interact
Learn Through Experience

A-MAZE-ING SHAPES

An interaction unit exploring plane geometry while designing mazes.

Judy Bippert is a credentialed teacher with a master's degree in educational administration. She has more than 25 years experience teaching junior high school mathematics, coordinating Gifted Education programs, teaching math methods to preservice teachers, and supervising teacher interns. Judy is a member of the faculty of the School of Teacher Education at San Diego State University in San Diego, California.

Louise Vandling is a credentialed teacher and administrator with a master's degree. She has more than 25 years of experience as an elementary teacher, administrator, district math mentor teacher and staff developer. Louise is currently a mathematics specialist at Casita Center, a math, science and technology magnet school in Vista Unified School District, Vista California and a math methods instructor at California State University, San Marcos.

10200 Jefferson Blvd.
P.O. Box 802
Culver City, CA 90232-0802
(800) 359-0961 • www.teachinteract.com
ISBN# 978-1-57336-125-5

PURPOSE

> "… unit offers a wide variety of activities and has many helpful ideas …"

A-MAZE-ING SHAPES enables students to recognize, investigate, understand, and enjoy the geometry all around us. The unit offers a wide variety of activities and has many helpful ideas for the teacher. The Curriculum and Evaluation Standards established by the National Council of Teachers of Mathematics (1989, page 112) state: "Students (in grades 5-8) discover relationships (in geometry) … by constructing, drawing, measuring, visualizing, comparing, transforming, and classifying." Using a straight edge, protractor, and compass, students investigate basic concepts of plane geometry. Working in groups, students study the properties of specific shapes and angles, and then design an A-MAZE-ING entrance, which includes these geometric elements, for the Pythagorean theme park **Geo World**.

A-MAZE-ING SHAPES enhances students' grasp of geometric concepts in the following ways:

Knowledge

- Learning how area and perimeter are interrelated
- Increasing understanding of geometrical shapes, their properties, and their relationship to each other
- Becoming aware of spatial relationships
- Learning specialized geometry vocabulary
- Using tools for geometric measurement and construction
- Choosing problem solving strategies

Skills

- Measuring and constructing geometric angles, shapes, and figures with mathematical tools
- Finding the area and perimeter of quadrilaterals
- Increasing spatial visualization abilities
- Identifying figures in a plane
- Using accurate vocabulary to describe shapes and geometric properties
- Collecting and organizing information
- Communicating mathematically, orally, and in writing
- Forming designs using geometric shapes
- Recognizing symmetry as a geometric property
- Constructing a maze
- Identifying patterns
- Solving problems
- Speaking publicly

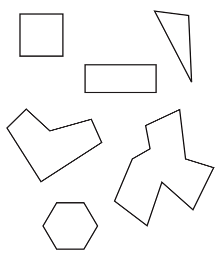
Attitudes

- Appreciating geometry and its real world applications
- Developing positive attitudes toward math
- Increasing self confidence in problem solving abilities
- Realizing the value of working with others toward a common goal

OVERVIEW - 1

A-MAZE-ING SHAPES challenges students individually and in groups to develop and use their knowledge of geometric properties as they face the task of developing a maze. Using triangles and quadrilaterals, students explore area and perimeter, study spatial relationship concepts, measure angles, construct shapes and prepare for their final project, the design of a maze that includes specific geometric shapes.

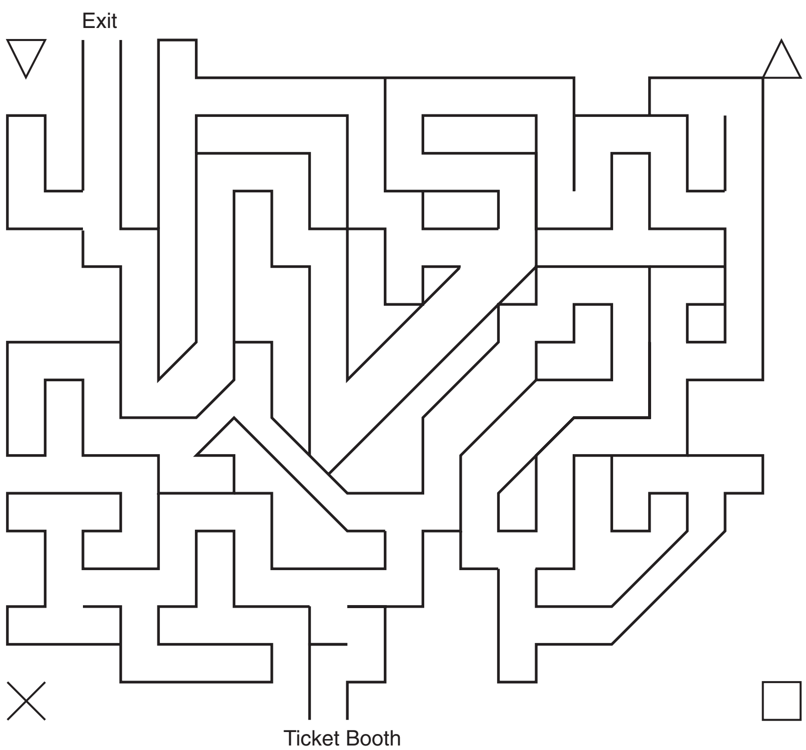
“… gain a thorough understanding of what



polygons are …”

A-MAZE-ING SHAPES teaches students about triangles and quadrilaterals, which have three and four sides respectively. Students will gain a thorough understanding of what polygons are and will be able to use related vocabulary when working with shapes and geometry. Students will understand that polygons are figures with many sides, and may be regular, with all sides of equal length and all interior angles of equal size, or non-regular, with non equal sides and angles.

The study of pathways (mazes) and networks belongs to the branch of geometry called topology. In ancient times the maze or labyrinth was an intricate structure of interconnecting passages through which it was difficult to find one’s way. These structures are said to have originated in Greek mythology and later were used as prisons or as protective barriers. A-MAZE-ING SHAPES connects the study of mazes and shapes by having students include geometric figures within their maze designs.



Assigned to study groups, students participate in specific geometric activities designed to help them gain the knowledge of geometry needed to construct their mazes. Students then design their mazes, and make formal presentations explaining how each maze contains the necessary geometric requirements.

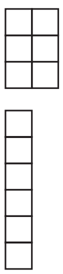
To further develop the ideas and concepts students investigate at school, A-MAZE-ING SHAPES offers information and activities for students to take home. These CHALLENGE CARDS deepen their understandings and strengthen the home-school connection. EXTENSION ACTIVITIES are included to expand concept development and make connections with other areas of the curriculum. Individual and group expectations are included to help facilitate cooperative work.

Assessment opportunities are embedded throughout A-MAZE-ING SHAPES. Students write about their understanding of geometrical concepts. Questions are provided to guide classroom discussions and for use as journal entries. Guidelines for teacher observation of student learning and a rubric for assessing the level of mastery are included.

The following are the key mathematical ideas developed throughout A-MAZE-ING SHAPES. You will assess students based on their understanding that:

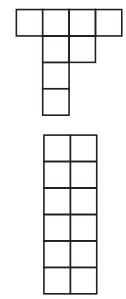
- Figures with the same area can have different perimeters and figures with the same perimeter can have different areas.
- Geometric figures have specific attributes and properties by which they are identified, classified, and named with appropriate language.
- Geometric figures can be composed of or broken down into other geometric figures.
- Relationships within and among geometric figures can be revealed through measuring and looking for patterns. Constant relationships can be expressed as formulas.
- Key terms and concepts help communicate meaning.
- Symbols are used to express geometric terms.
- An angle is defined by its measure in degrees
- A maze is a topological figure that contains a path with many blind alleys

"… same area can have



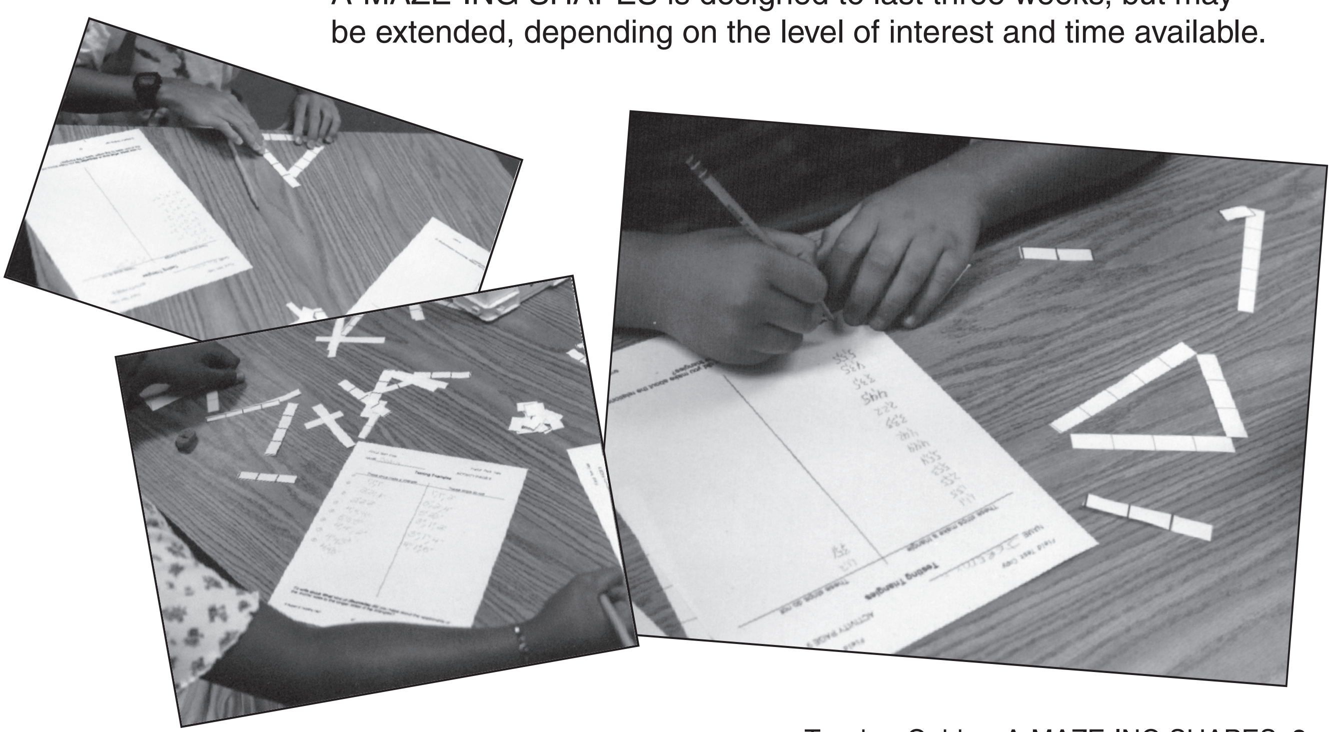
different perimeters …"

"… same perimeter can have



different areas …"

A-MAZE-ING SHAPES is designed to last three weeks, but may be extended, depending on the level of interest and time available.



One Week Before Beginning

1. Read through the entire Teacher Guide.
2. After studying the recommendations in this INITIAL PREPARATION section, decide how you will present this interactive unit to your students.

Teaching Tip

For the next three weeks students will be working in study groups. As you assign students to these groups, give careful consideration to having pairs who can work together.

Organize Your Classroom

1. Set up study groups of four students who will be working together. Most of the work will be done individually or in pairs. The sets of pairs in the study groups will also collaborate.
2. Encourage students to share their findings with or seek assistance from others in their study group.
3. Make a quick reference wall chart of the following to remind students how to work together.
   - **Student Expectations**
     TRY - Effort and participation count.
     ASK - Request help from table mates.
     HELP - Assist the group when requested.
     THINK - Go beyond what you already know.
     RESPOND - Use courtesy always.
   - **Study Group Expectations**
     EXPLORE - Look for all possibilities.
     DISCUSS - Listen to all possibilities.
     COOPERATE - Contribute ideas and suggestions.
     ENCOURAGE - Value each person's effort.

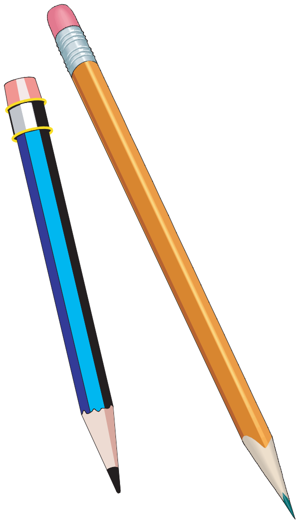
Duplication

Before starting A-MAZE-ING SHAPES, make copies of the **MASTER** pages which begin on page 55:
(**MASTER number**/quantity to duplicate are indicated in italics)

Teaching Tip

ACTIVITY A and ACTIVITY I can be made into overhead transparencies and students can use blank paper.

- PARENT LETTER—**M1** class set
- TEACHER OBSERVATION CHECKLIST—**M2** one copy per day
- ACTIVITIES A-I—**M3-M11** class set
- SHOW WHAT YOU KNOW—optional **M12-M14** class set
- CHALLENGE CARDS 1-14—optional **M15-M21** class set
- POLYGONS/NOT POLYGONS—**M22** overhead transparency
- Whole-page GRIDS A-C—**M23-M25** overhead transparency of 1-inch, 1/4-inch, and 1-centimeter grids plus two class sets of 1/4-inch and 1-centimeter grids
- Half-page GRIDS—**M26** overhead transparency and class set cut in half
- MAZES 1A-1E—**M27-M31** overlay transparency
- REGULAR POLYGONS—**M32** overhead transparency
- PROTRACTORS—optional **M33** class set
- QUIZZICAL QUADRILATERAL—**M34** overhead transparency
- MAZES 2A-2F—**M35-M40** overlay transparency
- CERTIFICATE—optional **M49** class set

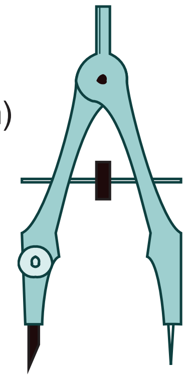


Optional EXTENSION ACTIVITIES:

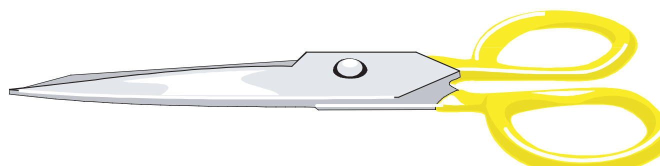
- EXTENSION DAY 1: **Fold A Shape—M41** class set
- EXTENSION DAY 8: **Angle Line Designs—M42** class set
- EXTENSION DAY 9: **Make A Protractor—M43-M44** class set
- EXTENSION DAY 12: **Triangle Tessellations—M45-M47** class set
- EXTENSION DAY 14: **Cartesian Coordinate GRID—M48** class set

Materials for Each Student

You will need several manipulatives:



- Calculator—optional
- Colored construction or tag paper (6-inch by 9-inch)
- Color tiles or 1-inch squares cut from grid paper—12 of one kind
- Compass
- Pencils will be needed every day
- colored pencils, crayons or markers
- Envelope
- Index card
- Paper strips of 1/2-inch width cut into 1-, 2-, 3-, 4-, and 5-inch lengths—three of each for Day 11
- Protractor
- Ruler
- Scissors
- Scratch papers
- Tape and glue

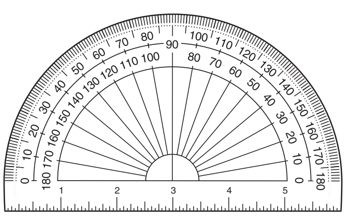


Optional EXTENSION ACTIVITIES:

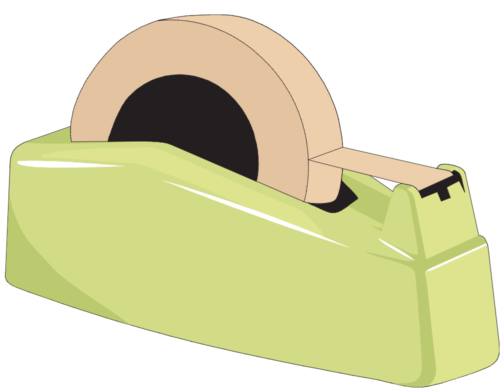
- Circular coffee filter
- String (9-inch length)

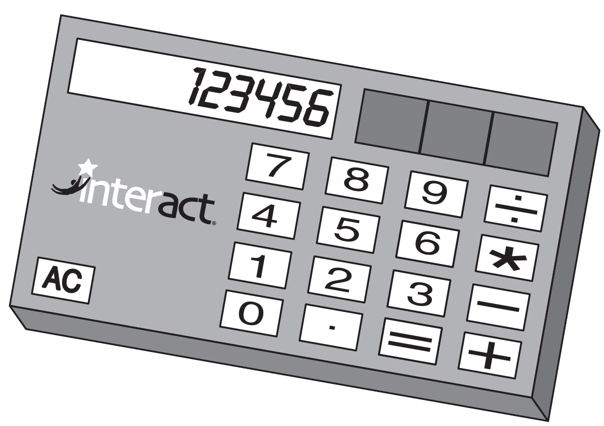
Teaching Tip

*If you do not have protractors in your classroom, use the PROTRACTOR—**M33** to make a*



transparent protractor for each pair of students and one for use on the overhead projector.





Planning Unit Activities

1. **Day 1** is for setting the stage and helping students understand the objectives of A-MAZE-ING SHAPES. Students in study groups explore how shapes fit together by creating polygons.
2. **Days 2-13** are background studies to develop the knowledge necessary for the final design. Some supplemental studies are included that could extend the length of the simulation either in amount of time on individual days or the total number of days.
3. On **Day 14** study groups design mazes.
4. On **Day 15** study groups present their final projects to the class. CERTIFICATES are included to acknowledge student efforts.

Teaching Tip

The EXTENSION ACTIVITIES may be used on days other than the days indicated.

Scheduling Time for EXTENSION ACTIVITIES

If, during the unit, you are able to add a few extra days, or perhaps add more time to an individual day, consider the optional EXTENSION ACTIVITIES provided to supplement the learning on Days 1, 8, 9, 12, and 14.

- EXTENSION ACTIVITY DAY 1: **Fold A Shape** helps students build a sense of spatial relationships.
- EXTENSION ACTIVITY DAY 8: **Angle Line Designs** builds the connection between geometry and art.
- EXTENSION ACTIVITY DAY 9: **Make A Protractor** helps students understand the relationship between the protractor and the number of degrees in a circle.
- EXTENSION ACTIVITY DAY 12: **Triangle Tessellations** builds the relationship between art, patterns, and geometry.
- EXTENSION ACTIVITY DAY 14: **Cartesian Coordinate GRID** adds another dimension to learning by having students identify the location of their geometric shapes using the Cartesian Coordinate GRID.

Teaching Tip

Interact encourages students to "write across the curriculum." Students will understand more clearly and learn more deeply if they communicate their mathematical understandings in their own words.

Discussion Questions and Student Writing

1. Daily Writing/Discussion Prompts are listed in the DAILY DIRECTIONS. Some ACTIVITY SHEETS, SHOW WHAT YOU KNOW SHEETS and CHALLENGE CARDS include writing prompts.
2. Decide how you want students to record their thinking and discoveries of A-MAZE-ING SHAPES. Two suggested ways:
   a. Keep a portfolio or folder for responses to all daily work. Have students respond on blank paper or the back of ACTIVITY pages and add these responses to their portfolio.
   b. Create a notebook (daily log or journal).

Vocabulary

1. A comprehensive vocabulary of geometric terms is provided for your reference on pages 17-19.
2. Key Vocabulary words are introduced each day in the DAILY DIRECTIONS and are listed in the STUDENT GUIDE.

Teaching Tip

Extra spaces are provided so students can define any other words you want to include.

3. Using their own words, students should write vocabulary definitions in their STUDENT GUIDES with appropriate pictures or symbols.

Challenge Cards
1. You can decide when and how often to use these activities.
2. The CHALLENGE CARDS are designed as student homework, extensions, or for learning centers. They extend and support the learning but are not necessary for daily objectives.
3. If more than one activity is offered on a CHALLENGE CARD, you may assign different activities to different study groups.
4. The CHALLENGE CARD activities correspond to the same number day (i.e., CHALLENGE CARD 1 goes with Day 1), but could also be used any time later in the simulation.
5. Since CHALLENGE CARDS are intended to extend student learning, you may want to use questions from them as part of your assessment.

Home-School Connection
1. This TEACHER GUIDE includes a sample letter designed to help families develop an understanding of the unit's objectives and to describe how students will be working together to accomplish their goals. You may decide to create your own family letter.
2. Each day CHALLENGE CARDS are available to be used as extensions or as problem solving experiences students can work on at home with their families.
3. Students are encouraged to challenge other students and their families to solve the maze designs.
4. The A-MAZE-ING SHAPES portfolios or journals can be shared with parents to demonstrate proficiency with geometric concepts.
5. Encourage students to invite families to attend the maze design presentations at the culmination of A-MAZE-ING SHAPES.

Teaching Tip

As an extension, individuals or small groups can complete short research papers, give oral presentations, or design posters on one or more of the mathematicians.

Students will enjoy and benefit from maze books to study and work through as they progress through this interaction unit.

Background Information
Pages 10-14 contain background information supporting the educational foundation for teaching geometric concepts, the history of geometry, and explanations of labyrinths, mazes, topology, and network theory. In addition it contains information on famous mathematicians who have contributed to the study of geometry. These pages may serve as a resource for student research, or simply as background information for you.

Literature Connection
Several books are listed in the BIBLIOGRAPHY (pages 15-16) and in the DAILY DIRECTIONS that will help tie in literature with the individual topics. You can use these books to enhance the introduction of topics, but they are not required to develop understanding.

“… several ways to assess student progress …”

Assessing Student Progress

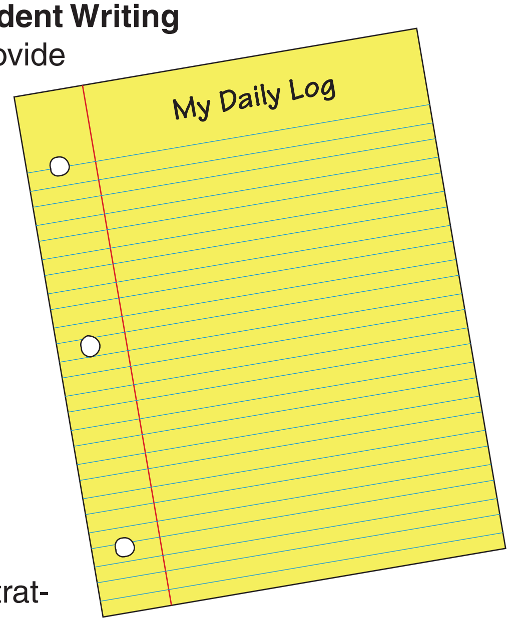
A-MAZE-ING SHAPES provides several ways to assess student progress and understanding. The need for the use of any or all of the assessment alternatives is based on your judgment as a teacher.

Teacher Observation Checklist

1. Predetermine which students you will be observing each day.
2. On the checklist you will need to complete the “ability to” columns with specific task(s) in which your students are participating.
3. As you observe each student, interview the student regarding the properties of the particular activity the student is doing based on what he/she knows about geometry. Also listen for students’ ability to correctly use geometric vocabulary.
4. You will not be able to observe every student every day; however, you should have an opportunity to observe every student at least every three to four days. Thus, you will observe every student a minimum of two to three times during the simulation.
5. You can assess students you do not observe doing a specific task by examining the level of understanding exhibited in their daily work, journal entries, and projects.

Discussion Questions and Student Writing

1. Daily log or journal entries provide a means for students to communicate their thinking as they progress through the series of geometry activities. As they record their daily experiences or write about their thinking at least once or twice a week, you will be able to evaluate the level of the students’ understanding of specific concepts.



2. Class discussion will give you insight about your students’ strategies and reasoning skills.
3. In addition to the specific discussion/writing prompts listed each day, the following questions can be used on any day for class discussion prompts, for individual student interviews, or for writing prompts.
   a. What new information did you discover today?
   b. How does this new information relate to previous lessons?
   c. Explain how you might use this lesson in a real life application.
   d. What did you value about today’s lesson?

> … journal entries, final student-created mazes, and final assessment letters …

Show What You Know

1. For specific days there are reproducible assessment questions which may be copied for students to complete.
2. You may wish to put these on an overhead transparency or chalkboard for students to respond in their journals.

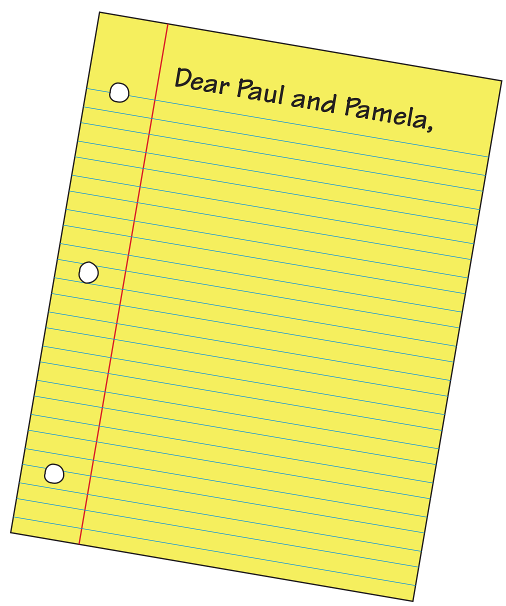
Rubric

Evaluate journal entries, final student-created mazes, and final assessment letters with the following scoring guide rubric (or one of your own):

- **4 points:** Fully accomplishes the purpose of the task; demonstrates clear understanding of geometric concepts and vocabulary; recorded work communicates thinking clearly using some combination of written, symbolic or visual means.
- **3 points:** Substantially completes the purpose of the task; displays essential grasp of the geometric concepts; recorded work communicates a large part of the thinking.
- **2 points:** Partially accomplishes the task with limited grasp of geometric concepts; recorded work may be incomplete, misdirected, or not clearly presented; strategies may be ineffectual or not appropriate.
- **1 point:** Little or no progress toward accomplishing the task; approach may lead away from task completion; shows little evidence of appropriate reasoning.

Final Assessment on Day 15

Ask each student to write a letter to Paul and Pamela explaining (proving) that they have included each design element in their maze by defining each one by properties/definition. Encourage creativity here. Allow students to write words and draw illustrations.



