREATED ANGLE CHALLENGE — *class set*
- Centimeter rulers — *class set*
- Heavy weight paper — *class set*
- Scissors — *class set*
- Tape (transparent) — *class set*
- White paper — *class set*

Procedure

1. Distribute the STUDENT-CREATED ANGLE CHALLENGE and discuss.
2. Before students begin this project, make a transparency of the **Sample Student-created Angle Challenge** and determine:
 - the concept(s)
 - the materials needed
 - the amount of time needed
 - the wording for the challenge

Through the class discussion your students will learn how to write better instructions for their own challenge.

3. Instruct students to create something that would challenge the other students in the room.
4. Students create written instructions and/or a worksheet and then work through the activity and modify it as needed.
5. Students exchange Challenges with a partner and complete the new activity.
6. **Optional Extension**
If you have a math night or want a special event in math, instruct students to develop instructions at school and invite parents in to work on the project using their child's instructions.

STUDENT-CREATED ANGLE CHALLENGE



STUDENT-CREATED ANGLE CHALLENGE

Create a challenge activity related to angles or angle measure. Try to make the activity interesting and challenging while remembering the skills and concepts you have experienced in this geometry unit.

Follow this format when designing your challenge:

Concepts

Materials

Time

Instructions

Challenge Information

In order to complete this challenge you must:

1. Develop the activity.
2. Construct a Challenge page. Write clear **Instructions** and provide **Challenge Information**.
3. Experience the activity yourself.
4. Generate an answer key.

STUDENT-CREATED ANGLE CHALLENGE

Create a challenge activity related to angles or angle measure. Try to make the activity interesting and challenging while remembering the skills and concepts you have experienced in this geometry unit.

Follow this format when designing your challenge:

Concepts

Materials

Time

Instructions

Challenge Information

In order to complete this challenge you must:

1. Develop the activity.
2. Construct a Challenge page. Write clear **Instructions** and provide **Challenge Information**.
3. Experience the activity yourself.
4. Generate an answer key.



SAMPLE STUDENT-CREATED ANGLE CHALLENGE

Actual student-created challenge

Concepts

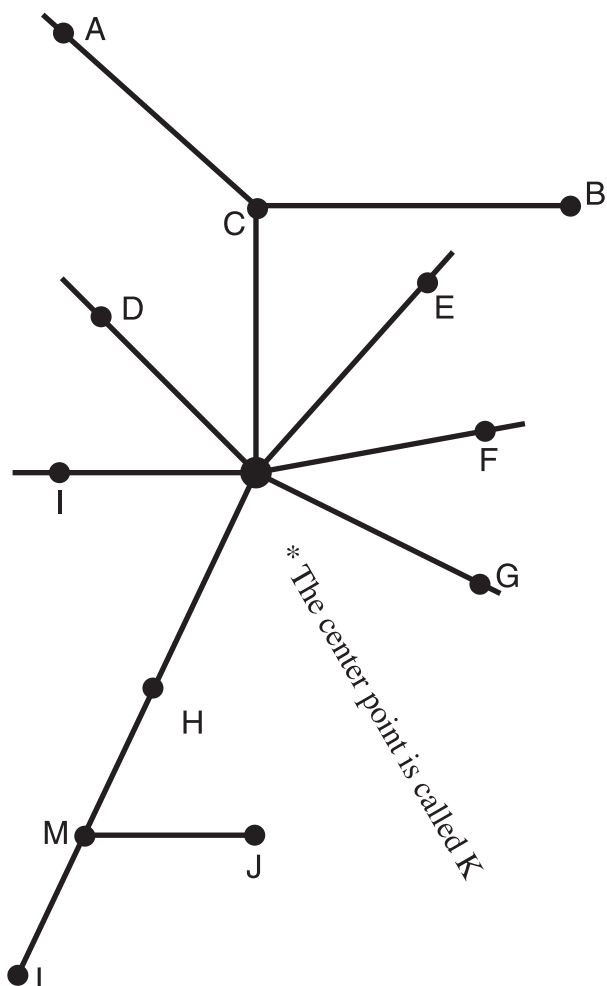
Materials

Time

Instructions

Challenge Information

1. How many acute angles?
List all acute angles.
2. How many obtuse angles?
List all obtuse angles.



Acute Angles

CKD
CKE
CKF
EKF
GKF
GKH
GKM
GKL
HKI
MKI
LKI
HMJ

Obtuse Angles

ACB
ACK
CKH
CKM
CKL
CKG
DKF
DKG
DKH
DKM
DKL
IKE
IKF
IKG
HKE
MKF
MKE
LKE
LKF
JML
HKF

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