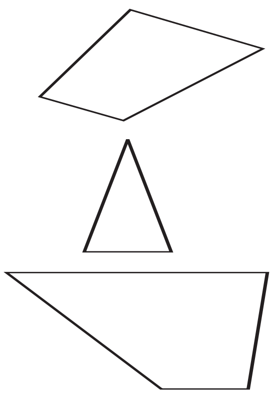
Answer Keys

Solutions for the ACTIVITIES and CHALLENGES are found on pages 51-54 of this TEACHER GUIDE.

BACKGROUND INFORMATION - 1

“

… manipulate, construct, identify,



and describe geometric shapes.

”

Van Hiele Developmental Model of Geometry

When we look at how children develop geometric concepts, the Dutch husband and wife team of Dina van Hiele-Geldof and Pierre Marie van Hiele offers a model to explain why some students can recognize a square but are unable to define its qualities, or why it may not be clear to some students why a square is also a rectangle.

Children first view shapes as total entities rather than by their parts or properties. They can recognize squares or rectangles because these are similar to shapes they have previously encountered. To begin to understand the characteristics of figures, students need opportunities to manipulate, construct, identify, and describe geometric shapes. Through observation and experimentation they learn to describe a class of figures by properties, to compare shapes, and to sort by single attributes.

In the next stage children build on their understanding of the characteristics of figures to establish the interrelationships of properties. Although students do not yet see the logical order for formal proofs, definitions become meaningful (e.g., they understand how the opposite angles of parallelograms are equal). Students need to develop and use definitions, identify minimum sets of properties that describe a figure, and study relationships while they look for implications and inclusions.

From this point students are able to move into more formal deduction and proof. Students are more able to express and exchange their emergent views about the structures they observe.

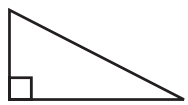
A Brief History of Geometry

In ancient times, both the Babylonians and Egyptians used geometry in land-surveying and building measurements. They used points, lines, areas, and volumes in visually appealing ways. The first formal geometry began with the Greeks. The writing of proofs became an art in which it was a matter of pride to be as economical as possible with the steps in reasoning and yet leave no loopholes. Greek mathematicians accumulated a repertoire of proved theorems, which could then be used together to formulate more advanced theorems. The basic Greek system of abstraction and proof remains intact today.

Famous Mathematicians

Thales: A salt and olive oil tycoon who operated along the coasts of Asia Minor from about 600 to 550 BCE, Thales was the first to lay down guidelines for the abstract development of geometry. In his travels he came in contact with the lore of the old mathematics and astronomy, and in his retirement he took them up as a hobby. He became the first to show a truly scientific approach to geometric problems, asking "why" as well as "how." Thales achieved fame because of his astronomical studies, mapping the stars and calculating the number of days in the year. He is recognized primarily for his scientific approach, the method of deduction which gave mathematics the philosophical foundation it needed.

"… in a right triangle



the square of the long side (hypotenuse) is equal to the sum of the squares of the other two sides …"

Pythagoras: One of Thales' star pupils was Pythagoras, who was encouraged to travel and enlarge his mathematical understanding. About 540 BCE Pythagoras founded a semi-religious, semi-mathematical cult in Crotona in Southern Italy. Along with mathematics, he taught his disciples to worship numbers and believe in reincarnation. The Pythagoreans explored number theory with a passion, discovering many relationships including the concept of perfect numbers, triangular numbers, irrational numbers, and the idea of harmonics. Pythagoras' best remembered teaching is the theorem that in a right triangle the square of the long side (hypotenuse) is equal to the sum of the squares of the other two sides ($c^2 = a^2 + b^2$). The Babylonians had discovered this property, but the Pythagorean school was the first to prove it.

Plato: This mathematician from Athens is believed to have traveled to Italy to study with the Pythagoreans who remained even after the Crotona school was disbanded and Pythagoras had died. He was the one who insisted that geometric proofs be demonstrated with no aids other than a straightedge and a compass. He demanded accurate definitions, clearly stated assumptions, and logical deductive proof.

Archimedes: Plato's work was carried on by Archimedes, the mathematician and scientist who was also an engineer. He was most famous for his skill in devising pulleys, pumps, and weapons to defend Syracuse against the Romans. He is ranked (with Newton and Gauss) as one of the three greatest mathematicians of all time. King Hieron of Syracuse asked Archimedes to find out if the royal goldsmith had cheated him by putting silver in his new gold crown. While taking his bath, Archimedes realized that if he weighed the crown in air and then while submerged in water, he could compare the two weights to determine the density of the metal from which the crown was constructed. He was so excited he rushed out of the bath and ran naked through the streets shouting, "Heureka! Heureka!" (I found it! I found it!).

"… Heureka! Heureka! …"

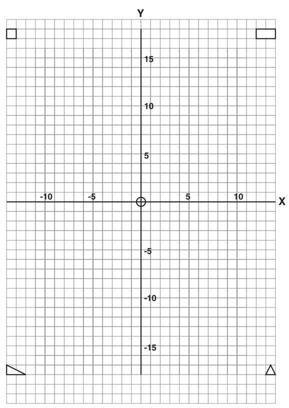
Euclid: The most famous of all the masters of geometry, Euclid, lived in about 300 BCE. He began compiling the theorems of his predecessors. He organized all known geometric thought and rewrote the proofs in clear terms in a work called The Elements.

Apollonius: Apollonius studied with the successors of Euclid. He eventually was called “The Great Geometer,” because he laid the foundations for a geometry of form and position, in contrast to the work of Archimedes, which was based on measurement.

Hypatia: The Greeks did their best to preserve the Archimedean tradition of creative inquiry in spite of the debauchery of the late Roman empire. Among the last of the few badgered Greeks was the beautiful, immensely learned woman mathematician, Hypatia, who lectured at the University of Alexandria. She was killed about 400 CE by a sectarian Christian mob. More than 1000 years passed before geometry was once again a serious area of study.

“

… Cartesian Coordinate



System …

”

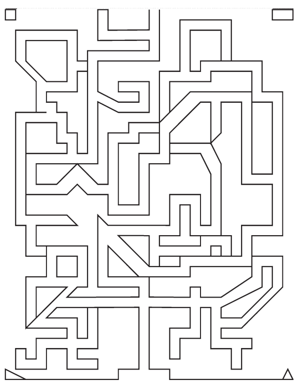
Descartes: One of the more recent mathematician giants was Rene Descartes, a French aristocrat born in 1596. At the age of 22, after earning a law degree, he began to develop “analytic” geometry. He looked for fundamental truths which could be used as basic axioms upon which other proofs could grow. While watching a fly crawling on the ceiling, he described the insect’s path in terms of its displacements along the perpendicular lines formed where the walls met the ceiling. This description was the beginning of the idea of a pair of numbers determining the position of a point on a surface, one number as a distance measured horizontally and the other number as a distance measured vertically. In his honor the rectangular coordinate system is often called the “Cartesian Coordinate System.” His method has been used to identify places on maps and atlases.

Euler: An 18th century Swiss mathematician, Leonhard Euler developed the concept of network theory. Investigating the many sided objects known as polyhedra, which might be described as networks of points and lines in three dimensions, he discovered that no matter how many faces a polyhedron has, there is a predictable relationship among the number of points, edges, and sides. It was his study of networks that led to the development of the field of topology.

Mazes and Labyrinths

Any intricately winding structure of interconnecting passages, through which it is difficult to find one’s way, can be called a labyrinth. A maze is a labyrinth that has branches leading into blind alleys or along looped paths. Long before geometric theories were formulated or described, ancient builders used labyrinths and mazes to protect their tombs, such as the mazes within the Egyptian pyramids. There are labyrinths carved in rock in several areas in Europe that date back to as early as 2000 BCE. The labyrinth seems to have often been associated with birth or death in many ancient cultures. One of the practical uses of a labyrinth in ancient times was as an entrance to a fortress, such as the one at Troy. Anyone entering was forced to travel a great distance while exposed to bombardment from the defenders. Because they have an entrance and/or exit, mazes can be said to have no inside; all the paths in a maze connect to the outside without crossing any boundaries.

"... mazes can be said



to have no inside ..."

The Minotaur: The Greek myth of the Minotaur describes a labyrinth and is supported by coins and rock carvings dating back to a few centuries before Christ. The wife of King Minos of Crete became infatuated with a white bull sent by Poseidon. The resulting offspring was the Minotaur, who had a human body with a bull's head. When the son of the king Minos was murdered, he blamed the people of Athens. He ordered them to send a tribute of seven youths and seven maidens every nine years. They were sent one by one into a vast and intricate labyrinth where they would be slain by the Minotaur. Theseus, the son of the king of Athens, was able to slay the Minotaur with the help of Ariadne, the daughter of Minos. She fell in love with Theseus, and gave him a ball of thread that he could unwind while he walked through the labyrinth and then retrace in order to find his way out. The myth was corroborated by 1899 excavations which uncovered a palace dating to 1600 BCE with many rooms and winding corridors, as well as frescos depicting bulls and labyrinths.

Church Labyrinths: Church labyrinths are carved or represented by tiles. The winding path allows anyone walking the path to think seriously (meditate) about his or her life. It can also symbolize the twists and turns of the Christian's life journey, spent avoiding or succumbing to temptations along the road to salvation.

Garden and Turf Mazes: Garden mazes, made of hedges or shrubbery, were very popular in 18th century Europe. One of the most famous is located at the Renaissance palace of Hampton Court, which still exists today just outside of London, England. Turf mazes are spiral whorls cut into the turf in many British villages. They are sometimes a path formed by a trench or by raised turf. The courses might have been used by Roman soldiers about the time of Christ as a kind of running track.

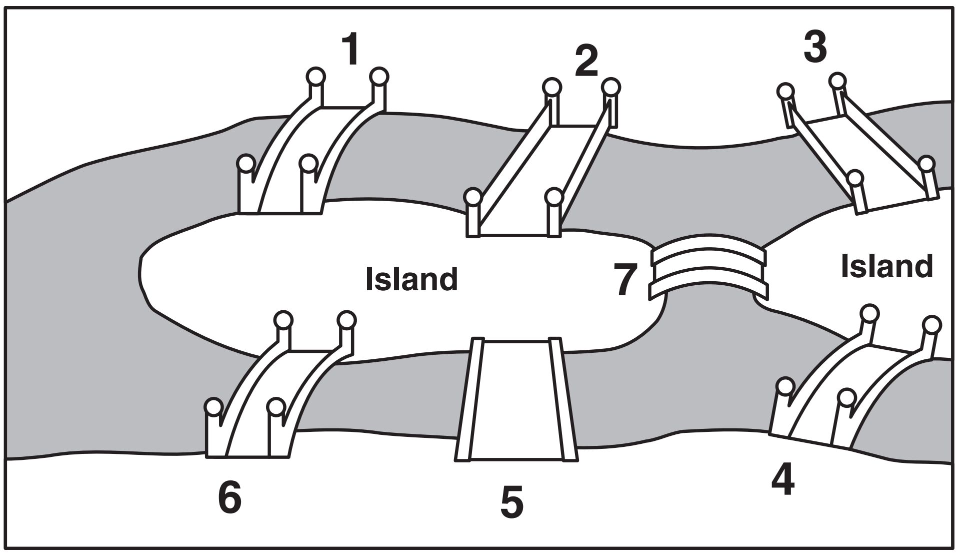
Topology

Topology: The branch of geometry concerned with the ways in which surfaces can be twisted, bent, pulled, stretched or otherwise deformed from one shape into another is called topology. An example is the reflection in a fun house mirror: each point in the reflection has a one-to-one correspondence with the original image. The study of pathways, mazes, and networks forms the branch of topology known as network theory.

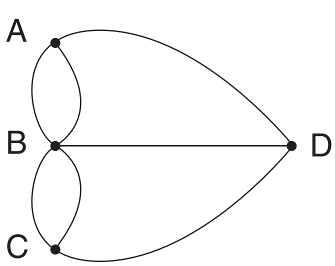
Network Theory: People who designate one-way streets and make other decisions regarding traffic flow in big cities work with network theory. Network theory, one of the most practical forms of topology, has applications to electrical circuitry and economics. It was originated over 200 years ago by Leonhard Euler, the Swiss mathematician who solved the "Bridges of Konigsberg" problem 100 years before topology had even been named.

The Bridges of Konigsberg: It had been a tradition among the townspeople of Konigsberg that the seven bridges could not be crossed in a continuous walk without recrossing the route at some point, but no one knew the explanation.

" … the seven bridges could not be crossed in a continuous walk without recrossing the route at some point … "



When Euler heard of the Konigsberg bridges, he wondered if he could find a route that would take someone over the city's seven bridges without crossing any bridge more than once. Euler worked with a map of the bridges and finally decided the problem was not solvable. He realized an important principle was involved and proceeded to demonstrate mathematically why such a walk was impossible. He drew a diagram to help visualize the bridge problem. Notice that vertices A, C, and D each have three pathways coming together. Vertex B has five pathways through it. Thus, there are four odd vertices, and the seven bridges cannot be crossed without retracing the route.

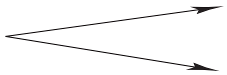


Euler developed two rules that would apply to other problems as well as the Konigsberg Bridge problem:

1) If a network contains only even vertices (with an even number of pathways coming together at one point), it can be traced in one continuous line, without retracing.

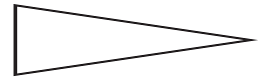
2) If a network contains exactly two odd vertices (with an odd number of pathways coming together at two points) it can be traced in one continuous line without retracing.

VOCABULARY - 1



Acute angle: an angle with less than 90°

Acute triangle: a triangle with all acute angles

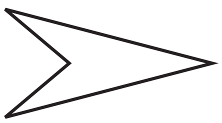


Angle: the space enclosed when two rays (or sides) come together

Area: a flat surface in a plane with a closed border; expressed in square units

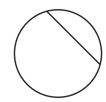
Axes: numbered lines on a grid used to locate points and functions on a grid or graph

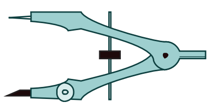
Bisect: to divide exactly into two equal parts



Chevron: a four-sided figure with one concave vertex whose two adjacent sides are congruent with each other, and with two opposite adjacent sides that are also congruent with each other; contains only one line of symmetry

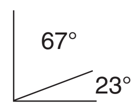
Chord: a line segment that connects two points on a circle



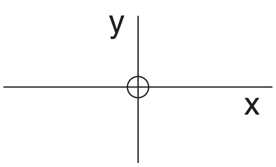


Compass: an instrument used to construct geometric figures

Complementary angles: two angles whose measures add up to 90°

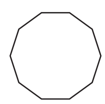


Congruent (≅)**:** angles, segments or figures having the exact same size and shape

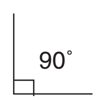


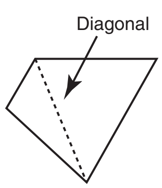
Coordinates: X and Y coordinates are counted off on axes to identify points and functions on a grid or graph

Decagon: a 10-sided polygon



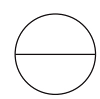
Degree (°)**:** unit of measure for an angle

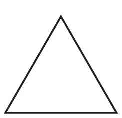




Diagonal: a line segment (that is not a side) that joins two vertices of a polygon

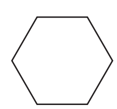
Diameter: a chord that passes through the center of a circle

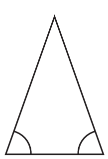




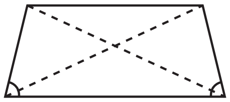
Equilateral triangle: a triangle where all sides are of equal length and all angles are congruent

Hexagon: a polygon with six sides

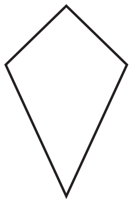




Isosceles triangle: a triangle with two sides congruent

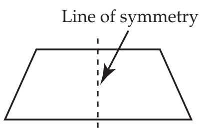


Isosceles trapezoid: a trapezoid where the diagonals are equal in length and there is exactly one line of symmetry; these properties cannot be generalized to any trapezoid.

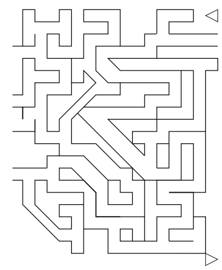


Kite: a four-sided figure with all vertices convex where two adjacent sides are congruent with each other and the two opposite adjacent sides are also congruent with each other; contains only one line of symmetry.

Line segment: a straight line named by its endpoints.

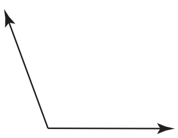


Line of symmetry: a line that divides a figure into two parts that match if the figure is folded along the line

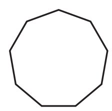


Maze: a confusing, intricate network of winding pathways with one or more blind alleys

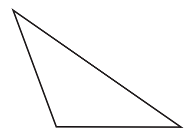
Midpoint: the point that separates a line segment into two congruent segments



Nonagon: a nine-sided polygon

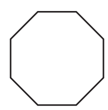


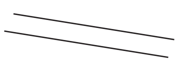
Obtuse angle: an angle with more than 90° but less than 180°



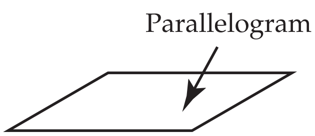
Obtuse triangle: a triangle with one obtuse angle

Octagon: an eight-sided polygon



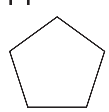


Parallel (II)**:** any lines in a plane that remain an exact distance apart and never intersect

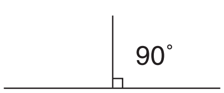


Parallelogram: a quadrilateral with opposite sides parallel and congruent

Pentagon: a five-sided polygon



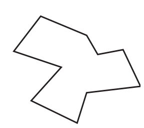
Perimeter: the distance around the outside of a plane figure



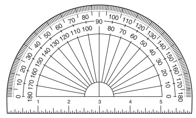
Perpendicular (⊥)**:** intersecting lines that form a 90° angle

Plane: a flat surface that continues infinitely in all directions

Polygon: a flat closed shape with three or more straight sides

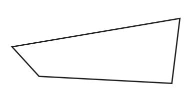


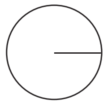
Point: an element with no dimension (size, shape, or extension); represented by a capital letter (as in point A)



Protractor: an instrument used to measure an angle in degrees

Quadrilateral: a four-sided polygon

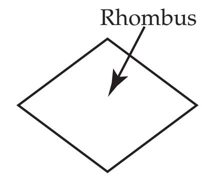




Radius: a straight line segment extending from the center to the periphery of a circle.

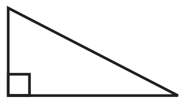
Ray: a straight line with one endpoint

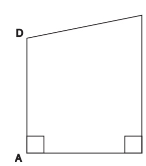
Rectangle: a quadrilateral that has opposite sides parallel and congruent, and all angles are 90°



Rhombus: a parallelogram with all sides congruent; has two diagonal lines of symmetry

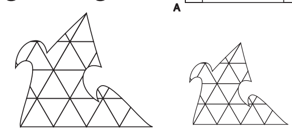
Right angle: an angle of exactly 90°



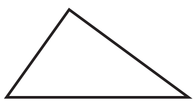


Right triangle: a triangle containing a right angle

Right trapezoid: a trapezoid containing two right angles

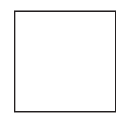


Scale: the ratio between the dimensions of a map or model and the thing it represents

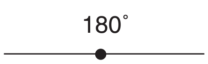


Scalene triangle: a triangle with all sides of different lengths

Similar figures: two figures of the same shape but different sizes

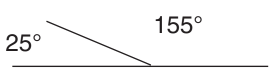


Square: a special kind of rectangle with four congruent sides and with four lines of symmetry

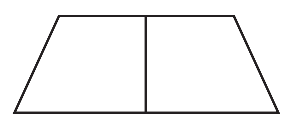


Straight angle: an angle that measures 180°

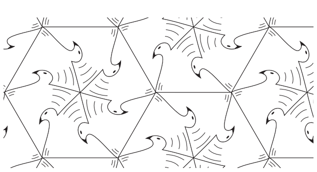
Straight line: the shortest distance between two points, named by any two points on the line; continues forever in both directions



Supplementary angles: two angles whose measures have a sum of 180°

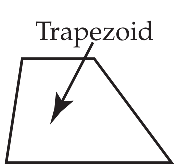


Symmetrical figure: a figure having two halves that are reflections of one another

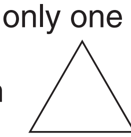


Tessellation: a design that covers a plane with no gaps or overlaps

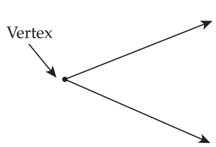
Topology: the study of those properties of geometric figures that remain unchanged even when distorted, as in how a maze is connected together



Trapezoid: a quadrilateral with only one set of parallel sides



Triangle: a three sided polygon



Vertex: the point where two rays meet to form an angle

Intended as example only. Alter as desired.

UNIT TIME CHART

Day 1	Day 2	Day 3	Day 4	Day 5
Seeing how Shapes Fit Together Introduce scenario ACTIVITY A CHALLENGE CARD 1 **Optional** EXTENSION ACTIVITY	**Comparing the Same Areas with Different Perimeters** ACTIVITY B SHOW WHAT YOU KNOW Day 2 CHALLENGE CARD 2	**Varying Area and Perimeter in Relationship to One Another** ACTIVITY C SHOW WHAT YOU KNOW Day 3 CHALLENGE CARD 3	**Investigating Area in Relationship to Perimeter & using Scale Drawings** ACTIVITY D CHALLENGE CARD 4	**Creating a Maze** CHALLENGE CARD 5

Day 6	Day 7	Day 8	Day 9	Day 10
Determining the Shapes of Quadrilaterals ACTIVITIES E, F, and G CHALLENGE CARD 6	**Investigating the Properties of Quadrilaterals** ACTIVITIES E, F, G, and H SHOW WHAT YOU KNOW Day 7 CHALLENGE CARD 7	**Determining the Types of Angles** QUIZZICAL QUADRILATERAL CHALLENGE CARD 8 **Optional** EXTENSION ACTIVITY	**Measuring Angles** SHOW WHAT YOU KNOW Day 9 CHALLENGE CARD 9 **Optional** EXTENSION ACTIVITY	**Determining the Types of Triangles** SHOW WHAT YOU KNOW Day 10 CHALLENGE CARD 10

Day 11	Day 12	Day 13	Day 14	Day 15
Investigating how Sides Determine Triangles ACTIVITY I CHALLENGE CARD 11	**Identifying Triangles** ACTIVITY I SHOW WHAT YOU KNOW Day 12 CHALLENGE CARD 12 **Optional** EXTENSION ACTIVITY	**Constructing Angles and Triangles Using a Compass** CHALLENGE CARD 13	**Designing Mazes Using Required Shapes** CHALLENGE CARD 14 **Optional** EXTENSION ACTIVITY	**Presenting Final Mazes**

Day 1: Seeing how Shapes Fit Together

Key Vocabulary

Plane, Polygon, Rectangle, Square, Triangle

Literature Connection

- Shapes, Shapes, Shapes
- Sadako and the Thousand Paper Cranes
- Grandfather Tang's Story

Materials for Teacher

1. TEACHER OBSERVATION CHECKLIST—**M2** copy
2. Chart of student and group expectations (see page 4)
3. POLYGONS/NOT POLYGONS—**M22** overhead transparency

Materials for Each Student

1. ACTIVITY A: **Create A Polygon**—**M3** copy
2. STUDENT GUIDE
3. Construction paper, scissors, glue, and ruler—one of each
4. CHALLENGE CARD 1: **Toothpick Puzzles**—optional **M15** copy
5. EXTENSION ACTIVITY: **Fold A Shape**—optional **M41** copy

Procedure

1. Assign students to study groups. See the INITIAL PREPARATION section, page 4, for suggestions.
2. Review the Chart of Student and Group Expectations (especially if your students are not used to working in cooperative groups).
3. Introduce the scenario of the simulation to students:

Read or Tell

"Geo World, a geometry theme park, will open next fall. Incorporating state-of-the-art computer generated thrills, each ride and exhibit will offer experiences found no where else on earth. Paul and Pamela Pythagoras, descendants of the great mathematician, are building the theme park using the royalties earned from his famous theorem.

Paul plans four entrances, with each entrance offering a different maze to challenge park visitors. All visitors must find their way through one of these mazes in order to enter the park.

Paul and Pamela are asking your help to design the mazes for each of the entrances. Use your creativity and design a maze, making it as challenging as possible. The maze will, of course, mainly be a pathway into the park.

Teaching Tip

Students can read along using their copy of the scenario in the STUDENT GUIDE.

The idea of the maze is a good motivator for students.

DAILY DIRECTIONS - 2

This entrance maze must also contain the following design elements:

- A rectangular perimeter of less than or equal to 640 meters
- An area of less than or equal to 26,000 square meters
- A minimum of four different types of quadrilaterals
- A minimum of one isosceles triangle
- A minimum of one equilateral triangle
- A minimum of one scalene triangle
- A minimum of one right triangle
- A minimum of two right angles
- A minimum of two acute angles measuring less than 60°
- A minimum of two obtuse angles measuring more than 100°
- The solution path through the maze must touch each of the required polygons

In order to design the entrance mazes designers need to understand all of these geometric requirements and how they can fit together.

Teaching Tip

If you plan to have every student present his or her own maze design on Day 15, you will need to modify this instruction.

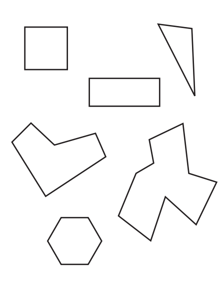
Beginning today you will meet with a study group of three other students. Together you will study and discuss the polygons as they are introduced. After learning about the polygons, each individual will be responsible for designing and building your own maze. Then your study group will select one design to submit for final approval. Your group will write a proposal to Pamela and Paul proving that your selected maze contains all of the required design elements. Your group will also present your maze to the whole class and will demonstrate how it contains the required design elements. Good luck.

4. To clarify for students what defines a polygon, use examples like the ones below **M22** on the board or overhead to demonstrate the attributes of polygons. Help students come up with the definition of a polygon (a closed shape in a plane with straight sides).

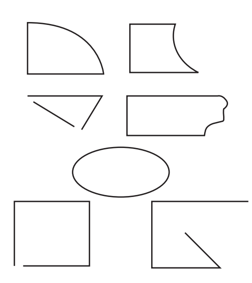
If you have a map of the United States available, you may want to have students look at it.

- *Are any of the states polygons?*
- *What states are polygons?*
- *What states are not? Why?*

These are polygons.



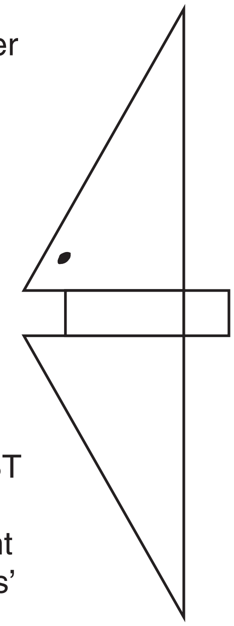
These are not polygons.



Teaching Tip

This activity should be left open-ended to allow you to assess the students' understanding of triangles, squares, and rectangles, and their ability to measure.

5. Give each student a copy of ACTIVITY A: **Create A Polygon**. Review these instructions with the students:
   a. Cut the construction paper to a size no larger than 3" x 5." The reason they have a bigger piece to start with is to allow for mistakes.
   b. Cut the rectangle to form a square, a rectangle and two triangles.
   c. DO NOT tell them exactly how to do this because there are a variety of ways in which the rectangle can be cut into the four pieces.



Assessment

1. Use the TEACHER OBSERVATION CHECKLIST as you walk around the classroom and observe students at work. This activity offers an excellent opportunity for pre-assessment of your students' ability to measure and their understanding of basic geometric shapes.
   a. It would be appropriate to complete the columns to read "the ability to measure" and "the ability to understand shapes."
   b. The following are student behaviors to look for:
      - Can they use a ruler to measure equal sides on the square?
      - Do they understand that a square has right angles and sides of equal length?
      - Do they understand that a triangle has three sides?
      - Can they construct a square, triangle, rectangle?
   c. For those students whom you are not able to observe, the finished projects will allow you to assess their ability to measure and their understanding of the shapes.
2. Writing Prompts
   a. Write definitions of **Key Vocabulary** words in the STUDENT GUIDES.
   b. How do you know a shape is a square?
   c. How do know when a shape is a rectangle?
   d. How do you know when a shape is a triangle?
   e. ACTIVITY A: **Create A Polygon** Write about your polygon and tell how you made it.

Optional

- CHALLENGE CARD 1: **Toothpick Puzzles**
- EXTENSION ACTIVITY DAY 1: **Fold A Shape**

DAILY DIRECTIONS - 4

Day 2: Comparing the Same Areas with Different Perimeters

Key Vocabulary

Area, Line segment, Perimeter

Literature Connection

- How Big is a Foot?
- Length

Materials for Teacher

1. TEACHER OBSERVATION CHECKLIST—**M2** copy
2. ACTIVITY B: **Paul's Perplexing Plan**—optional **M4** overhead transparency
3. Whole-page GRID A (1-inch)—**M23** overhead transparency
4. Color tiles or 1-inch squares—six of one kind

Materials for Each Student

1. ACTIVITY B: **Paul's Perplexing Plan**—**M4** copy
2. STUDENT GUIDE
3. Color tiles or 1-inch squares cut out from Whole-page GRID A—**M23**/12 of one kind
4. Whole-page GRID A (1-inch)—**M23** two copies
5. Pencils and crayons or markers—one of each
6. SHOW WHAT YOU KNOW Day 2—optional **M12** copy
7. CHALLENGE CARD 2: **Fencing**—optional **M15** copy

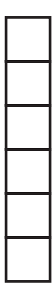
Teaching Tip

This activity can also be done using centimeter cubes and centimeter grid paper.

Procedure

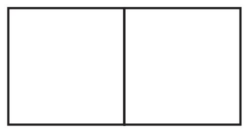
1. Provide each student with twelve square tiles, grid paper, and ACTIVITY B: **Paul's Perplexing Plan**.
2. Read the problem together.
3. Using the whole-page GRID transparency and six tiles or squares, demonstrate how to determine area and perimeter:
   a. Placing six tiles on six squares of the grid, given that each tile represents one square foot, demonstrate that in a 1 x 6 area there would be six square feet of land.
   b. Square units are used for area because each one covers an entire flat square of surface.
   c. Show how to count the perimeter then label: Write "Area equals 6 square units" and "Perimeter equals 14 units."
   d. This is a good time to explain that we can write these using a short-cut method:
   (**A** stands for area and **P** represents perimeter.)

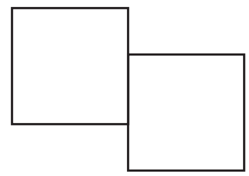
   A = 6 square units and P = 14 units.

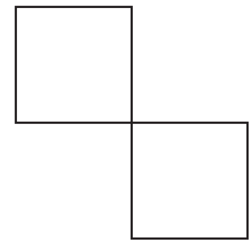


f. Demonstrate how to record a shape made of 1-inch tiles on the centimeter grid paper on the ACTIVITY sheet.

Like this: **Not like this or this:**







4. Allow students to work individually on ACTIVITY B, then compare findings with a partner and/or their group.
5. At the end of the period, have groups share their findings with the whole class.

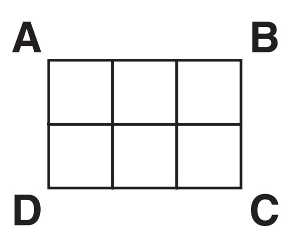
Teaching Tip

Be sure that students understand that the squares should match up with whole sides together.

The smallest possible perimeter is 14. The largest perimeter is 26. All perimeters are even numbers. If a student finds an odd numbered perimeter or one greater than 26, have him or her count the number of squares and perimeter lengths again.

Assessment

1. Use the TEACHER OBSERVATION CHECKLIST to:
   a. Observe the ability of students to create and record a variety of shapes using 12 squares
   b. Identify the correct perimeter for each shape, and
   c. Recognize the greatest and smallest perimeters.
2. Writing Prompts
   a. Write definitions of **Key Vocabulary** words in the STUDENT GUIDES.
   b. ACTIVITY B: **Paul's Perplexing Plan** Write a letter to Paul and tell him what would be the best arrangement for his area of 12 square feet to have the smallest perimeter. Be sure to draw him a picture so he will understand your explanation.
3. Review how to label rectangles and line segments. Using SHOW WHAT YOU KNOW Day 2, either as a handout, on a chalkboard, or as an overhead transparency, have students describe what they would do to find the perimeter of a rectangle ABCD.



A = 6	AB = 3
P = 10	BC = 2
	CD = 3
	DA = 2

4. The students' grid papers from the activity, with multiple layouts, and their letters and diagrams will indicate their understanding of the area and perimeter concepts.

Optional

- CHALLENGE CARD 2: **Fencing**

Day 3: Varying Area and Perimeter in Relationship to One Another

Materials for Teacher
1. TEACHER OBSERVATION CHECKLIST—**M2** copy
2. Whole-page GRID B (1/4-inch)—**M24** overhead transparency
3. Half-page GRIDS (4- x 6-inch)—**M26** overhead transparency

Materials for Each Student
1. ACTIVITY C: **Investigating Area & Perimeter**—**M5** copy
2. Whole-page GRID B (1/4-inch)—**M24** copy
3. Half-page GRIDS (4- x 6-inch)—**M26** copy
4. Scissors
5. SHOW WHAT YOU KNOW Day 3—optional **M12** copy
6. CHALLENGE CARD 3: **Finding Cost**—optional **M16** copy

Read or Tell

Procedure
1. Set the scene for today's activities:

   "You will be making a recommendation to Paul and Pamela as to what dimensions the entrance maze needs to be in order to have a perimeter no larger than 640 meters, and an area no larger than 26,000 square meters. During the next two days we will be investigating the relationship of area to perimeter."

2. Tell students that they will learn to develop a formula for describing facts about rectangular shapes:
   a. Using the overhead, have students look at one 4 x 6 GRID. Ask the following questions:
      - What is the area?
      - How did you get it?
      - Is there an easier way than counting to arrive at 24?
      - Do you see a relationship between the lengths of the sides and the area?
   b. Show a 3 x 5 rectangle. Again ask the previous four questions. Lead students through the questions to reach the conclusion that for a rectangle, A = L x W.
   c. Then look again at the 4 x 6 GRIDS:
      - What is the perimeter?
      - How did you find it?
      - Did you count?
      - Is there an easier way?
   d. Lead students through questions to reach the conclusion that P = 2 (L + W) and show how this formula fits a rectangle. Ask them if this formula will work for any of the four-sided shapes they can make, or just a rectangle.
   e. Then use figure A of ACTIVITY C to verify the method.

Teaching Tip

You can make a 3 x 5 rectangle for the overhead by covering a portion of a 4 x 6 GRID.

Figure A

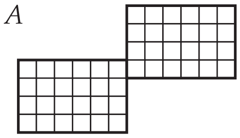


Figure B

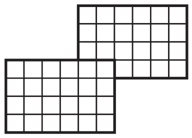
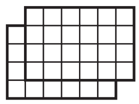


Figure C



3. Distribute ACTIVITY C: **Investigating Area & Perimeter.**
   a. Demonstrate how to find the area and perimeter of example A on the worksheet by using the overhead transparency of two 4- by 6-inch GRIDS.
   b. Overlap the overhead transparencies of two 4- by 6-inch GRIDS to show how to put shape B together.
   c. Have students complete the area and perimeter for B and C.
   d. Tell students they will be using their 4- by 6-inch GRIDS to:
      - Find as many whole number perimeters as they can;
      - Draw each figure they find and record the area and perimeter of each figure on their grid paper;
      - Note the largest and smallest perimeters; and
      - Note the largest and smallest areas without completely covering one rectangle with the other.
   e. Demonstrate figure B or C on the 1/4-inch GRID overhead transparency. Be sure students notice that the rectangles are overlapping. Write the area and perimeter on the figure.
   f. Allow students to work on ACTIVITY C.
   g. Have students share their findings with their partner/group.
      - Did they find different areas and perimeters?
      - Same? Larger or smaller?
   h. Have groups share some of their findings with the whole class.

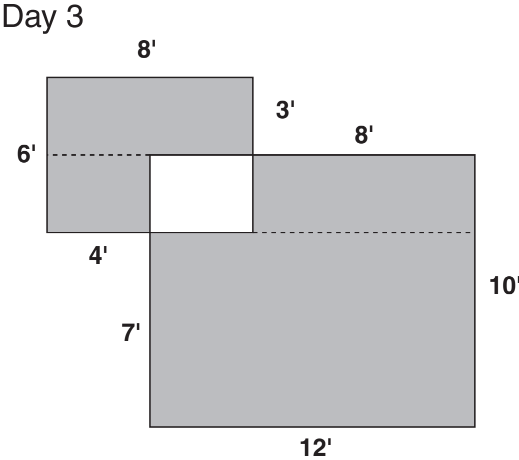
Assessment

1. Using the TEACHER OBSERVATION CHECKLIST, assess the students' ability to find the area of a polygon and their ability to find the perimeter of a polygon.
2. Writing Prompts
   a. ACTIVITY C: **Investigating Area & Perimeter** Compare the shape you created that has the largest perimeter and the shape you created with the largest area.
      - Are they the same shape?
      - What can you say about the relationship between the area and the perimeter of these shapes?

Teaching Tip

If you separate the two rectangles it is easier to see that the area of the white portion must be subtracted from both rectangles. The answer is 144 square feet. Another way to approach this is to subdivide the figure into smaller rectangles.

3. SHOW WHAT YOU KNOW Day 3
   Students should explain how they found the area of the shaded region. Show this figure on the board or overhead. Distribute copies of the figure to students and remind them to look at the overlapping rectangles.



Optional

- CHALLENGE CARD 3: **Finding Cost**

Day 4: Investigating Area in Relationship to Perimeter

Key Vocabulary

Scale

Literature Connection

- Gulliver's Travels
- Jim and the Beanstalk

Materials for Teacher

1. TEACHER OBSERVATION CHECKLIST—**M2** copy
2. Whole-page GRID C (1-centimeter)—**M25** overhead transparency

Materials for Each Student

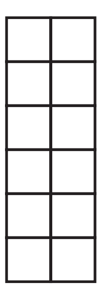
1. ACTIVITY D: **Maximizing Area**—**M6** copy
2. STUDENT GUIDE
3. Whole-page GRID C (1-centimeter)—**M25** copy
4. CHALLENGE CARD 4: **Area & Perimeter**—optional **M16** copy

Read or Tell

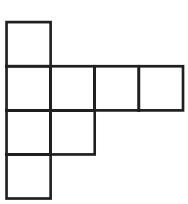
Procedure

1. Set the scene for today's activities:

   "We are going to help Paul and Pamela to determine the best dimensions of the entrance maze. Paul and Pamela want to see how you "measure up" to the challenge. We know the perimeter will be no larger than 640 meters, with an area no larger than 26,000 square meters. Since you cannot draw that large an area on paper, your maze is going to be a scale drawing of the entrance maze. Scale refers to the ratio of the size of the maze design compared to the size of the actual entrance. On our drawings 1 centimeter equals 10 meters of actual length. Paul and Pamela have requested that all scale drawings submitted for consideration fit on a single piece of 8.5" x 11" paper. We are going to begin by using a smaller number of units to see how to represent a large area with a small drawing."



2. Using your overhead transparency of 1-centimeter GRID, show areas as in these examples, and demonstrate how the perimeter remains 16 while the area changes.



Teaching Tip

3a. Students should discover that the smallest area will be a rectangle with one side being only one unit wide, and that the largest area will be a square.

3. Distribute ACTIVITY D: **Maximizing Area** and the 1-centimeter GRID paper.
   a. Challenge the students to find the largest rectangular area possible and smallest rectangular area possible using perimeters of 16 and 24.
   b. After students have completed the first two perimeters, have them share their results.
   c. They should discover the shapes needed for the largest and smallest area.
   d. Have students work in their study groups to figure out the largest and smallest area when the perimeter is 100 units.
   e. The students will be limited as to the ways they can draw rectangles with a perimeter of 100. With such a large perimeter students will have to make a conjecture based on their discoveries with smaller perimeters.
   f. Have study groups report their findings for rectangles with a perimeter of 100 to the total class and explain their thinking.
   g. For discussion:
      - How many centimeters would you use to represent 640 meters?
      - What is the maximum area that you can use?
      - In your study groups you need to determine what shapes you could use that would give you a 640 meter perimeter.

Assessment

1. Using the TEACHER OBSERVATION CHECKLIST, assess students' ability to:
   a. find a variety of rectangular shapes with a given perimeter, and
   b. identify the largest and smallest area possible with a given perimeter.
2. Writing Prompts:
   a. Write definitions of **Key Vocabulary** words in the STUDENT GUIDES.
   b. ACTIVITY D: **Maximizing Area** Compare the dimensions of the rectangles with the greatest area. What is true about the rectangles that have the greatest area?
   c. As a follow-up to the discussion outlined in 3g above, have students write a letter to Paul and Pamela telling them what the dimensions of the entrance area should be to give them the maximum area, and to draw a picture so Paul and Pamela understand their descriptions.

2c. The perimeter of the entrance maze can be 640 meters. 160 m x 160 m results in an area of 25,600 square meters.

Optional

- CHALLENGE CARD 4: **Area & Perimeter**

Day 5: Creating a Maze

Teaching Tip

For background information on mazes, network theory, and topology, see pages 12-14 of this TEACHER GUIDE.

Key Vocabulary

Maze, Topology

Literature Connection

- Amazing Mazes
- Fish Mazes

Materials for Teacher

1. TEACHER OBSERVATION CHECKLIST—**M2** copy
2. MAZES 1A-1E—**M27-M31** overlay transparency

Materials for Each Student

1. STUDENT GUIDE
2. Whole-page GRID C (1-centimeter)—**M25** copy
3. Colored pencils and ruler
4. CHALLENGE CARD 5: **Networks**—optional **M17** copy

Procedure

Read or Tell

1. Set the stage for maze design:

   "Our goal is to help Paul and Pamela develop a maze that includes all of their required design elements. In order to do that we must learn how to make a maze with one solution path. Later, as we learn more about the design elements that Paul and Pamela require we will examine ways to include these design elements in the final maze. Between now and then you will want to practice making mazes and try them out on your classmates and family members."

2. Demonstrate how to develop a maze using these steps:
   a. Start with the MAZE 1-A GRID **M27** overlay.
   b. Using MAZE 1-B **M28** overlay, line up the four symbols in the corners so the grids fit together. Demonstrate how to make a circuitous path by always going from square to square directly, and only going through the corners to make angles.
   c. After the path is completed, using MAZE 1-C **M29** overlay, demonstrate how to draw around the path to make it a road. It would be helpful for students to use colored pencil (lightly) for this step to show clearly where the solution path is. Take away MAZE 1-B **M28** overlay so that students can see the solution pathway.

d. Demonstrate how to make dead-end avenues by erasing segments and adding pathways that lead nowhere, using MAZE 1-D **M30** overlay. Emphasize the importance of having an end point in the dead-ends. The boundaries (sides) of the dead-end avenues must not cross the original pathway or the solution could be blocked.
e. Using MAZE 1-E **M31** overlay, demonstrate how the completed maze will look with no grid lines by removing all other MAZE 1 overlays.

Teaching Tip

Pencils are recommended so barriers can be erased to add dead-ends.

You might want to have maze books available for students to try to solve additional mazes. (See BIBLIOGRAPHY, pages 15-16)

3. Students should begin work on developing their own maze designs. Consultation within study groups is encouraged, but every student must develop his or her own maze. After students have experimented with drawing mazes on the grid papers, encourage them to try designing a maze on plain paper over the grid so that the grid lines don't inhibit their creativity; or to try creating a maze freehand.
4. After the mazes have been completed, allow students to challenge others in their study groups to try to solve them. Members of the group will need to check each other to make sure there is a way to solve the maze.
5. Most students will probably not finish a very involved maze in one day. Students should be encouraged to continue working on mazes during the rest of the simulation. As students are introduced to new polygon shapes and types of angles, encourage them to think of ways to add them to their mazes. These polygons and angles comprise the design elements of the simulation maze.

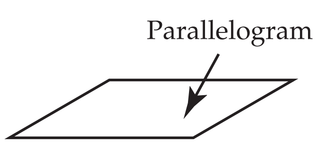
Assessment

1. Using the TEACHER OBSERVATION CHECKLIST, observe students' ability to create a maze pathway and add dead-ends.
2. Writing Prompts
   a. Write definitions of **Key Vocabulary** words in the STUDENT GUIDES.

Optional

- CHALLENGE CARD 5: **Networks**

Day 6: Determining the Shapes of Quadrilaterals

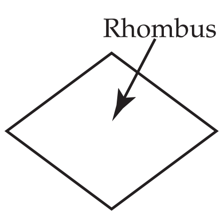


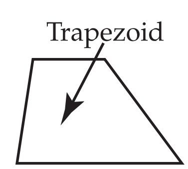
Key Vocabulary

Parallelogram, Quadrilateral, Rhombus, Trapezoid

Literature Connection

- The Village of Round and Square Houses
- The Shiek of Shapes





Materials for Teacher

1. TEACHER OBSERVATION CHECKLIST—**M2** copy
2. REGULAR POLYGONS—**M32** overhead transparency

Materials for Each Student

1. ACTIVITY E: **Trapezoids**—**M7** copy
2. ACTIVITY F: **Rhombuses**—**M8** copy
3. ACTIVITY G: **Parallelograms**—**M9** copy
4. STUDENT GUIDE
5. CHALLENGE CARD 6: **Rectangles**—optional **M17** copy

Procedure

Read or Tell

1. Set the scene for today's activities:

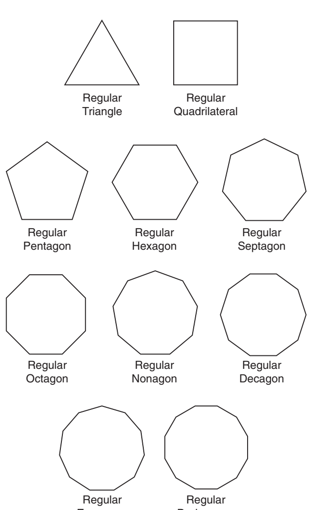
   "A variety of triangles and quadrilaterals are required as part of the maze entrances. These are two types of polygons. There are many other types of polygons and their names are determined by the number of sides. Triangles (three sides) and quadrilaterals (four sides) are the two types required by Paul and Pamela. Other polygons you may be familiar with are the pentagon (five sides) and hexagon (six sides). Names of the quadrilaterals we will explore today are the rhombus, parallelogram and trapezoid."

Teaching Tip

Use the overhead transparency of the polygon shapes as you read this, to display the different types of shapes.

For further discussion, students could locate these shapes in the classroom or the environment.

2. Review definitions students developed on day one for square and rectangle, using student vocabulary lists. Ask what makes a square a special rectangle (because in addition to meeting the definition of rectangle, the four sides are of equal length).
3. Have students complete the three ACTIVITY pages. Students should be encouraged to discuss the properties of the shapes with the members of their group.
4. When this activity is completed, have the students discuss their findings and use what they have learned to label all the shapes except the triangles on page 7 in their STUDENT GUIDES.



Assessment

1. Using the TEACHER OBSERVATION CHECKLIST, look for the ability to correctly identify a rhombus, trapezoid, and parallelogram.
2. Writing Prompts
   a. Complete definitions of **Key Vocabulary** words in the Student Guides.
   b. ACTIVITY E: **Trapezoids** What are the properties of a trapezoid?

c. ACTIVITY F: **Rhombuses** What are the properties of a rhombus?
d. ACTIVITY G: **Parallelograms** What are the properties of a parallelogram?
e. Can a square be a rhombus? How do you know?

Teaching Tip

Higher level students should be asked to compare and contrast a rhombus, square, and parallelogram.

Optional
- CHALLENGE CARD 6: **Rectangles**

Day 7: Investigating the Properties of Quadrilaterals

Key Vocabulary

Chevron, Congruent (≅), Diagonal, Kite, Line of symmetry, Parallel (||), Perpendicular (⊥), Right angle (∟), Symmetrical figure

Literature Connection
- Shape
- A Cloak for the Dreamer

Materials for Teacher
1. SHOW WHAT YOU KNOW Day 7—optional **M13** overhead transparency

Materials for Each Student
1. ACTIVITY E: **Trapezoids**—from day 6
2. ACTIVITY F: **Rhombuses**—from day 6
3. ACTIVITY G: **Parallelograms**—from day 6
4. ACTIVITY H: **Quadrilateral Characteristics**—**M10** copy
5. STUDENT GUIDE
6. SHOW WHAT YOU KNOW Day 7—optional **M13** copy
7. CHALLENGE CARD 7: **Shapes**—optional **M18** copy

Procedure
1. Explain why students need to study quadrilaterals today:

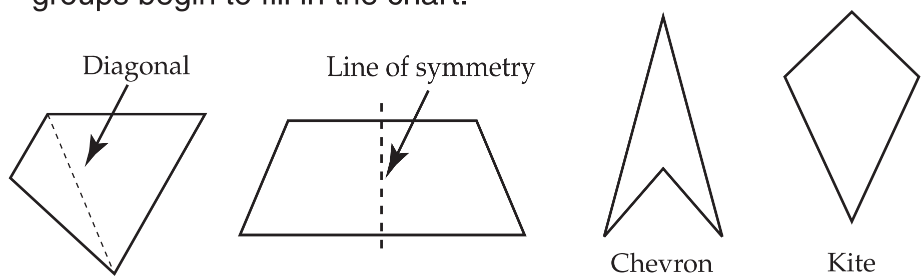
Read or Tell

"In your final presentation you will need to prove that you have at least four different types of quadrilaterals in your maze. In order for you to be able to do that you are going to work together in your study groups to identify the properties of quadrilaterals."

Teaching
Tip

Students may not be familiar with kites and chevrons. Point out that the difference is that in a chevron one vertex is concave, but in a kite all vertices are convex.

2. Review the definition of diagonal lines, lines of symmetry, congruence, and any other unfamiliar terms before you have study groups begin to fill in the chart.

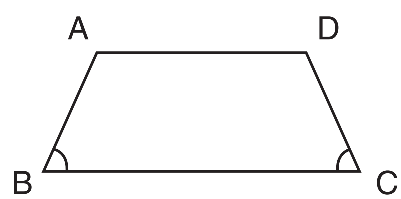


3. Have the students complete ACTIVITY H: **Quadrilateral Characteristics** using the ACTIVITY pages from Day 6 and the pictures on page 7 in the STUDENT GUIDE. Students will work as a group, but each member will need a copy of the chart to record the information. This will be useful when each student needs to prove that he or she has included all the different shapes in their final maze.

Assessment
1. TEACHER OBSERVATION CHECKLIST
2. Writing Prompt
   a. Write definitions of **Key Vocabulary** words in the STUDENT GUIDES.
3. SHOW WHAT YOU KNOW Day 7 Identify and list the properties of this quadrilateral, including diagonals, congruent angles, and lines.

Teaching
Tip

SHOW WHAT YOU KNOW Day 7 could also be displayed as an overhead transparency or on the chalk board.



   a. This shape is a trapezoid. Properties students may mention include:
      - One set of parallel lines
      - Diagonals bisect each other
      - Vertical line in middle is line of symmetry
      - Only one line of symmetry
      - Diagonals are NOT perpendicular
      - ∠ABC and ∠BCD are congruent
      - ∠BAD and ∠ADC are congruent
      - Opposite angles of the diagonals are congruent

Optional
- CHALLENGE CARD 7: **Shapes**

Day 8: Determining the Types of Angles

Key Vocabulary

Acute angle, Angle, Obtuse angle, Point, Ray, Vertex

Materials for Teacher

1. TEACHER OBSERVATION CHECKLIST—**M2** copy
2. Index card

Materials for Each Student

1. STUDENT GUIDE
2. Index card
3. CHALLENGE CARD 8: **Rays**—optional **M18** copy
4. EXTENSION ACTIVITY: **Angle Line Designs**—optional **M42** copy

Read or Tell

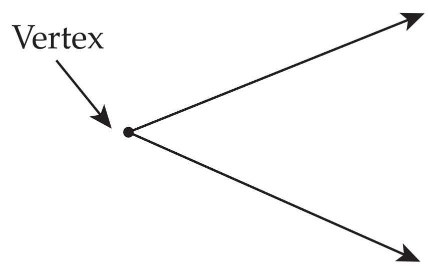
Teaching Tip

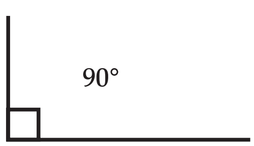
This lesson is designed to give students a sense of these different types of angles and to deepen their conceptual understanding. Tomorrow they will use standard measure for exact degrees.

Procedure

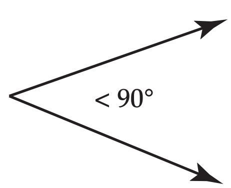
1. Set the scene for learning about angles:

"Paul and Pamela have required that you include acute, obtuse, and right angles in your maze. In order to find out what these mean, we first must define an angle. A ray is a straight line with one endpoint. When two rays come together at the same endpoint they form an angle and the point where they come together is called a vertex.

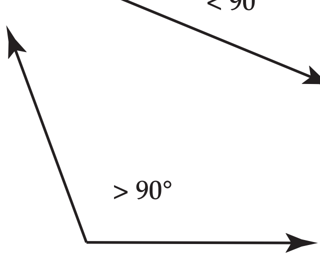




"Angles are measured in degrees. The symbol for degrees is °. A right angle measures 90° and forms a square corner. We use a small square to indicate a right angle. The two sides (rays) of a right angle are perpendicular to one another.



"An acute angle is an angle less than 90°.

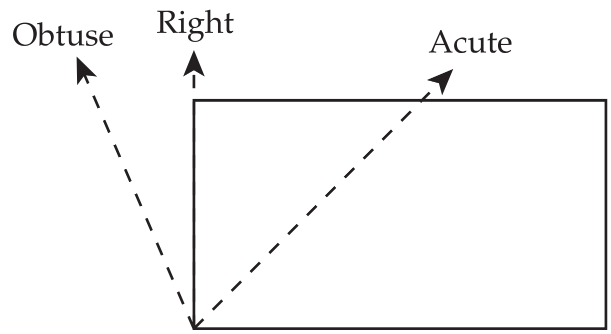


"An obtuse angle is an angle that measures greater than 90°.

Teaching **T**ip

On the overhead use an index card or a piece of paper with square corners to help identify the different angles. Demonstrate the various angles and how to check with the card or a piece of paper.

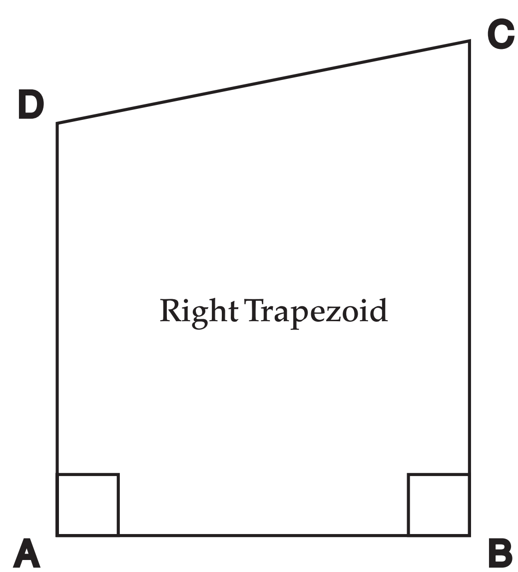
"When an index card is placed so one edge is on one ray and the corner is at the vertex, a right angle is formed if both rays are in line with the index card. An acute angle will have one ray hidden under the index card, and an obtuse angle will have one ray pointing away from the index card."



2. To give your students a kinesthetic understanding of the different angles, have students hold their arms, bent at the elbow in various positions to provide examples of the different angles.
   a. Have student groups create a chart with columns labeled: **Acute Angle**, **Obtuse Angle**, and **Right Angle**.
   b. Using an index card or a square corner of a piece of paper, ask students to find examples of these three types of angles in the classroom.
   c. As they find examples of each angle in the classroom, have them list the examples within the appropriate column.
   d. After each student has discovered some examples of each type of angle in the classroom have them come back together as a total group and discuss their findings and how they knew if their examples were acute, right or obtuse angles.
   e. Record the student examples as groups share their findings with the class. If an example is incorrect or if there is disagreement, have a student check the example again.

2c. This could also be done by creating a class chart on paper, chalkboard or overhead.

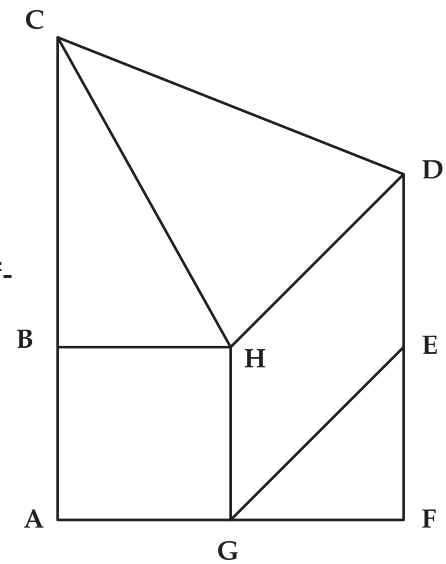
3. Explain that in geometry angles in a plane figure are identified using the letters from one ray, the vertex and then the other ray, in that order. **Be sure to stress:** The vertex is always the middle letter. For instance the angle at point D would be labeled ∠ADC.



Teaching Tip

The angle at A in Quizzical Quadrilateral could be represented by:
∠CAF
∠CAG
∠BAF or
∠BAG

4. Have students locate QUIZZICAL QUADRILATERAL on page 8 in the STUDENT GUIDE.
   a. Ask them to continue their charts by listing the angles in the quadrilateral under the appropriate headings. For instance, ∠AGE would be listed under "obtuse."
   b. Ask students to analyze and identify the QUIZZICAL QUADRILATERAL shape based on their knowledge of the properties, or by referring to ACTIVITY H. (It is a Right trapezoid.)
5. Bring students back to the total class to share their findings about the angles in the QUIZZICAL QUADRILATERAL.



Assessment
1. Use the TEACHER OBSERVATION CHECKLIST as you observe students' ability to identify and correctly label the different types of angles.
2. Writing Prompt
   a. Write definitions of **Key Vocabulary** words in the STUDENT GUIDES.

Optional
- CHALLENGE CARD 8: **Rays**
- EXTENSION ACTIVITY: **Angle Line Designs**
  If students have never done this before, you will need to lead them through the procedure.

***Angle Line Designs** will help build the connection between geometry and art.*

Day 9: Measuring Angles

Key Vocabulary
Degrees, Diameter, Protractor

Materials for Teacher
1. PROTRACTORS—**M33** overhead transparency
2. QUIZZICAL QUADRILATERAL—**M34** overhead transparency

*You can use the PROTRACTORS—**M33** to make an overhead transparent protractor if you don't already have one.*

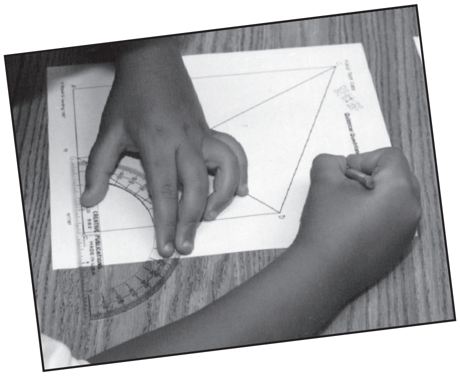
*If you don't have protractors for students, copy the PROTRACTORS—**M33** onto transparencies and cut out one for each student to use.*

Materials for Each Student
1. Angle charts from Day 8
2. STUDENT GUIDE
3. Protractor, scratch papers, scissors
4. SHOW WHAT YOU KNOW Day 9—optional **M13** copy
5. CHALLENGE CARD 9: **Quadrilaterals**—optional **M19** copy
6. EXTENSION ACTIVITY: **Make A Protractor**—optional **M43-M44** copy

Read or Tell

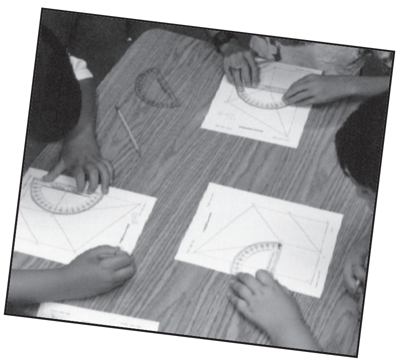
Procedure

1. Set the scene for measuring angles:



"Paul and Pamela have required that you have angles less than 60° (degrees) and greater than 100°. Let's find out what that means. A degree is a unit of angle measurement. Degrees are measured using a protractor. There are 360° in a full circle or all the way around a given point. A right angle has 90° and forms a square corner, an acute angle measures less than 90°, an obtuse angle measures more than 90° and less than 180°.

"A protractor is a tool used to measure angles in degrees. Usually it is shaped like a half circle or semicircle. The outside edge is numbered by 10's from 0 to 180 in both directions. To measure an angle with a protractor, place the center mark on the vertex of the angle and line up the straight edge of the protractor with one ray (the baseline ray) of the angle. There are two scales on the semi-circular part of the protractor. Find which scale on the protractor reads zero for the baseline ray. Follow this scale around to the second ray of the angle to read the number of degrees in the angle."



2. On the Overhead, demonstrate how to measure with a protractor. Sometimes the side of an angle may be too short to cross the numbers on the protractor. Demonstrate how to extend the line using a straight edge so that it crosses the numbers on the protractor.
3. Explain the next step:

Read or Tell

"We are now going to use a protractor to measure the angles of the QUIZZICAL QUADRILATERAL in your STUDENT GUIDE."

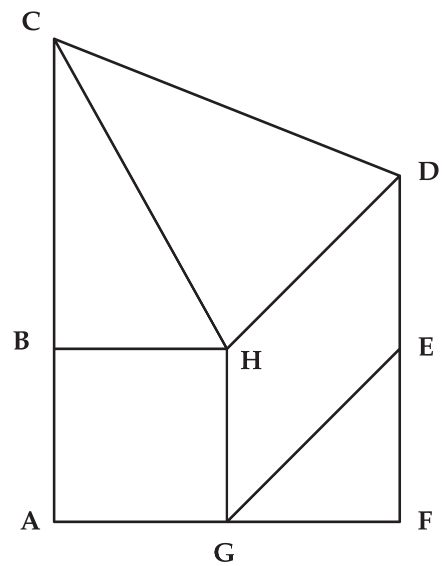
4. Have students measure and record the number of degrees in each angle of the QUIZZICAL QUADRILATERAL from their STUDENT GUIDES. (For example, ∠EDH = 45°.)
5. Have students compare the angle measurements with their angle charts completed on Day 8. Once they measured the angles, did the students find any angles that had been placed in the wrong category (e.g. an angle of 100° in the right angle column)?

Read or Tell

6. Reinforce the concept development:

    "We've been studying quadrilaterals. What do we know about quadrilaterals? (a closed figure with four straight sides) How many can you identify in the QUIZZICAL QUADRILATERAL?"

7. As students identify them, list them on the overhead or board. Solutions:
    ABHG
    ACDF
    ACHG
    BCDH
    DEGH
    DFGH



Read or Tell

8. Allow students to discover:

    "As you were listing the degrees in the angles of the quadrilaterals, did anyone notice anything about the number of degrees in the angles of a quadrilateral?"

9. Allow students time to compare the number of degrees listed in the quadrilaterals.
10. Lead a class discussion on the total number of degrees found in the four angles of each quadrilateral (360°)

Read or Tell

11. Demonstrate the property:

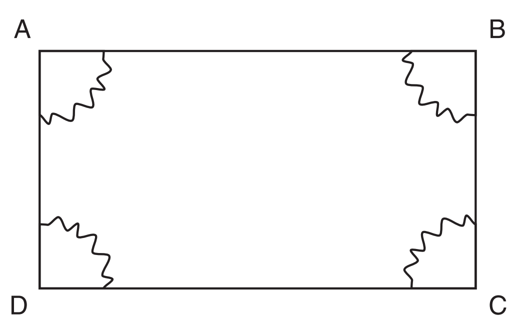
    "Let's try a further exploration of the sum of the angles in a quadrilateral."

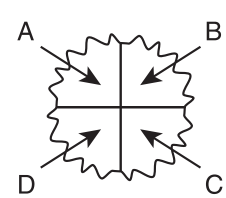
Teaching Tip

Try this yourself first before you give instructions.

Tear off each corner, then place the four vertices together.

12. Using a square or rectangular piece of paper, demonstrate how students are to rip the four corners off the quadrilateral and put the four corners together, with all the vertices meeting in the center.
13. They should see that they form a circle or go all the way around the center point and total 360°.





Read or Tell

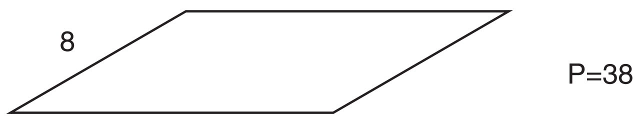
14. Ask:

 "Will this work with ALL quadrilaterals?"

15. Have each student cut any kind of quadrilateral (odd shapes or regular shapes) from a piece of scratch paper, tear off the four corners and put them together.
16. Have students compare their different quadrilateral shapes and what they found. Did the four corners from each type of quadrilateral form a circle around the vertex? Ask students what statement they can make about the total number of degrees of any quadrilateral.

Assessment

1. Writing Prompt
   a. Write definitions of **Key Vocabulary** words in the STUDENT GUIDES.
2. SHOW WHAT YOU KNOW Day 9 optional
   a. Write down everything you know or can figure out about this quadrilateral.



Teaching Tip

Properties of this parallelogram include:
- *Shorter sides are 8*
- *Longer sides are 11*
- *One line of symmetry*
- *Opposite angles are congruent*
- *Two sets of parallel lines*

If you are able to add another day or more time to the simulation, this EXTENSION ACTIVITY will help students understand the relationship between the protractor and the number of degrees in a circle.

Optional

- CHALLENGE CARD 9: **Quadrilaterals**
- EXTENSION ACTIVITY: **Make A Protractor**
  This is a teacher led activity. Follow the instructions in this TEACHER GUIDE.

Day 10: Determining the Types of Triangles

Key Vocabulary

Acute triangle, Equilateral triangle, Isosceles triangle, Obtuse triangle, Right triangle, Scalene triangle

Literature Connection

Greedy Triangle

Materials for Teacher

1. Sample triangles for demonstration
2. SHOW WHAT YOU KNOW Day 10—optional **M14** overhead transparency

Materials for Each Student

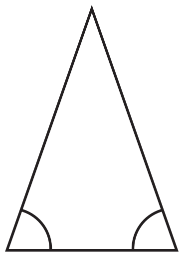
1. STUDENT GUIDE
2. Protractor, scissors
3. SHOW WHAT YOU KNOW Day 10—optional **M14** copy
4. CHALLENGE CARD 10: **Triangles**—optional **M19** copy

Read or Tell

Procedure

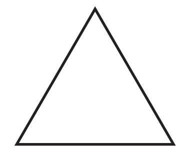
1. Set the scene:

> "Paul and Pamela have required that there be four different types of triangles as part of the design of the entrance mazes. We need to investigate these triangle types and figure out how to make them."

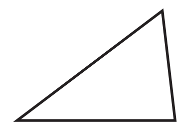


Isosceles

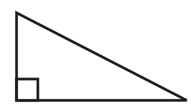
2. Give the students the following information:
   a. A triangle is a flat, closed figure with three straight sides.
   b. If all three sides are the same length, it is an **equilateral** triangle.
   c. If only two sides are the same length, it is an **isosceles** triangle.
   d. If none of the sides are the same length, it is a **scalene** triangle.
   e. A **right** triangle has one angle that measures 90°, or a right angle.



Equilateral



Scalene



Right

Teaching Tip

Scalene Triangles and Isosceles Triangles can also be Right Triangles

3. Refer students to page 7 in the STUDENT GUIDE.
   a. Have students identify and label the four types of triangles.
   b. Have students use a protractor to measure and label the angles of each of the triangles.
4. Investigate the angles:

Read or Tell

> "What do you notice about the angles in each triangle? Do you see any patterns? Do you think all triangles of one type have the same angle measurements? You know how to classify triangles based on the relationship of the length of the sides. Do you think you can classify a triangle by its angle measurement?"

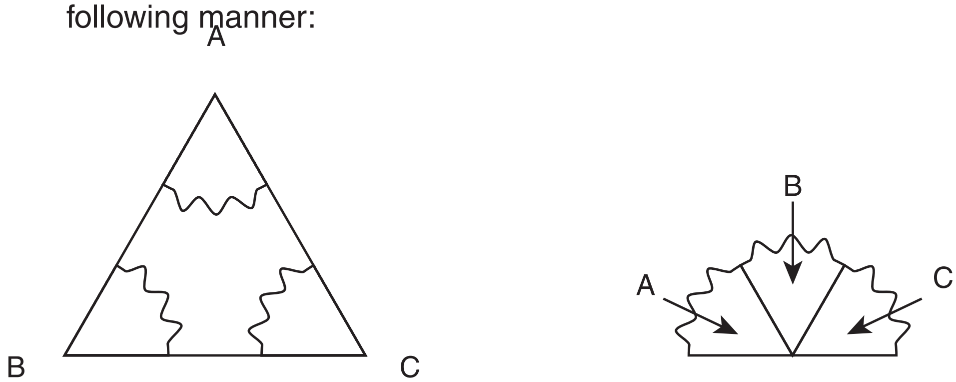
5. Here are some of the things students might mention:
   a. The sum of the three angles is 180°.
   b. In a right triangle, one of the angles is 90° and the other two add up to 90°.
   c. In an isosceles triangle two of the angles will always be the same measure (congruent) (≅)
   d. In an equilateral triangle, all the angles measure 60°.
   e. In a scalene triangle, none of the angles are congruent.

Read or Tell

6. Continue the lesson:

    "When we were looking at quadrilaterals, we found that when we put together the angles contained within a quadrilateral they total 360°. Let's see what happens when we investigate the angles contained within triangles."

7. Have students cut out a triangle from a piece of scratch paper.
8. Encourage group members to cut different types of triangles.
9. Then have them tear off the vertices of the triangles as they did with the quadrilaterals, and lay them down next to each other so the three vertices come together in the following manner:



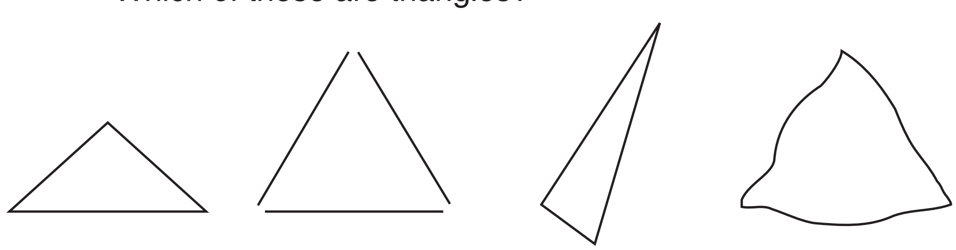
Read or Tell

10. Draw conclusions:

    "What did you find out about the angles of a triangle? (they form a straight line) How many degrees are there in the angles of a triangle? (180°)"

Assessment

1. Writing Prompts
   a. Write definitions of **Key Vocabulary** words in the STUDENT GUIDES.
   b. Have students draw three shapes that are triangles and three shapes that are not and explain how they know that the figures either are or are not triangles
   c. SHOW WHAT YOU KNOW Day 10—optional
      Which of these are triangles?



(A) (B) (C) (D)

SHOW WHAT YOU KNOW Day 10 offers an opportunity to assess whether students under-stand properties of a triangle. For those students who don't, take some time to reinforce the idea that a triangle has three straight sides in a closed figure. You may want students to copy the four triangle samples into their journals.

Optional
- CHALLENGE CARD 10: **Triangles**

Day 11: Investigating how Sides Determine Triangles

Materials for Teacher
1. TEACHER OBSERVATION CHECKLIST—**M2** copy
2. Paper strips of 1/2-inch width cut into 1-, 2-, 3-, 4-, and 5-inch lengths for demonstrating

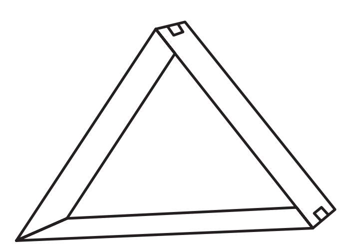
Materials for Each Student
1. ACTIVITY I: **Testing Triangles**—**M11** copy
2. STUDENT GUIDE
3. Paper strips of 1/2-inch width cut into 1-, 2-, 3-, 4-, and 5-inch lengths—three of each
4. Envelope
5. CHALLENGE CARD 11: **What Do You Know?**—Optional **M20** copy

Read or Tell

Procedure
1. Set the scene:

   "Paul and Pamela want four different kinds of triangles in the entrance mazes. Do you think any three lengths of your pathway can form a triangle for your maze? For the next two days you will be working with your study group to investigate which whole number combinations of lengths will form a triangle. Since mathematics is the science of patterns, see if you can find a pattern in your discoveries."

2. If you have not already done so, have students cut paper strips of 1/2-inch width into three strips in each of the following lengths: 1", 2", 3", 4", and 5".
3. Distribute ACTIVITY I: **Testing Triangles**. Demonstrate how the combination of 1", 1" and 1" will form a triangle, using paper strips on the overhead projector. Next demonstrate the combination of 1", 1" and 2". Be sure that students notice that sides measuring 1", 1" and 2" will **not** form a triangle.
   a. Show students how to list their combinations on their record sheet as follows:



Teaching
Tip

Encourage students to find the sum of the lengths of the two shorter sides and compare that sum with the length of the third side. They should discover that in order to make a triangle, the sum of the lengths of the two smaller sides must be greater than the third side.

4. Have student groups experiment with remaining combinations and record their discoveries. You may want the students in the study group to split up the combinations.

These strips make a triangle	These strips do not make a triangle
1", 1", 1"	1", 1", 2"
2", 2", 1"	

5. Have students save their strips in an envelope for tomorrow's activity.

Assessment
1. Use the TEACHER OBSERVATION CHECKLIST as you observe students' ability to correctly identify those combinations that will make a triangle and those that will not.
2. Writing Prompts
   a. Look at the combinations that make triangles and those that do not make triangles. Write a letter to Paul and Pamela to explain why some combinations work and others do not.
   b. What kind of discoveries did you make about the relationship of the shorter sides to the longer sides of the triangles?
   c. Paul and Pamela have been asked to include in GEO WORLD a triangular building with sides of 20', 30', and 60'. Write a letter explaining whether or not this building or a triangle of this shape can be constructed, and how you know.

Optional
- CHALLENGE CARD 11: **What Do You Know?**

Day 12: Identifying Triangles

Materials for Teacher
1. TEACHER OBSERVATION CHECKLIST—**M2** copy
2. Paper strips of 1/2-inch width cut into 1-, 2-, 3-, 4-, and 5-inch lengths for demonstrating as on Day 11

Materials for Each Student
1. ACTIVITY I: **Testing Triangles**—**M11** copy
2. STUDENT GUIDE
3. Paper strips from Day 11, pencil, ruler, protractor, and plain paper
4. SHOW WHAT YOU KNOW Day 12—optional **M14** copy
5. CHALLENGE CARD 12: **How Many?**—Optional **M20** copy
6. EXTENSION ACTIVITY: **Triangle Tessellation**—optional **M45-M47** copy

Read or Tell

Procedure

1. Set the scene:

> "Since you are going to need to put the four types of triangles into your maze you are going to need to be able to draw them. Look at your TESTING TRIANGLES chart and find those combinations that will make triangles. In your study group, divide up these combinations. Using your paper strips from yesterday, construct each of the combinations, then draw them. Measure and label the angles. Identify and label each triangle as right, scalene, isosceles, or equilateral."

2. Demonstrate with three 1-inch paper strips how to construct then draw a triangle. Have students identify the type of triangle.
3. Have the students draw the triangles using the paper strips as a straight edge. Ask the students to evaluate the triangles that they have made as part of a class discussion. Students should discuss the following:
   a. How are the triangles alike?
   b. How are they different?
   c. Do any triangles fit in more than one category?
   d. Where in our environment do we find these different triangles?

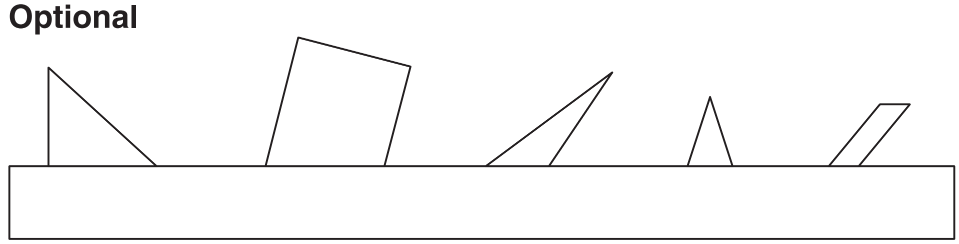
Teaching Tip

Some triangles may have more than one label (i.e., right isosceles triangle).

Assessment

1. Use the TEACHER OBSERVATION CHECKLIST to observe students' ability to correctly identify and label triangles and angles.
2. SHOW WHAT YOU KNOW Day 12: Identify each triangle or quadrilateral that is partly hidden from view.
   (Students will probably identify and label a right triangle, rectangle, scalene triangle, isosceles triangle; and a parallelogram; other responses could be correct.)

Optional



- CHALLENGE CARD 12: **How Many?**
- EXTENSION ACTIVITY: **Triangle Tessellations**
  If you can add another day, or more time to the simulation, this teacher-led ACTIVITY will help build the relationship between art, patterns, and geometry. Follow the directions in this TEACHER GUIDE.

Day 13: Constructing Angles and Triangles Using a Compass

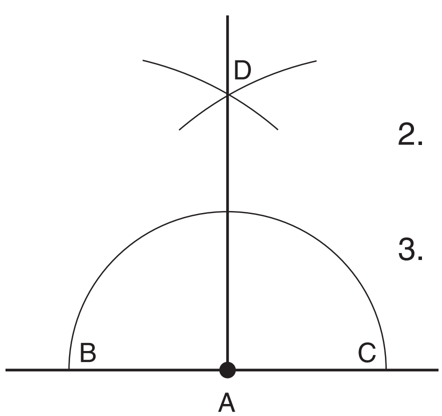
Key Vocabulary
Bisect, Compass, Midpoint, Radius

Materials for Teacher
1. TEACHER OBSERVATION CHECKLIST—**M2** copy
2. PROTRACTORS—**M33** overhead transparency
3. Compass for use on the overhead projector

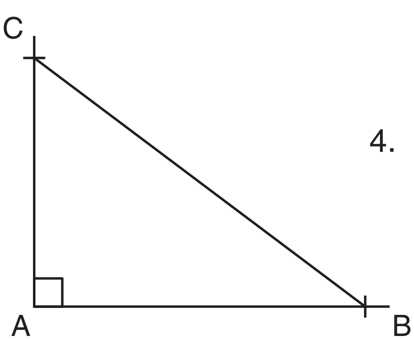
Materials for Each Student
1. STUDENT GUIDE
2. Compass, protractor, and ruler
5. CHALLENGE CARD 13: **Multiple Triangles**—optional **M21**

Procedure
1. Set the scene:

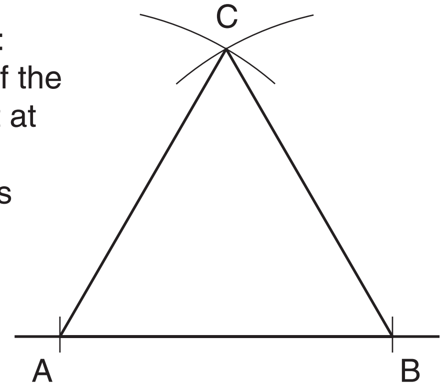
> "Today we are going to learn how to accurately construct the geometric design elements required for the entrance maze. You need to build at least four different types of quadrilaterals, one isosceles triangle, one equilateral triangle, one scalene triangle, and one right triangle. There also must be a minimum of two each: right angles, acute angles measuring less than 60°, and obtuse angles measuring greater than 100°. We are going to use compasses and a straight edge to make precise angles and figures."



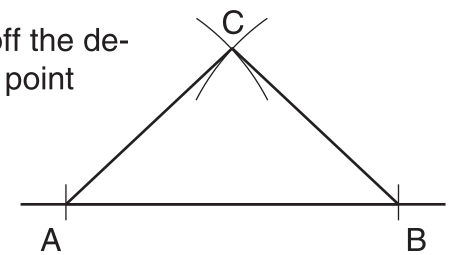
2. Using a compass and straight edge, demonstrate on the overhead the following methods for constructing right angles, right triangles, equilateral triangles, isosceles triangles, and squares.
3. Constructing a right angle or perpendicular:
   a. From desired point A, swing an arc from B to C.
   b. Place the compass on point B, extend the radius, and create an arc above A
   c. Keeping the radius the same, swing an arc from C that intersects the previous arc above A and label the intersection D.
   d. Connect points A and D.
4. Constructing a right triangle:
   a. Construct two lines perpendicular to each other.
   b. Measure the desired edge length along each line and mark that length with a point. Connect the points. If the two sides of the right triangle are equal, a right isosceles triangle is formed.



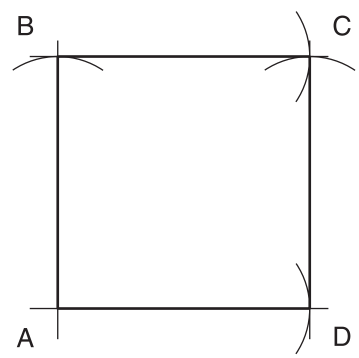
5. Constructing an equilateral triangle:
   a. Draw a straight line and mark off the desired edge length with a point at each end (AB).
   b. Spread the arms of the compass to that same length, then swing intersecting arcs from each end point.
   c. Draw two lines connecting the two points of the first edge with the point where the two arcs intersect (C).



6. Constructing an isosceles triangle:
   a. Draw a straight line and mark off the desired length for the base with a point at each end (AB).
   b. Spread the compass in desired length for the sides and swing intersecting arcs from both ends of the base (C).
   c. Connect the points.



7. Constructing a square:
   a. Construct two perpendicular lines AB and AD. Using the desired length for one side of the square as a radius, place the point of the compass on A and swing an arc through points B and D.
   b. Using that same radius length from points B and D, swing intersecting arcs at C.
   c. Connect the points.



8. It is now time to help students apply these construction techniques to satisfy the design elements called for in the maze instructions. Point out how students can use compasses and protractors to construct these figures that Paul and Pamela require. Have students practice making all of the figures needed for the entrance maze.

Teaching Tip

Make sure students understand that the base need not be a horizontal line at the bottom. It can be vertical, slanted, or even at the top of the triangle.

Assessment
1. Using the TEACHER OBSERVATION CHECKLIST, walk around the room as you observe students' ability to use a compass and straight edge and construct angles and shapes.
2. Writing Prompt
   a. Write definitions of **Key Vocabulary** words in the STUDENT GUIDES.

Optional
- CHALLENGE CARD 13: **Multiple Triangles**

Day 14: Designing Mazes Using Required Shapes

Materials for Teacher

1. TEACHER OBSERVATION CHECKLIST—**M2** copy
2. MAZE 2A-2F—**M35-M40** overlay transparencies

Materials for Each Student

1. Whole-page GRID B (1/4-inch)—**M24** copy
2. CHALLENGE CARD 14: **Pathways**—optional **M21** copy
3. EXTENSION ACTIVITY: **Cartesian Coordinate GRID**—optional **M48** copy

Procedure

1. Demonstrate how to create a maze around the required geometric shapes using the MAZE 2A-2F overlay transparencies.
   a. The scale on this maze construction is 1/4 inch = 5 meters. Use MAZE 2A—**M35** overlay.
   b. Demonstrate drawing the different types of shapes and angles using MAZE 2B—**M36** overlay.
   c. Show that the design elements can either be separate and the maze goes around them or the design elements can actually be incorporated as part of the pathways.
3. Steps in creating the maze:
   a. Decide what scale you will be using. If you use 1/4-inch = 5 meters you will have a larger area for the maze. If students prefer a smaller maze they can use 1/4-inch = 10 meters.
   b. On the grid (MAZE 2A—**M35** overlay), mark the corners of the rectangular entrance maze. Remember the perimeter cannot be larger than 640 meters (32 inches if using a 1/4-inch = 5 meter scale).
   c. Draw required shapes on grid master paper. (MAZE 2B—**M36** overlay)
   d. Draw the maze pathway (solution) on the grid being sure it includes or touches each shape. Be sure to draw it very lightly so you can erase. (MAZE 2C—**M37** overlay)
   e. Draw sides of pathway very lightly so you can erase to create dead-ends. (MAZE 2D—**M38** overlay)
   f. Check that your solution path touches all required shapes and angles.
   g. Make erasures so that you can add dead-ends, being sure not to cross over the original solution path as you demonstrate (MAZE 2E—**M39** overlay). Remove MAZE 2D—**M38** overlay to allow students to see how to work. You will need to add one or two dead ends to this transparency.

Teaching Tip

3c. There are small shapes on the four corners of the grid that will ensure that your maze overlay transparencies line up correctly.

3d. Using colored pencil to outline the solution path will clarify the solution and simplify adding the dead ends.

h. All the work should be done in pencil first, then, when the maze is completed, students can erase their solution path and darken the barriers using felt pen. (MAZE 2F—**M40** overlay). To eliminate the grid lines, students can then trace their mazes onto plain paper.

4. Allow students time to work on their mazes. Have them exchange mazes within their study groups to check that the maze actually has a start, a finish, and one solution path. Also they need to check that each maze includes all the required design elements.
5. Each group must then decide which one(s) they want to submit.

Teaching Tip →

5. You may want each student to present his or her maze instead of just one per group.

2a. This letter may be completed by the end of class today or at the end of Day 15.

Assessment

1. Use the TEACHER OBSERVATION CHECKLIST to observe students' ability to include the required geometric shapes. You may need to ask students where they placed particular shapes.
2. Writing Prompts
   a. As a final assessment, students should write a letter to Paul and Pamela explaining how they know that they have met all the requirements including the correct area and perimeter. They need to tell how they know that they have included each one of the design elements by describing the shape/angle and its properties.

Optional

- CHALLENGE CARD 14: **Pathways**
- EXTENSION ACTIVITY: **Cartesian Coordinate GRID**

Key Vocabulary

Axes, Coordinates

If your students are familiar with coordinate graphing you can add another dimension to the learning by having students (individuals, pairs, or study groups) identify the location of their geometric shapes using the Cartesian Coordinate GRID.

Materials

- **Cartesian Coordinate GRID**— **M48** overhead transparency

Procedure

1. This is a teacher led procedure.
   a. For higher ability students an additional challenge would be to locate the geometric figures based on their coordinates.
   b. A copy of the GRID can be placed beneath the maze and students can give coordinates for locations of each required element.
   c. To demonstrate, make a transparency of the Cartesian Coordinate GRID—**M48** and use it with the final MAZE 2F—**M40** overlay.
   d. With the students, identify the coordinates of the various geometric design elements.

Day 15: Presenting Final Mazes

Materials

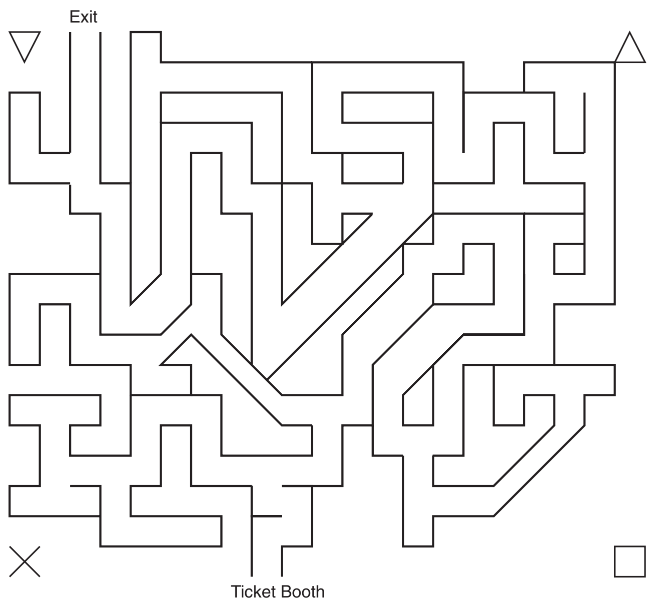
1. STUDENT CERTIFICATES—optional **M49** class set
2. Completed MAZES

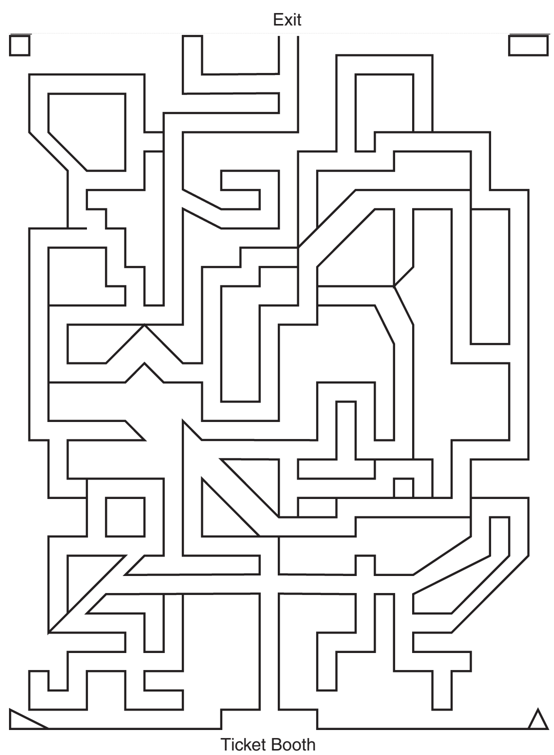
Teaching Tip

To enhance the presentations, make overhead transparencies of the student-created mazes for use in their presentations.

Presentations

1. Each group or individual presents their MAZE selection to the class, and tells why theirs should be selected to be submitted to Pamela and Paul on the basis of the established requirements.
2. This is an opportunity to invite families to help celebrate their student's learning and successful completion of the mazes.





ACTIVITY SOLUTIONS - 1

ACTIVITY A: **Create A Polygon**

Results will be individually determined.

ACTIVITY B: **Paul's Perplexing Plan**

The least amount of fencing would be 14', for a 3' x 4' fenced area.

ACTIVITY C: **Area & Perimeter**

1. *6" and 4"* 2. *20"* 3. *24 square inches*

Other answers will vary.

ACTIVITY D: **Maximizing Area**

- **Perimeter = 16 units**

Greatest area:	*16 square units*	Dimensions:	*4 units*	*by*	*4 units*
Smallest area:	*7 square units*	Dimensions:	*7 units*	*by*	*1 unit*

Other possible areas with a perimeter of 16 are:

Area:	*15 square units*	Dimensions:	*5 units*	*by*	*3 units*
Area:	*12 square units*	Dimensions:	*6 units*	*by*	*2 units*

- **Perimeter = 24 units**

Greatest area:	*36 square units*	Dimensions:	*6 units*	*by*	*6 units*
Smallest area:	*11 square units*	Dimensions:	*11 units*	*by*	*1 unit*

Other possible areas with a perimeter of 24 are:

Area:	*35 square units*	Dimensions:	*7 units*	*by*	*5 units*
Area:	*32 square units*	Dimensions:	*8 units*	*by*	*4 units*
Area:	*27 square units*	Dimensions:	*9 units*	*by*	*3 units*
Area:	*20 square units*	Dimensions:	*10 units*	*by*	*2 units*

- **Perimeter = 100 units**

Greatest area:	*625 square units*	Dimensions:	*25 units*	*by*	*25 units*
Smallest area:	*49 square units*	Dimensions:	*49 units*	*by*	*1 unit*

There are many other possible areas with a perimeter of 100. Have students share results within pairs or groups to verify the possibilities.

ACTIVITY E: **Trapezoids**

1. *Options A, C, and E*

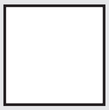
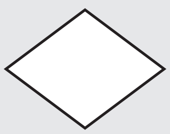
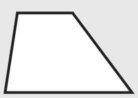
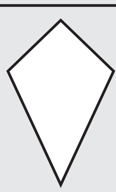
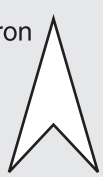
ACTIVITY F: **Rhombus**

1. *Options B and E*

ACTIVITY G: **Parallelograms**

1. *Options A and C*

ACTIVITY H: **Quadrilateral Characteristics**

Fill out answers for each quadrilateral below:	Number of pairs of congruent lines	Number of pairs of parallel lines	opposite angles congruent *(yes or no)*	Diagonals bisect each other *(yes or no)*	Diagonals perpendicular *(yes or no)*	Other properties *(congruent diagonals, right angles, lines of symmetry, etc.)*
Square 	*2*	*2*	*yes*	*yes*	*yes*	*4 lines of symmetry, 4 right angles, congruent diagonals*
Rectangle 	*2*	*2*	*yes*	*yes*	*no*	*2 lines of symmetry, 4 right angles, congruent diagonals*
Rhombus 	*2*	*2*	*yes*	*yes*	*yes*	*2 lines of symmetry*
Parallelogram 	*2*	*2*	*yes*	*yes*	*no*	
Trapezoid 	*0**	*1*	*no*	*no*	*no*	
Kite 	*2*	*0*	*1 pair*	*no***	*yes*	*1 line of symmetry*
Chevron 	*2*	*0*	*1 pair*	*no***	*yes*	*1 line of symmetry*

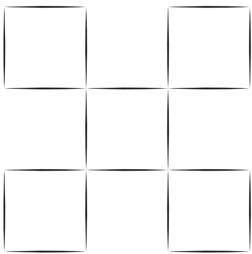
** 1 for isosceles trapezoid*
*** one diagonal bisects the other but not the reverse*

ACTIVITY I: **Testing Triangles**

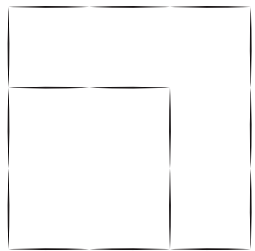
Results will vary.

CHALLENGE CARD 1: **Toothpick puzzles**

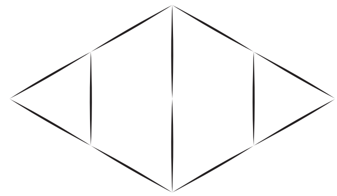
1a.



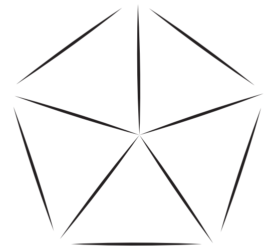
1b.



2.



3.



CHALLENGE CARD 2: **Fencing**

1. *7" x 13"*
2. *32' x 32'*
3. *6 blocks x 12 blocks*

CHALLENGE CARD 3: **Finding Cost**

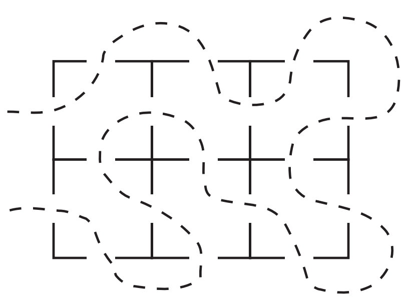
A. *$9.60* B. *$6.00* C. *$10.56* D. *$7.68*

CHALLENGE CARD 4: **Area & Perimeter**

1. *Answers will vary. Be sure students use proper methods to arrive at their answers.*
2. *Answers will vary. Again, be sure students use proper methods to arrive at their answers.*
3. *6" x 8" x 12"*

CHALLENGE CARD 5: **Networks**

1. *More than one solution is possible. Below is one example:*



2. *More than one solution is possible for figure 2a. Below is one example: For figure 2b there are nine pathways.*

Figure 2a

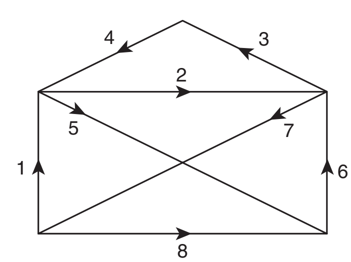
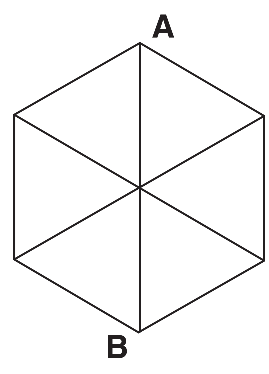


Figure 2b

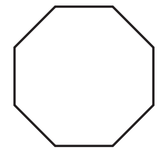


CHALLENGE CARD 6: **Rectangles**

1. *204 (sum of squares of 1 through 8*
2. *36*

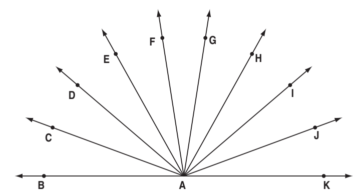
CHALLENGE CARD 7: **Shapes**

2. *20 diagonals*



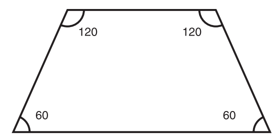
CHALLENGE CARD 8: **Rays**

1. *45 angles*
2. *∠BAC is acute (24°), ∠EAK is obtuse (128°), ∠DAG is acute (59°)*



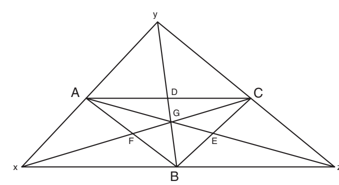
CHALLENGE CARD 9: **Quadrilaterals**

1. *60°, 120°, 60°, isosceles trapezoid*



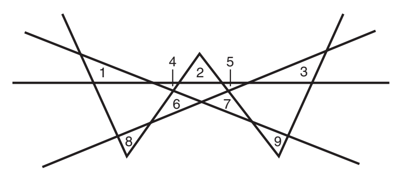
CHALLENGE CARD 10: **Triangles**

1. *47 different triangles.*



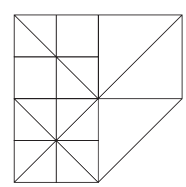
CHALLENGE CARD 11: **What Do You Know?**

1. Answers could include:
   - *Right isosceles triangle*
   - *Angles are 90°, 45°, and 45°*
   - *Two of the sides are 8 cm*
   - *One line of symmetry*
2. *Answer drawing:*



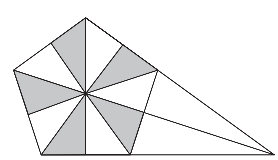
CHALLENGE CARD 12: **How Many?**

1. *12 squares, 17 (non-square) rectangles, 30 (non-rectangular) trapezoids, 28 triangles*
2. *5 triangles*



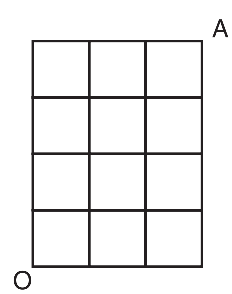
CHALLENGE CARD 13: **Multiple Triangles**

1. *28 triangles.*



CHALLENGE CARD 14: **Pathways**

1. *There are 35 different ways.*
   This problem is the same as: How many paths are there from O to A in the figure? The path must always be along the horizontal or vertical lines towards A.



Dear parents,

For the next three weeks your child will be participating in a math interaction unit called ***A-MAZE-ING SHAPES.*** *In this unit students will be conducting investigations and completing activities to develop their understanding of plane geometry.*

Working individually, in pairs, or in teams of four, your student will explore geometric concepts and construct mazes. The challenge is to construct an A-MAZE-ING entrance for the Pythagorean theme park, GEO WORLD, which must include specific geometric designs. The study of pathways (mazes) is connected to the branch of geometry called topology, and relates to network theory, which is utilized in circuit design and modern city planning.

By the end of this interaction unit, each student will develop a maze that includes various geometric figures. In order for students to fully understand the geometric shapes and angles required, we will be exploring them in class. Students are encouraged to practice making mazes on their own throughout the unit and may challenge family members to solve them.

Along with the class work, ***A-MAZE-ING SHAPES*** *includes several "challenges" designed to extend the learning and to further develop students' spatial sense and understanding of geometry. From time to time, your child will be bringing home these challenges and puzzles. Although students may not be able to solve all the challenge problems completely , please encourage your child to try them. Any member of the family can share in the Challenge Activities, and I encourage parents and brothers and sisters to tackle them together.*

If you have any questions about this interactive unit, please contact me.
Thank you.

TEACHER OBSERVATION CHECKLIST

Student Name	Cooperative Participation in Group	Understanding of Task	Ability to	Ability to	Comments

Rating scale
\+ (date) = Student fully accomplished the task
✓ (date) = Student substantially accomplished the task
– (date) = Student has partial or limited progress toward accomplishing the task
0 (date) = Student shows little or no progress toward accomplishing the task

ACTIVITY A: **Create A Polygon**

Name: ______________________________ Date: ________________

1. Use construction paper and cut a rectangle no larger than 3" by 5".
2. Cut the rectangle so that you have a square, a rectangle, and two triangles.
3. Use the four pieces to find as many different polygons as you can without overlapping the shapes.
4. Glue your favorite polygon on this paper.

To write about: Describe your polygon and tell how you made it.

ACTIVITY B: **Paul's Perplexing Plan**

Name: ______________________________________ Date: ____________________

Paul is planning a tribute to Euclid, master of geometry, at GEO WORLD. He wants a polygon shape with an area of 12 square feet and the sides in whole foot lengths. He needs to find out how much fencing is needed to go around the perimeter of this display. Since fencing is expensive, he wants to buy the least amount he can.

If each tile is one square foot of land and each edge is one foot, show on grid paper all the different arrangements he could have for this exhibit of GEO WORLD. Be sure to figure the perimeter for each arrangement.

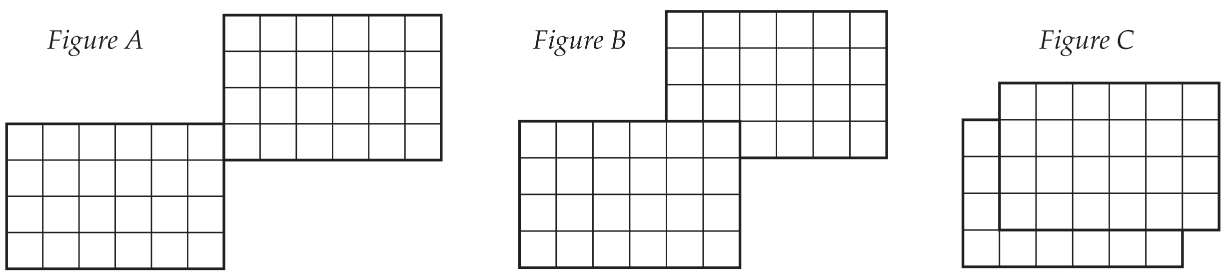
To write about: Write a letter to Paul and tell him what would be the best arrangement for his area of 12 square feet to have the smallest perimeter. Be sure to draw him a picture so he will understand your explanation.

ACTIVITY C: **Area & Perimeter**

Name: ______________________________________ Date: ____________________

Use grid paper to cut out two 4 inch x 6 inch rectangles

1. What is the length and width of each rectangle? ___________________________
2. What is the perimeter of each of the rectangles? ___________________________
3. What is the area of each rectangle? ______________________________________
4. The rectangles can be put together to form a variety of shapes. Each shape has its own area and perimeter.



5. Place your rectangles together in a variety of ways.
6. How does the arrangement of the two rectangles affect the perimeter?
7. How does it affect the area?
8. Find as many whole number perimeters as you can.
    a. Draw your findings on grid paper.
    b. Record the perimeter and area of each figure.
9. Questions about perimeters
    a. What is the largest perimeter?
    b. What is the smallest perimeter?
10. Questions about areas
    a. What was the largest area?
    b. What is the smallest area?

To write about: Compare the shape you created that has the largest perimeter and the shape you created with the largest area. Are they the same shape? What can you say about the relationship between the area and the perimeter of these shapes?

ACTIVITY D: **Maximizing Area**

Name: ______________________________________ Date: ____________________

Use a piece of grid paper to determine the largest and smallest rectangular area you could have with the following perimeters.

Perimeter = 16 units

Greatest area: ____________ Dimensions:___________ by____________

Smallest area: ____________ Dimensions: __________ by____________

Other possible areas with a perimeter of 16 are:

Area: ____________________ Dimensions:___________ by____________

Area: ____________________ Dimensions: __________ by____________

Perimeter = 24 units

Greatest area: ____________ Dimensions:___________ by____________

Smallest area: ____________ Dimensions: __________ by____________

Other possible areas with a perimeter of 24 are:

Area: ____________________ Dimensions:___________ by____________

Area: ____________________ Dimensions: __________ by ___________

Area: ____________________ Dimensions:___________ by____________

Area: ____________________ Dimensions: __________ by ___________

Perimeter = 100 units

Greatest area: ____________ Dimensions:___________ by____________

Smallest area: ____________ Dimensions: __________ by ___________

Other possible areas with a perimeter of 100 are:

Area: ____________________ Dimensions:___________ by____________

Area: ____________________ Dimensions: __________ by ___________

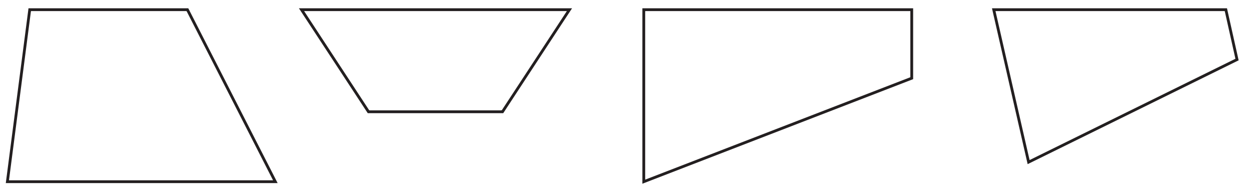
Area: ____________________ Dimensions:___________ by____________

To write about: Compare the dimensions of the rectangles with the greatest area. What can you say about rectangles that have the greatest area?

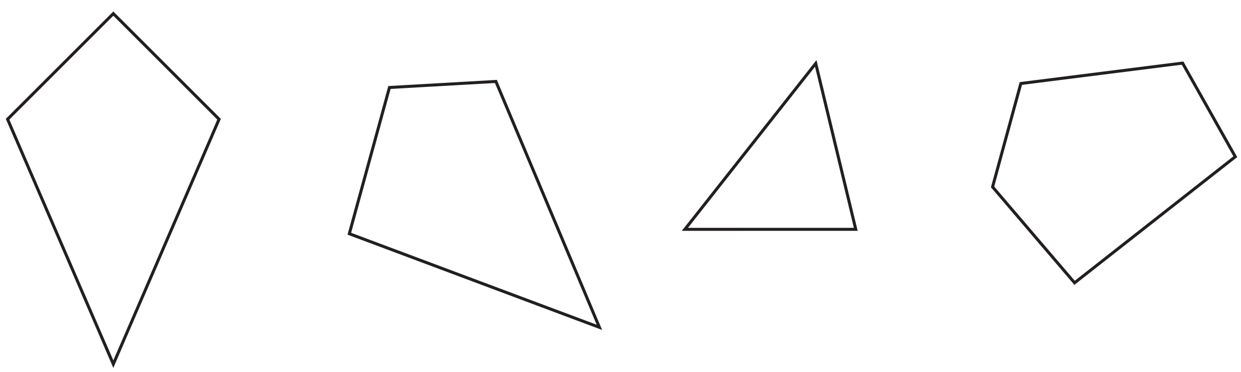
ACTIVITY E: **Trapezoids**

Name: ______________________________ Date: ____________________

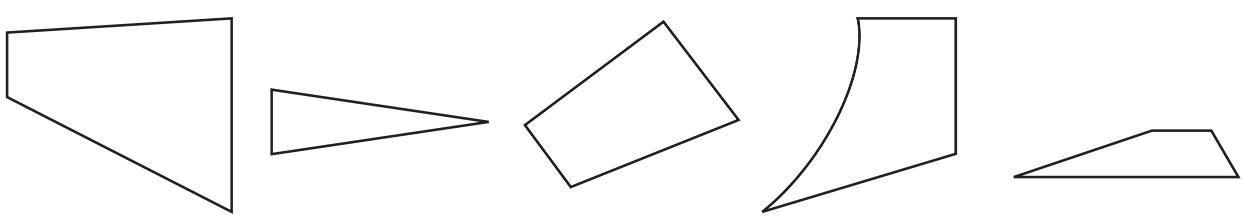
These are trapezoids:



None of these are trapezoids:



1. Which of these are trapezoids?



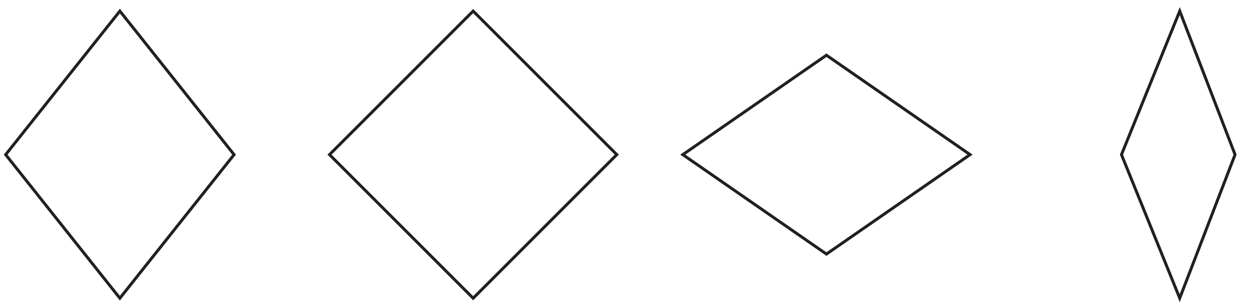
A B C D E

To write about: What are the properties of a trapezoid?

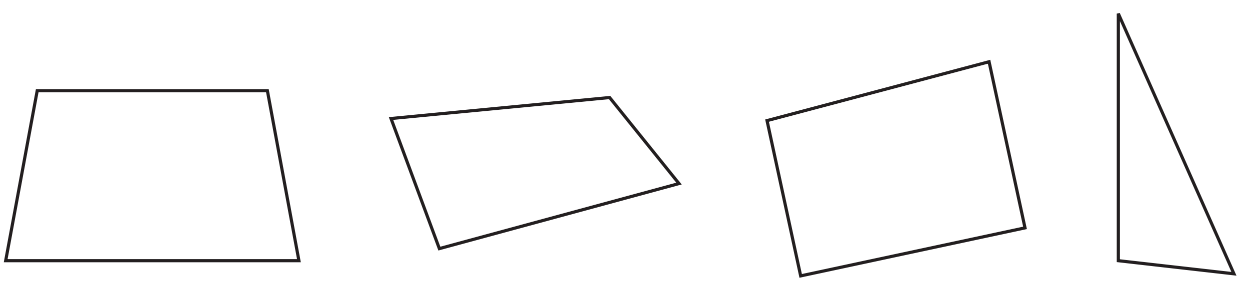
ACTIVITY F: **Rhombuses**

Name: ______________________________ Date: ____________________

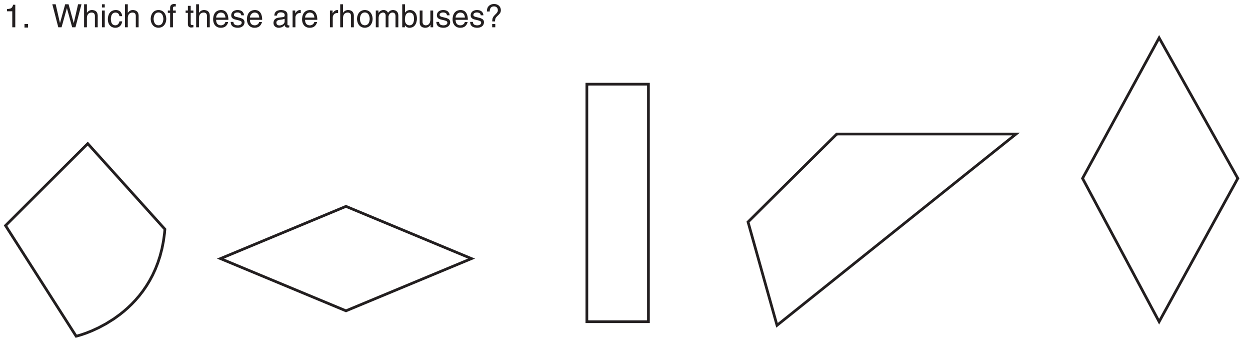
These are rhombuses:



None of these are rhombuses:



1. Which of these are rhombuses?



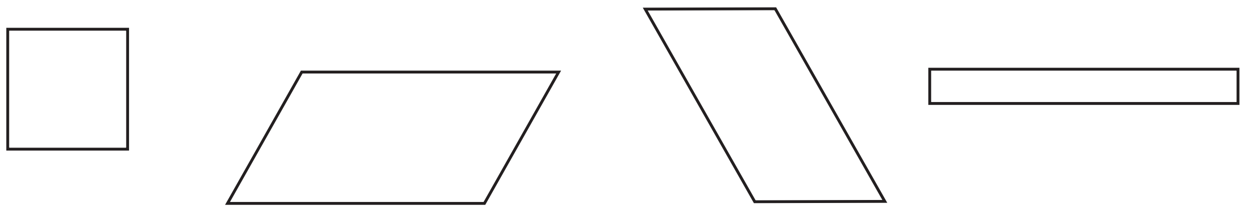
A B C D E

To write about: What are the properties of a rhombus?

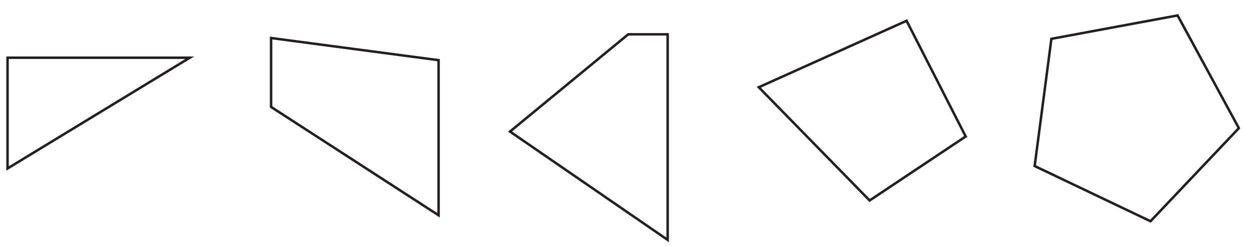
ACTIVITY G: **Parallelograms**

Name: ________________________________ Date: ____________________

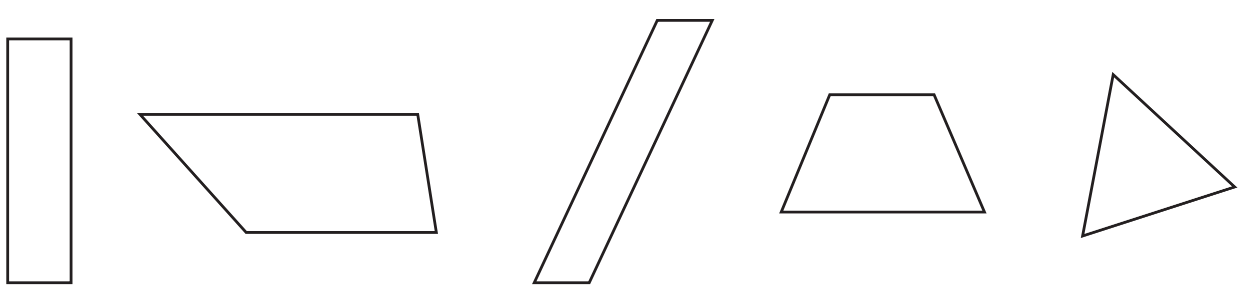
These are parallelograms:



None of these are parallelograms:



1. Which of these are parallelograms?



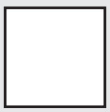
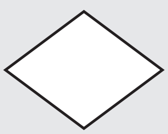
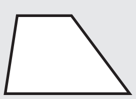
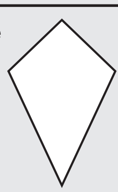
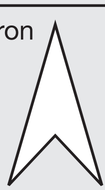
A B C D E

To write about: What are the properties of a parallelogram?

ACTIVITY H: **Quadrilateral Characteristics**

Name: ______________________________ Date: ____________________

Answer the following questions to identify and define each quadrilateral below:

Fill out answers for each quadrilateral below:	Number of pairs of congruent lines	Number of pairs of parallel lines	Opposite angles congruent (yes or no)	Diaognals bisect each other (yes or no)	Are diagonals perpendicular? (yes or no)	Other properties (congruent diagonals, right angles, lines of symmetry, etc.)
Square 						
Rectangle 						
Rhombus 						
Parallelogram 						
Trapezoid 						
Kite 						
Chevron 						

ACTIVITY I: **Testing Triangles**

Name: ______________________________ Date: ____________________

These strips make a triangle	These strips do not make a triangle

SHOW WHAT YOU KNOW: **Day 2**

Describe what you would do to find the perimeter of the rectangle ABCD.

A = ________________

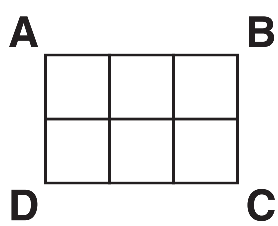
P = ________________

AB = ________________

BC = ________________

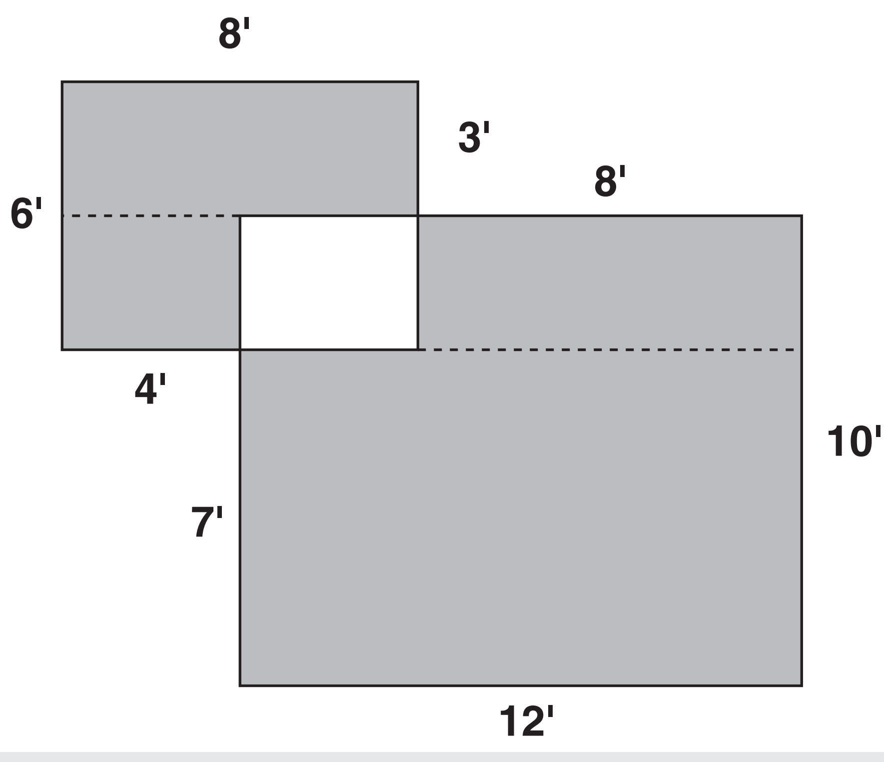
CD = ________________

DA = ________________



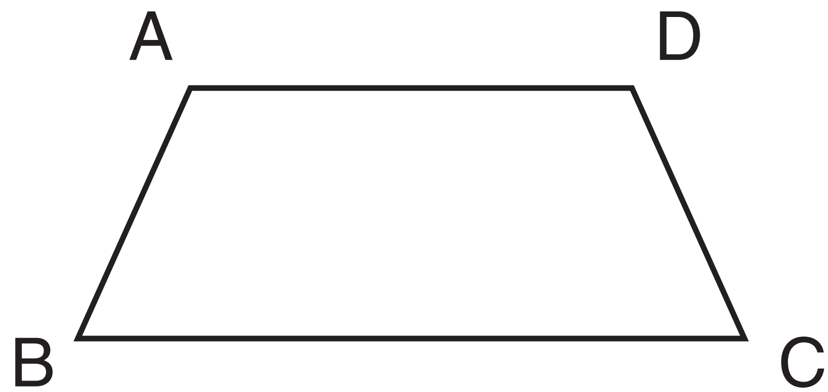
SHOW WHAT YOU KNOW: **Day 3**

What is the area of the shaded region. Explain how you found it.



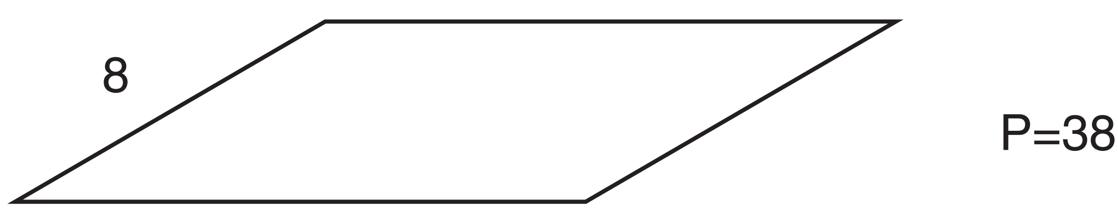
SHOW WHAT YOU KNOW: **Day 7**

Identify and list the properties of this quadrilateral.
Include diagonals, congruent angles, and lines.



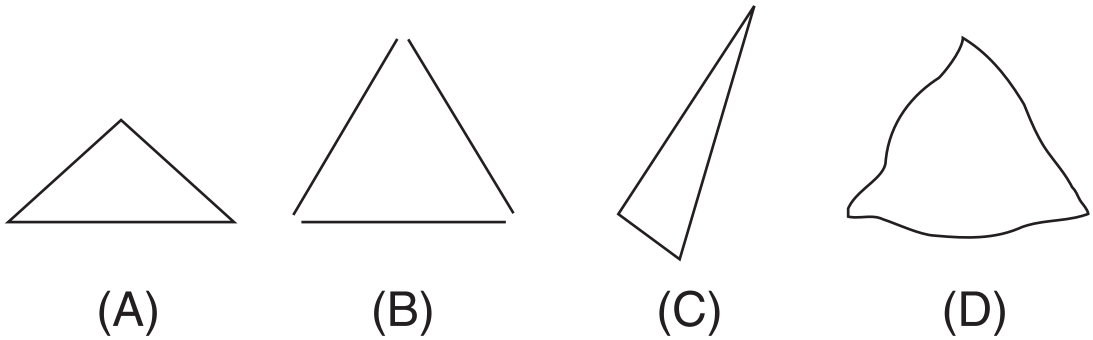
SHOW WHAT YOU KNOW: **Day 9**

Write down everything you know or can figure out about this quadrilateral.



SHOW WHAT YOU KNOW: **Day 10**

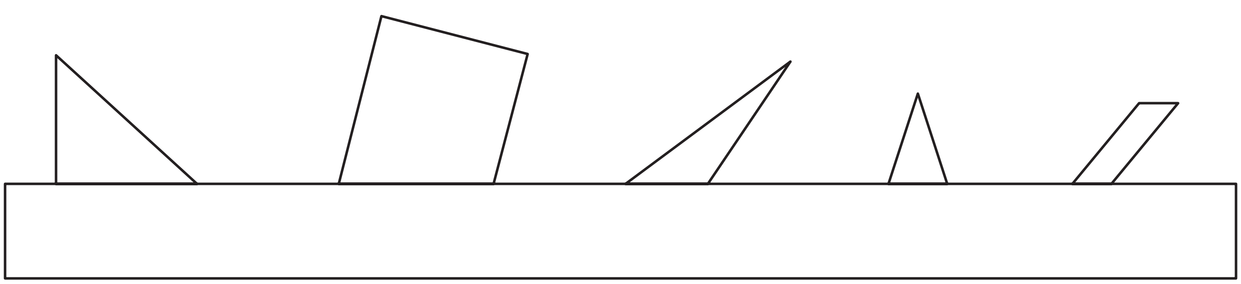
Which of these are triangles?



Michelle circled figures A and B. She says figures C and D are not triangles.
Write a letter to Michelle telling her whether or not you agree with her and why.

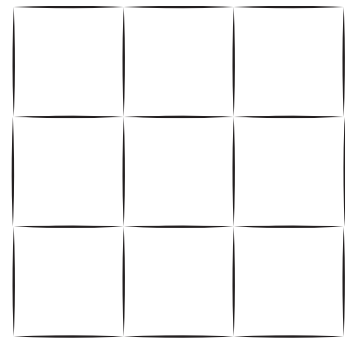
SHOW WHAT YOU KNOW: **Day 12**

Each of the shapes hidden from view is a triangle or a quadrilateral.
Sketch in the missing part and name the shape. Tell how you identified each shape.

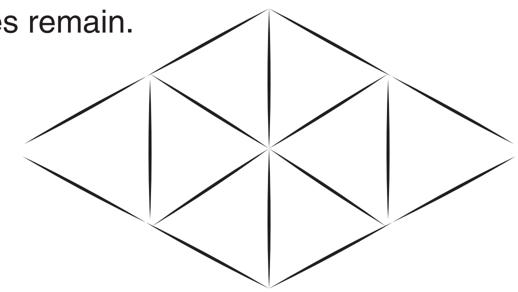


CHALLENGE CARD 1: **Toothpick Puzzles**

1. Use 24 toothpicks to make 9 squares:
   a. Take away 4 toothpicks, and leave 5 equal squares
   b. Take away 1/3 of the toothpicks and leave only two squares



2. Use 16 toothpicks to make the following design:
   a. Remove 4 toothpicks so that only 4 triangles remain.



3. Use 10 toothpicks to make 5 separate closed shapes of equal size.

CHALLENGE CARD 2: **Fencing**

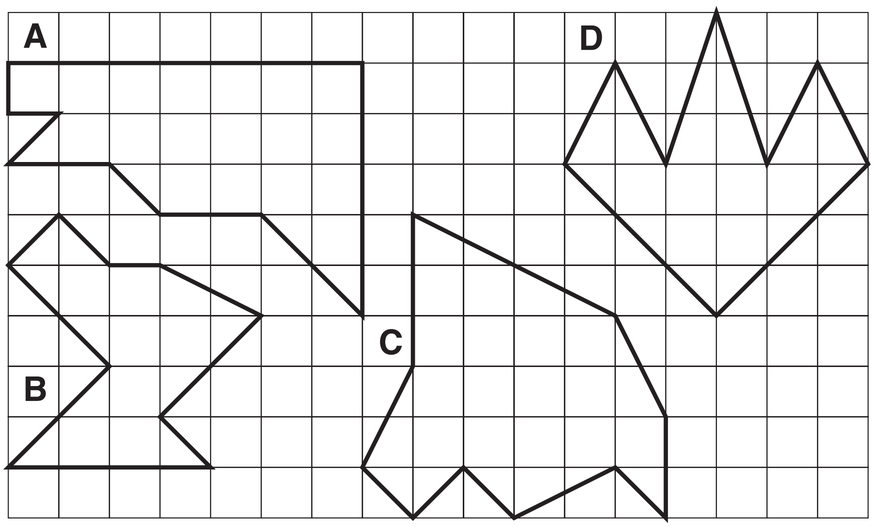
1. The length of a rectangle is 6 inches longer than the width. If the perimeter is 40 inches, what are the dimensions of the rectangle?

2. To make a yard for storing construction material, Paul will use a fence that is already on the property as one side of the yard. If he has 96 yards of fencing, what are the dimensions of the rectangular yard with the largest area he can make?

3. Pamela likes to keep in shape so every morning she runs a total distance of 36 city blocks. She always runs in a rectangular path. Today her path was twice as long as it was wide. How many blocks long and how many blocks wide was today's rectangular path?

CHALLENGE CARD 3: **Finding Cost**

1. The following shapes are being considered for a floor plan of one of the GEO WORLD exhibits. Each square represents 1 square foot. If building costs are $.48 a square foot, how much would each shape cost to build?

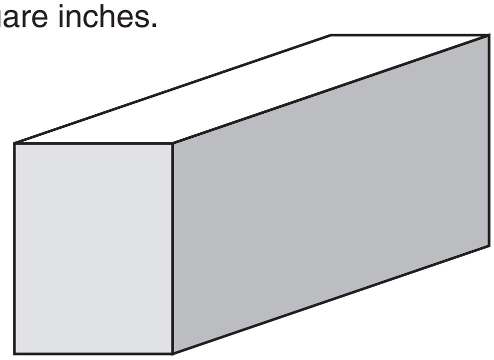


CHALLENGE CARD 4: **Area & Perimeter**

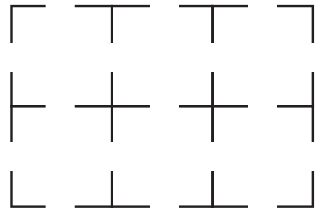
1. Find and list objects in the classroom or at home with the following perimeters (You may round off to the nearest inch/foot or as close as you can get.): 24", 32", 40", 64", 3', 10', 16', 22'.

2. Find objects around the classroom or at home with the following area (You may round off to the nearest inch/foot or as close as you can get.): 18 square inches, 24 square inches, 100 square inches, 9 square feet, 16 square feet, 24 square feet.

3. The side of a rectangular box has an area of 96 square inches. Its top is 72 square inches and its end is 48 square inches. What are the dimensions of the box?



CHALLENGE CARD 5: **Networks**



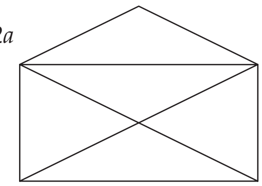
1. This house of doors has been designed as an exhibit to promote visitors ability to solve network problems. People going through it must walk through every door exactly once, being sure to enter through the first door and leave through the last door.

2. The following two figures are other interesting pathways being considered as exhibits for the park:

   Fig. 2a: • Visitors will need to walk on a continuous line without retracing any lines. See if it can be done.
   - Use your pencil and see if you can trace the path with one continuous line without retracing any line and without lifting your pencil.
   - Use arrows to show the direction your pencil is tracing.

   Fig. 2b: • Start at point A on the hexagon.
   - If visitors can only move on a downward path so that each path covered leaves them in a relatively lower position, how many pathways are there from point A to point B?

Figure 2a



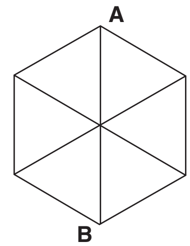
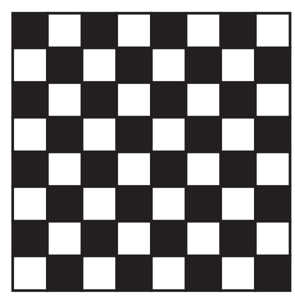


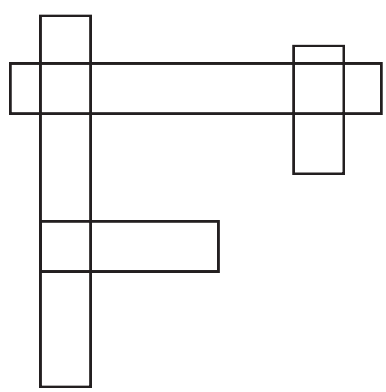
Figure 2b

CHALLENGE CARD 6: **Rectangles**

1. How many squares of all sizes are there on a checkerboard?



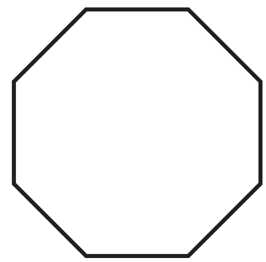
2. How many rectangles are in the figure below?



CHALLENGE CARD 7: **Shapes**

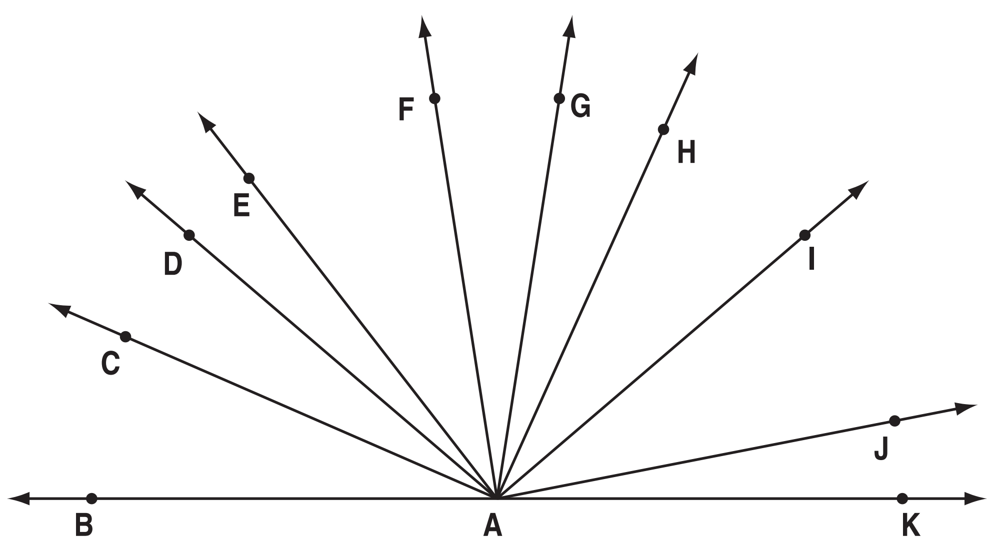
1. Make a mobile using the 7 different types of quadrilaterals defined in Quadrilateral Characteristics.

2. How many diagonals can be drawn in a regular octagon?



CHALLENGE CARD 8: **Rays**

1. How many angles are formed by the 10 rays with a common vertex in this illustration?



2. Identify as acute, right, or obtuse, and then measure the following angles:
   ∠BAC, ∠EAK, ∠DAG

CHALLENGE CARD 9: **Quadrilaterals**

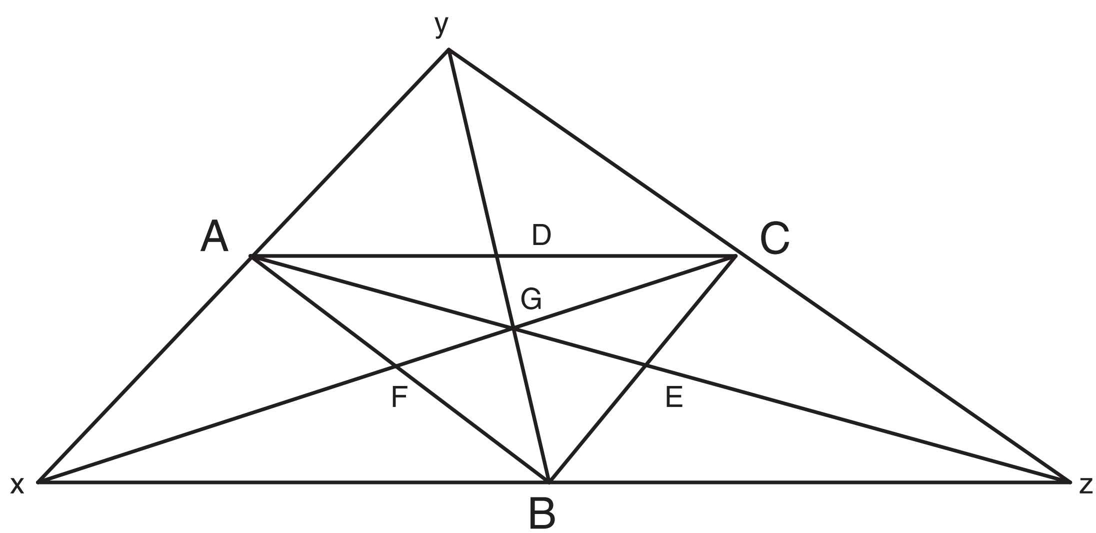
You have a symmetrical quadrilateral (with one line of symmetry) with an obtuse angle of 120 degrees.

How big are the other angles?
What does the figure look like?
What is the name of the figure?

With your group, create a quadrilateral puzzle similar to the puzzle above (giving hints about it's angles) and present it to another study group to solve.

CHALLENGE CARD 10: **Triangles**

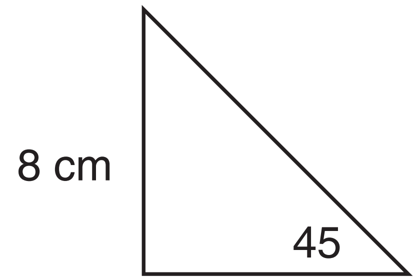
1. How many different triangles are there in this figure?



2. What is the measure of the largest angle less than 180° in this figure? The smallest angle?

CHALLENGE CARD 11: **What Do You Know?**

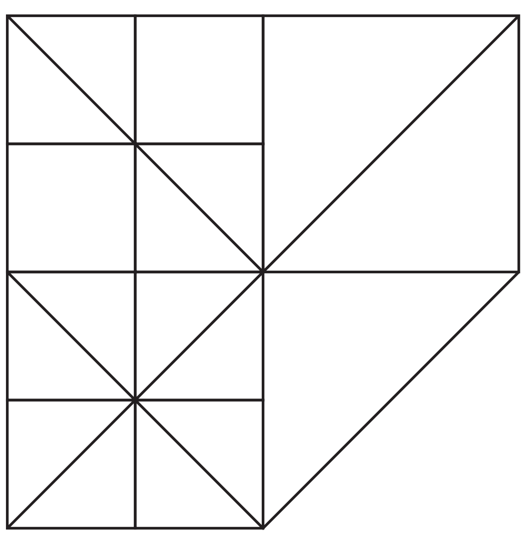
1. Write down everything you know or can figure out about this triangle:



2. Draw a large letter W as below.
   Can you draw three straight lines through the letter so that nine separate, non-overlapping triangles are formed? Triangles must stand alone and not include any other shapes within the sides.

CHALLENGE CARD 12: **How Many?**

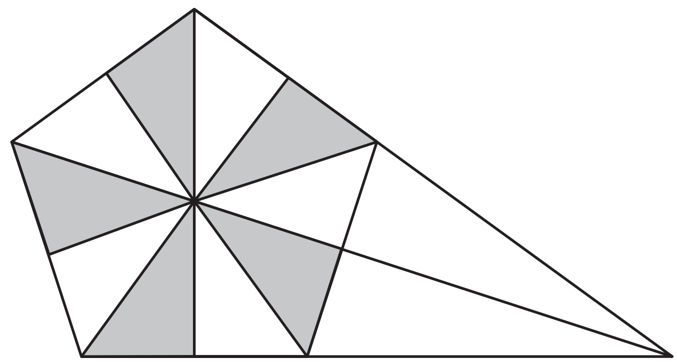
1. How many of each shape can you find:
   - Squares
   - Rectangles
   - Trapezoids
   - Triangles



2. How many isosceles triangles can be made with a perimeter of 24 inches if each side must be a whole number of inches?

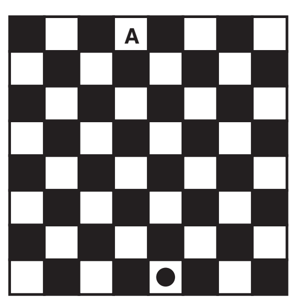
CHALLENGE CARD 13: **Multiple Triangles**

1. How many triangles can you count in this picture?



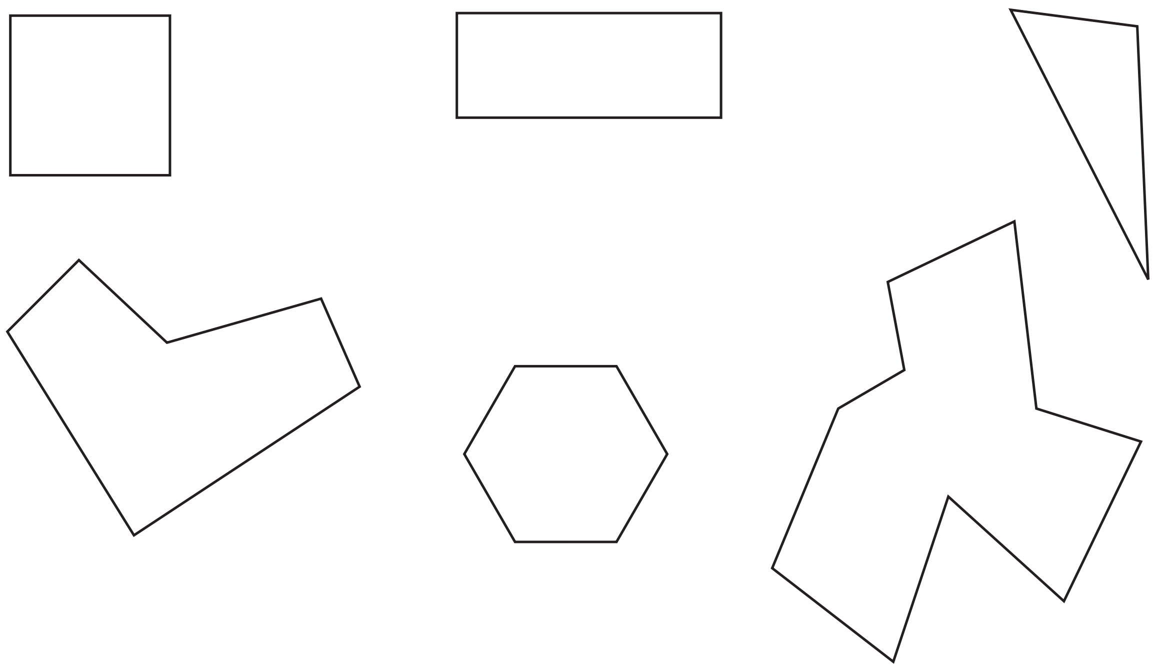
CHALLENGE CARD 14: **Pathways**

The checkerboard in this picture contains one checker.
The checker can only move "up" the board along the white squares.
In how many ways can this checker reach the square marked A?

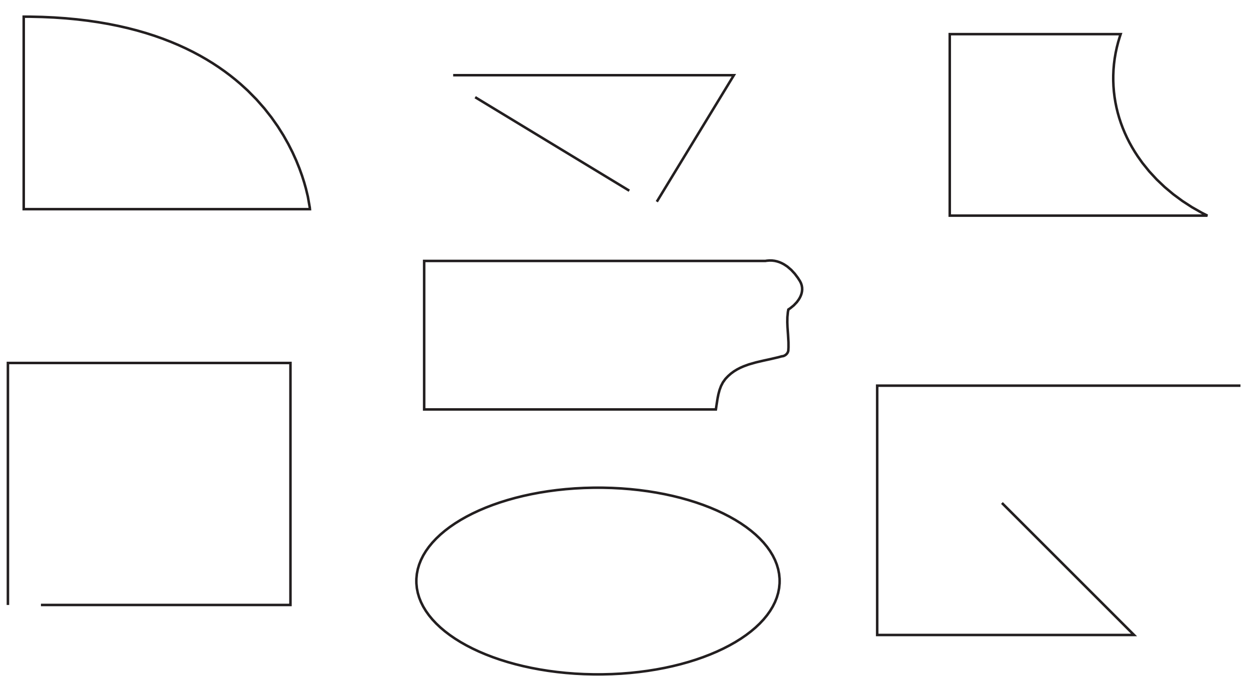


POLYGONS/NOT POLYGONS

These are polygons:



These are not polygons:



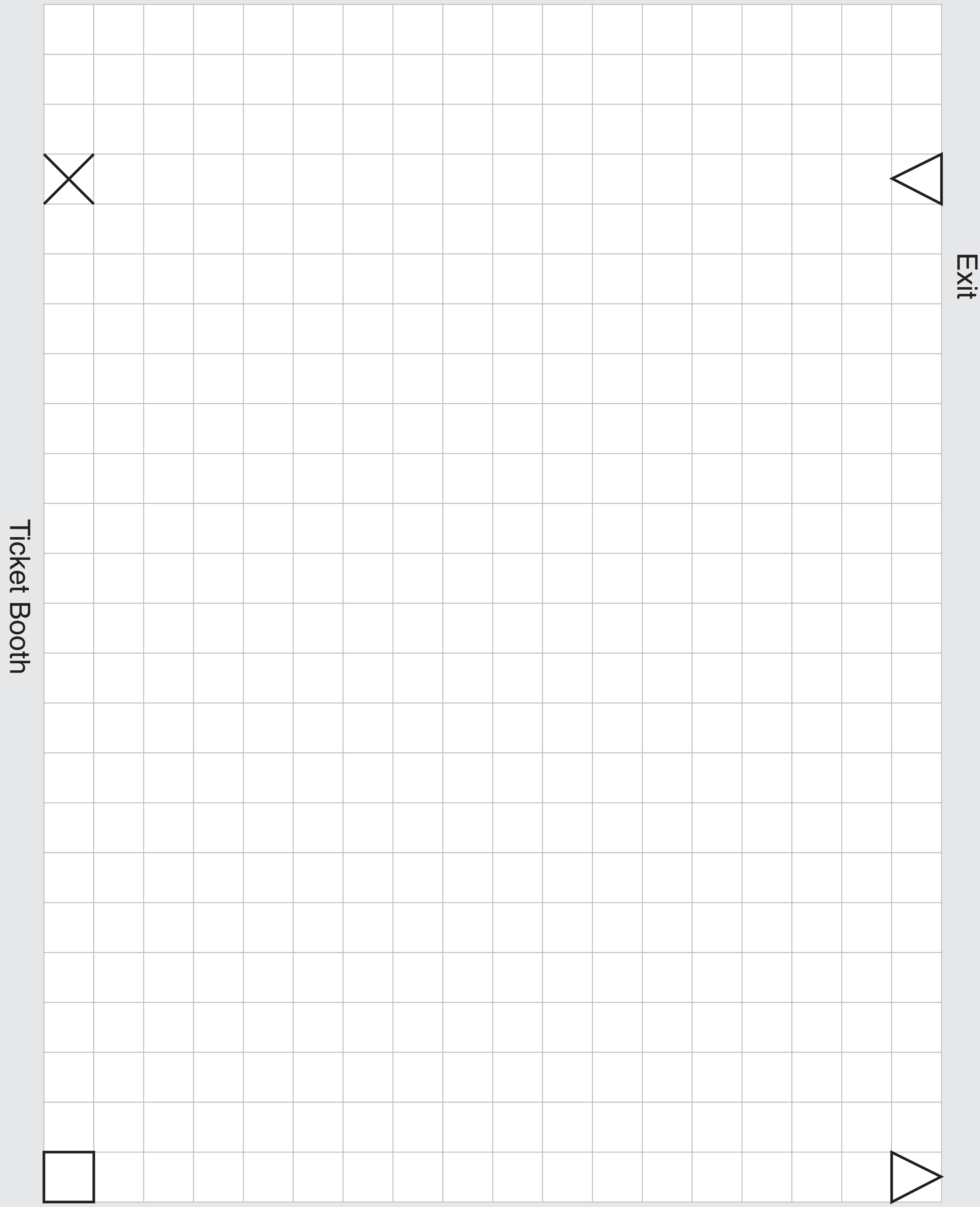
1-INCH GRID

1/4-INCH GRID

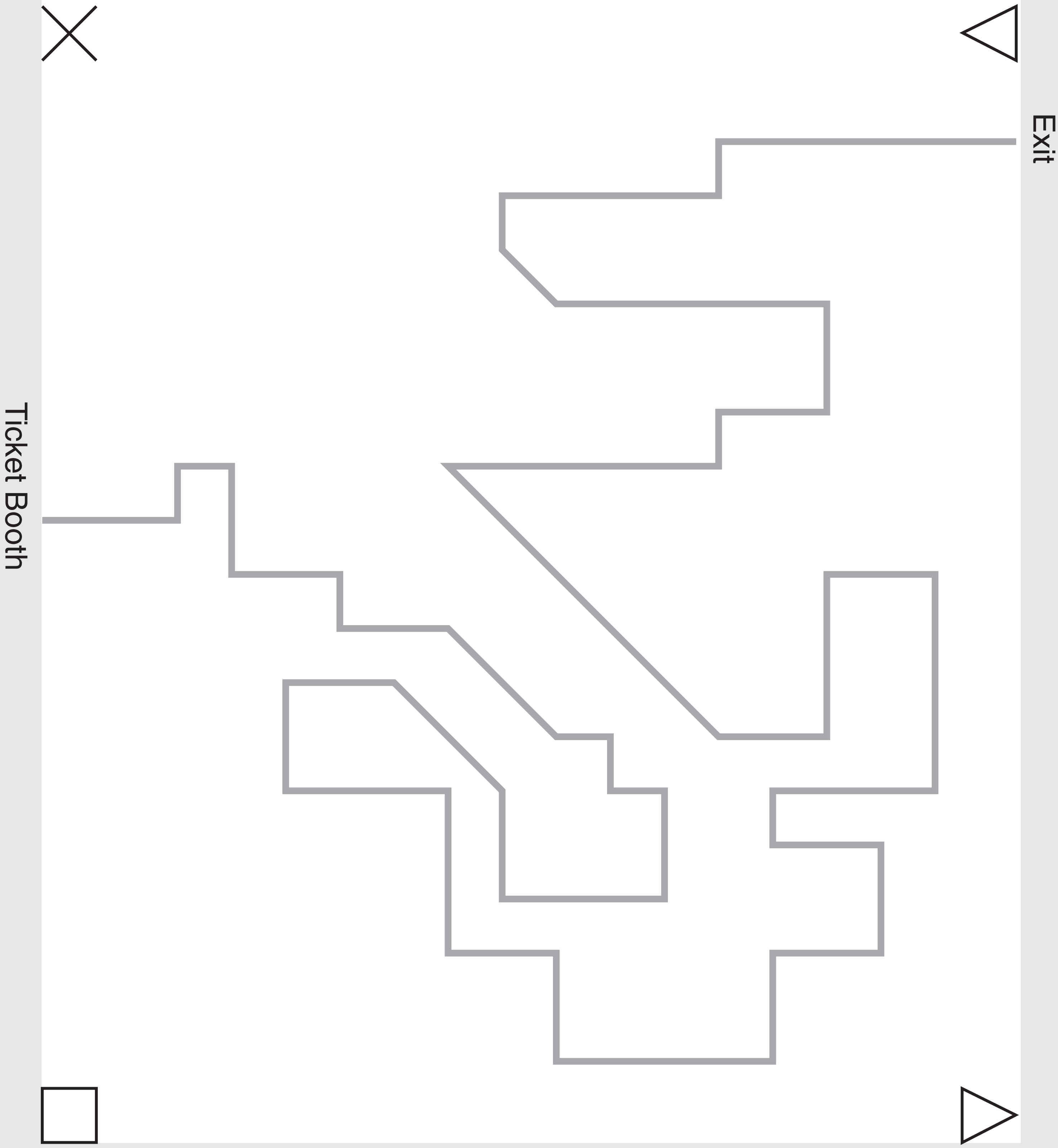
1-CENTIMETER GRID

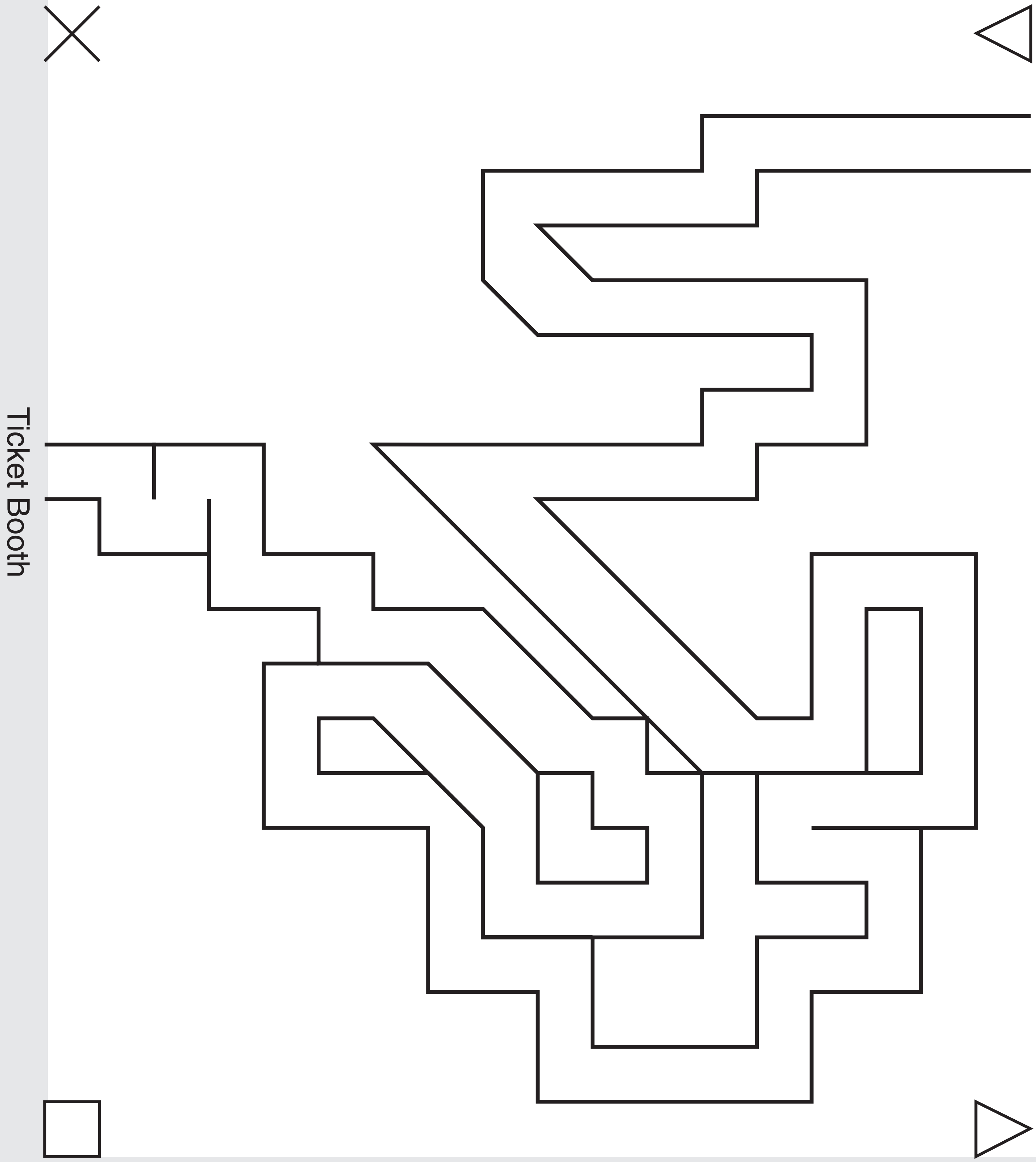
4-INCH BY 6-INCH GRIDS

MAZE 1A

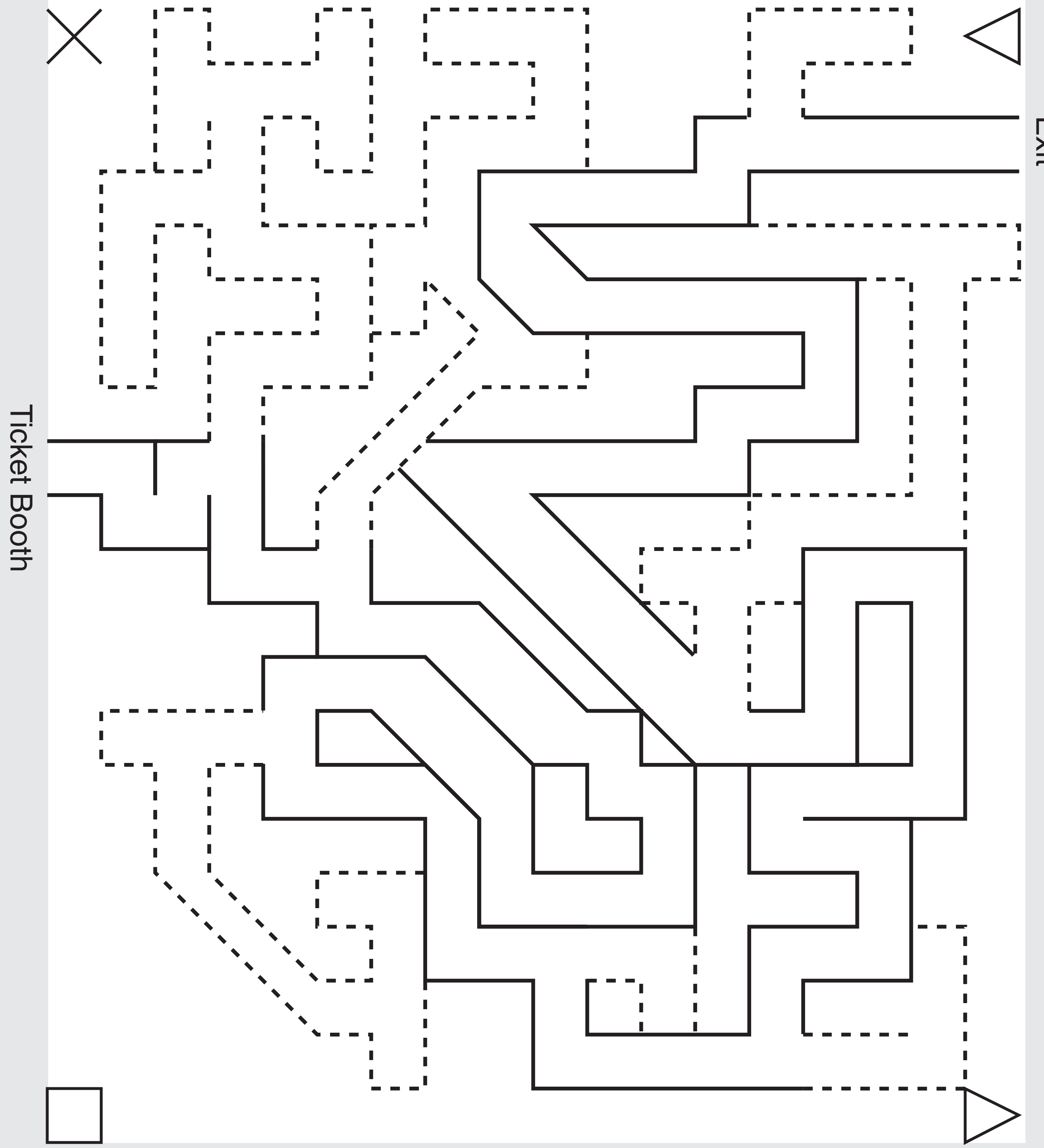


MAZE 1B

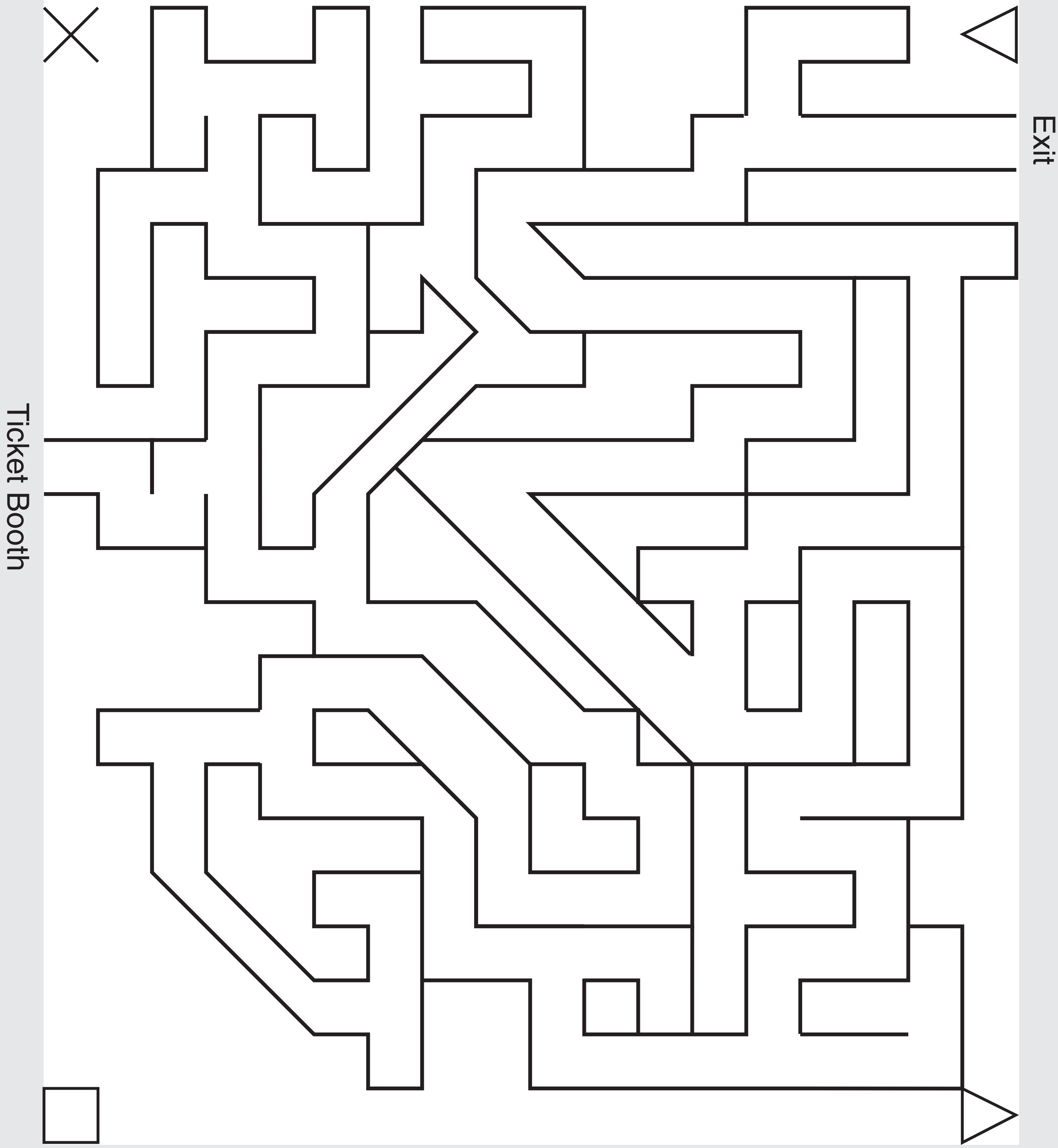



Exit
Ticket Booth

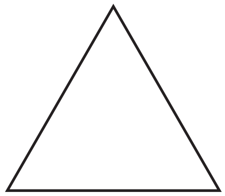
MAZE 1D



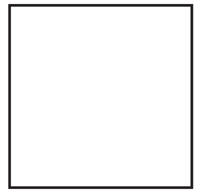
MAZE 1E



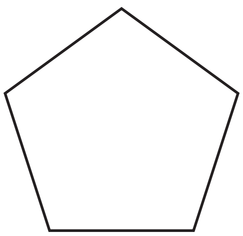
REGULAR POLYGONS



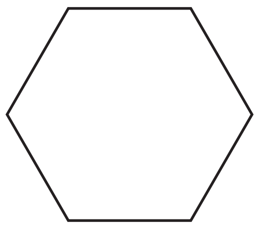
Regular
Triangle



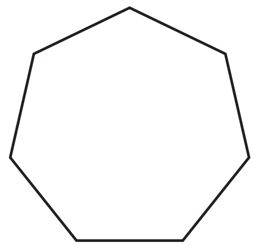
Regular
Quadrilateral



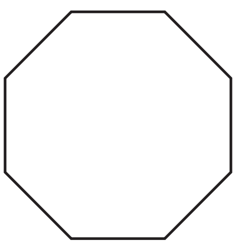
Regular
Pentagon



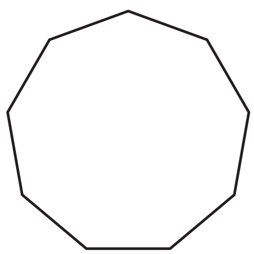
Regular
Hexagon



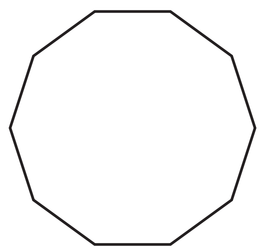
Regular
Septagon



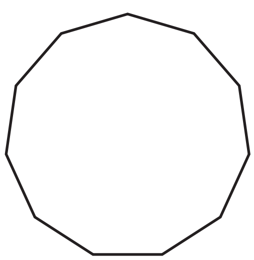
Regular
Octagon



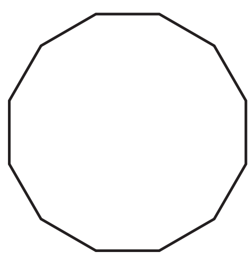
Regular
Nonagon



Regular
Decagon

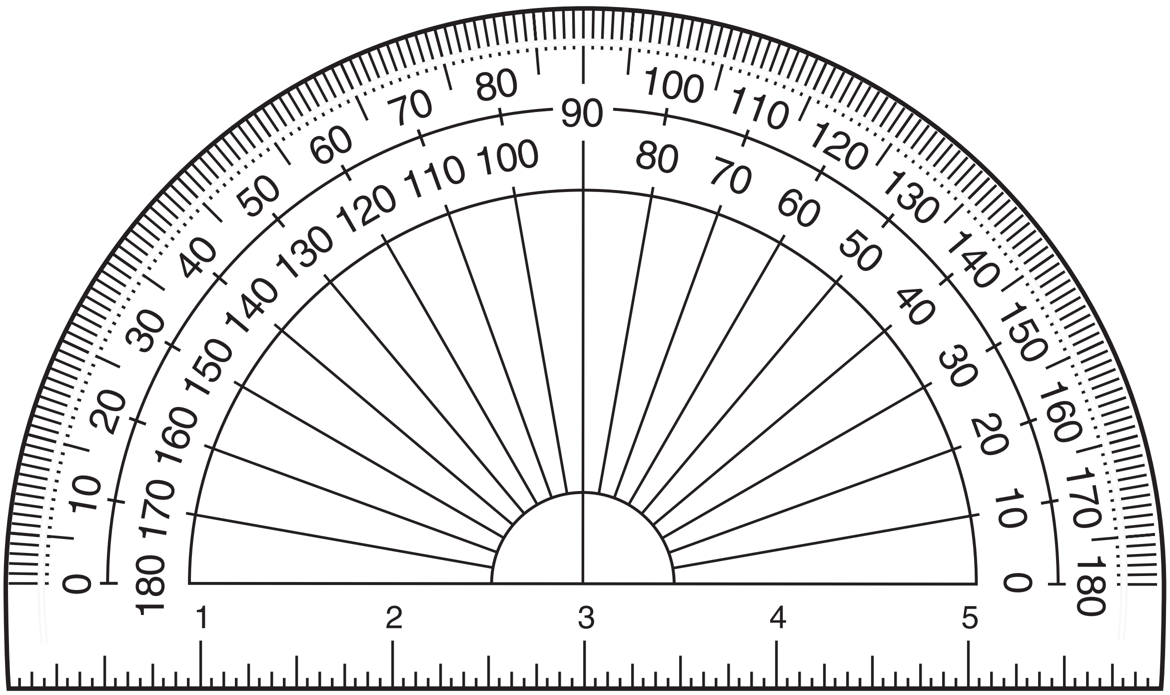


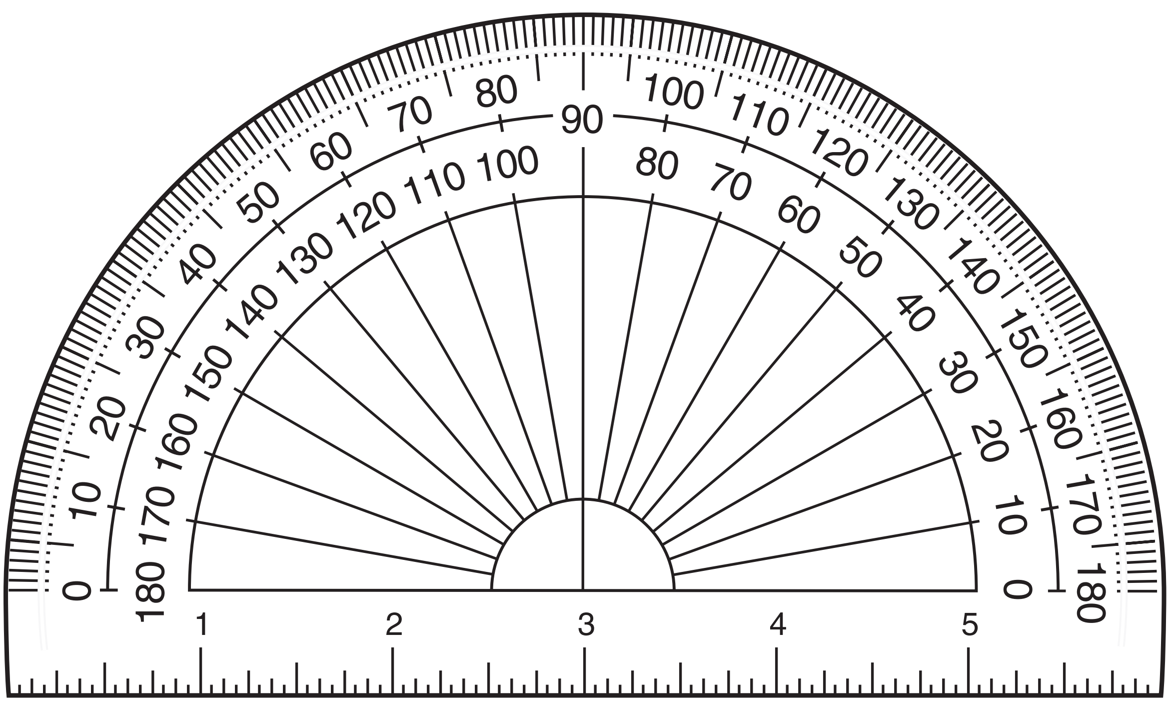
Regular
Enneagon



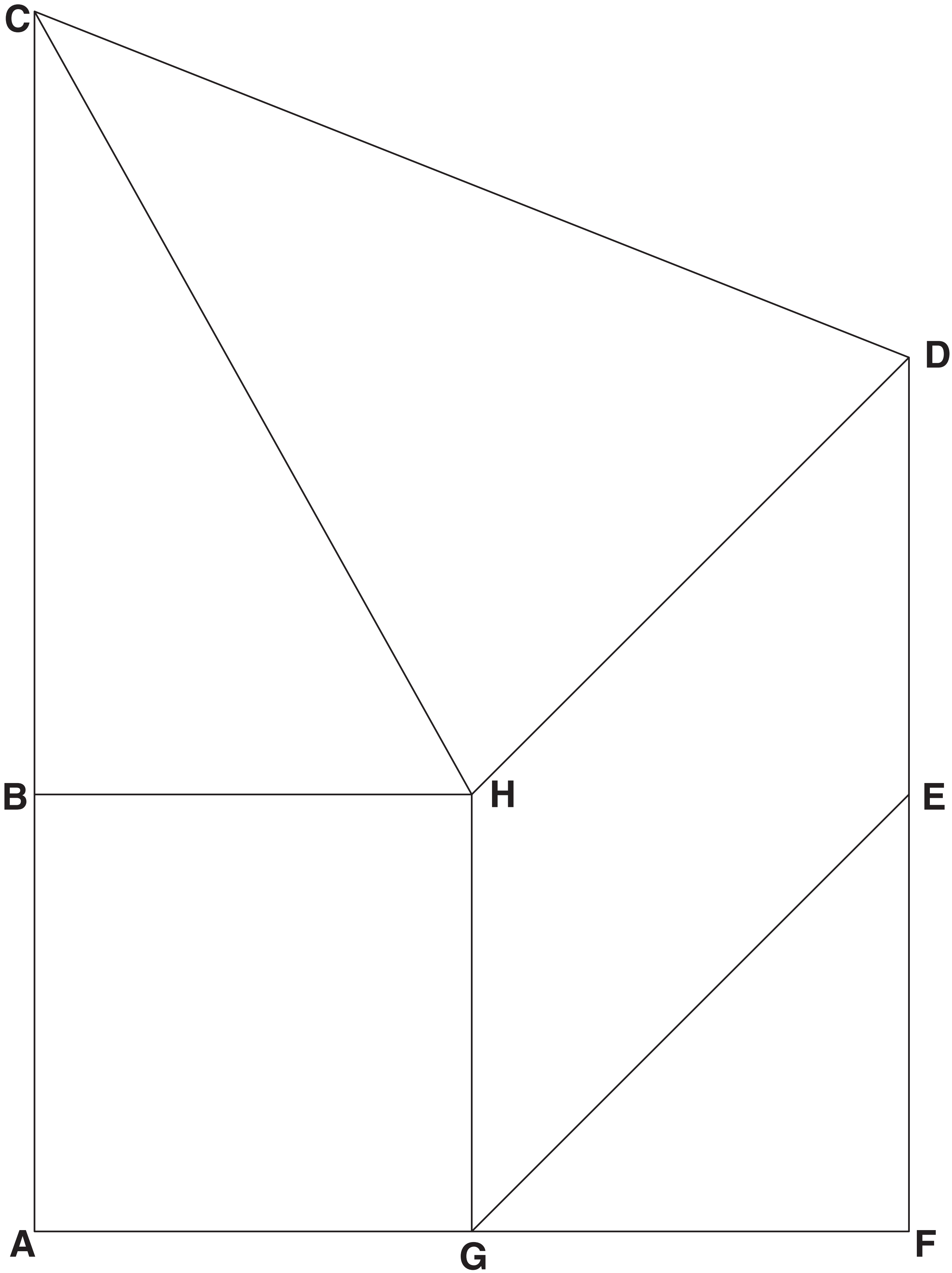
Regular
Dodecagon

PROTRACTORS

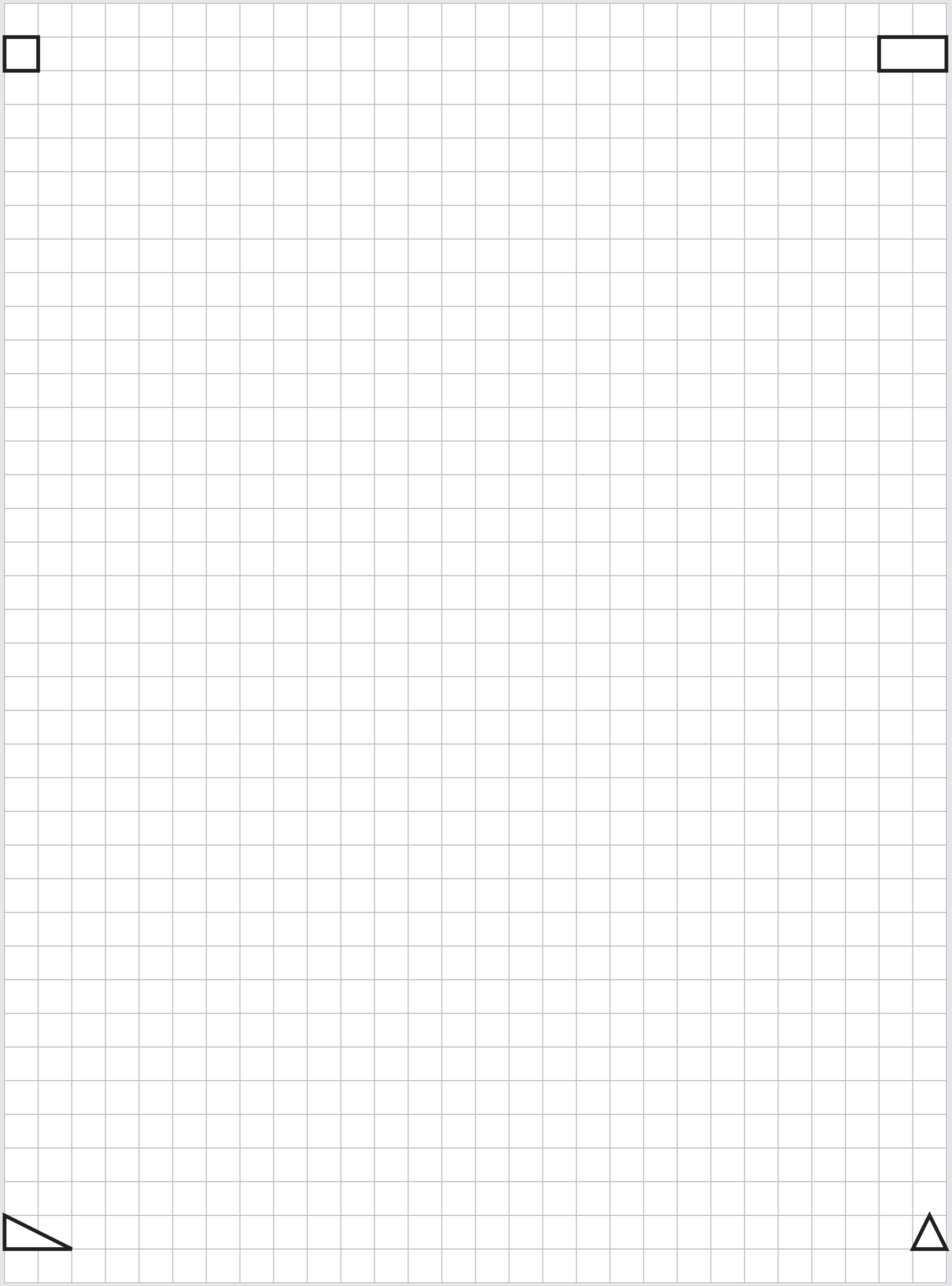


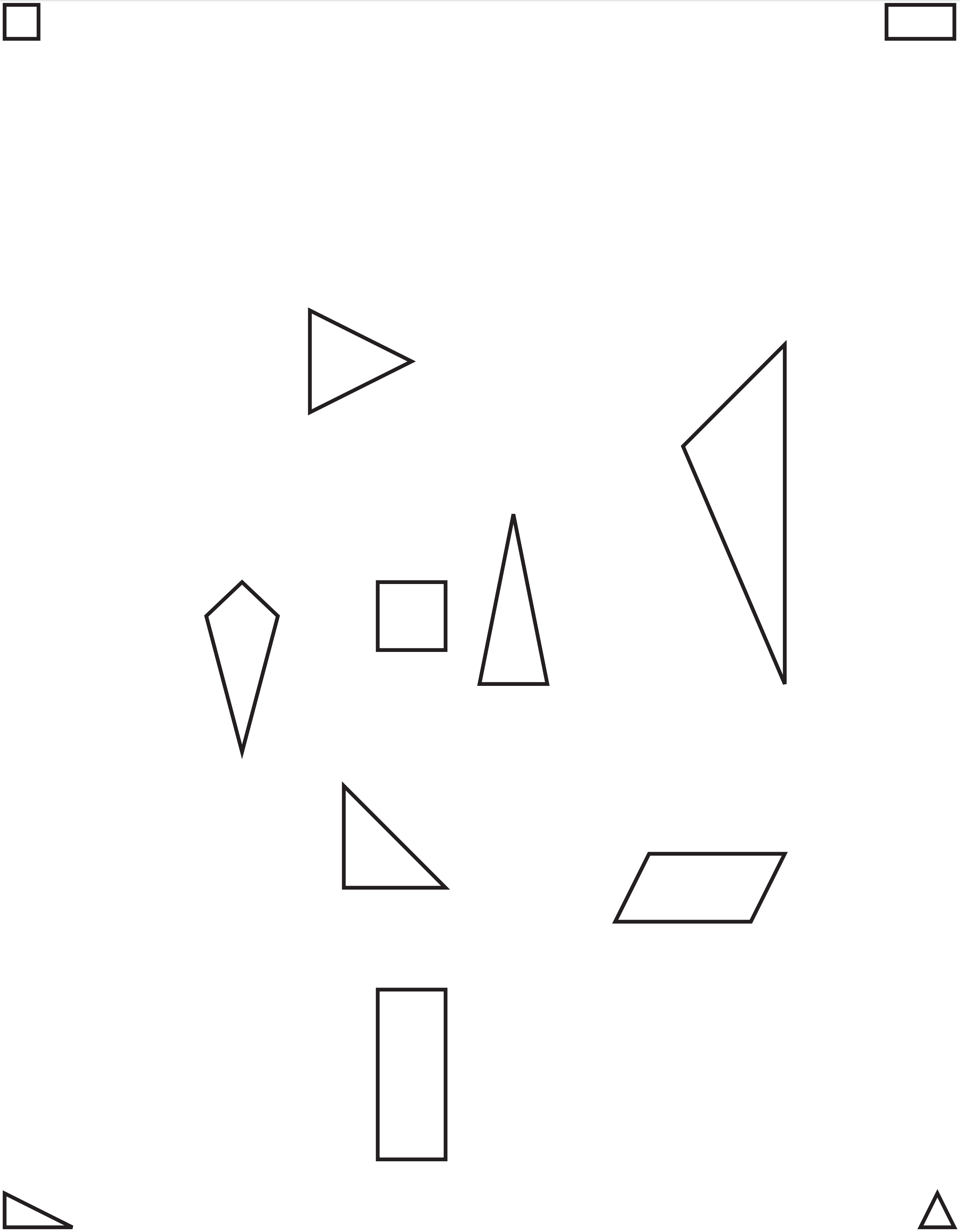


QUIZZICAL QUADRILATERAL

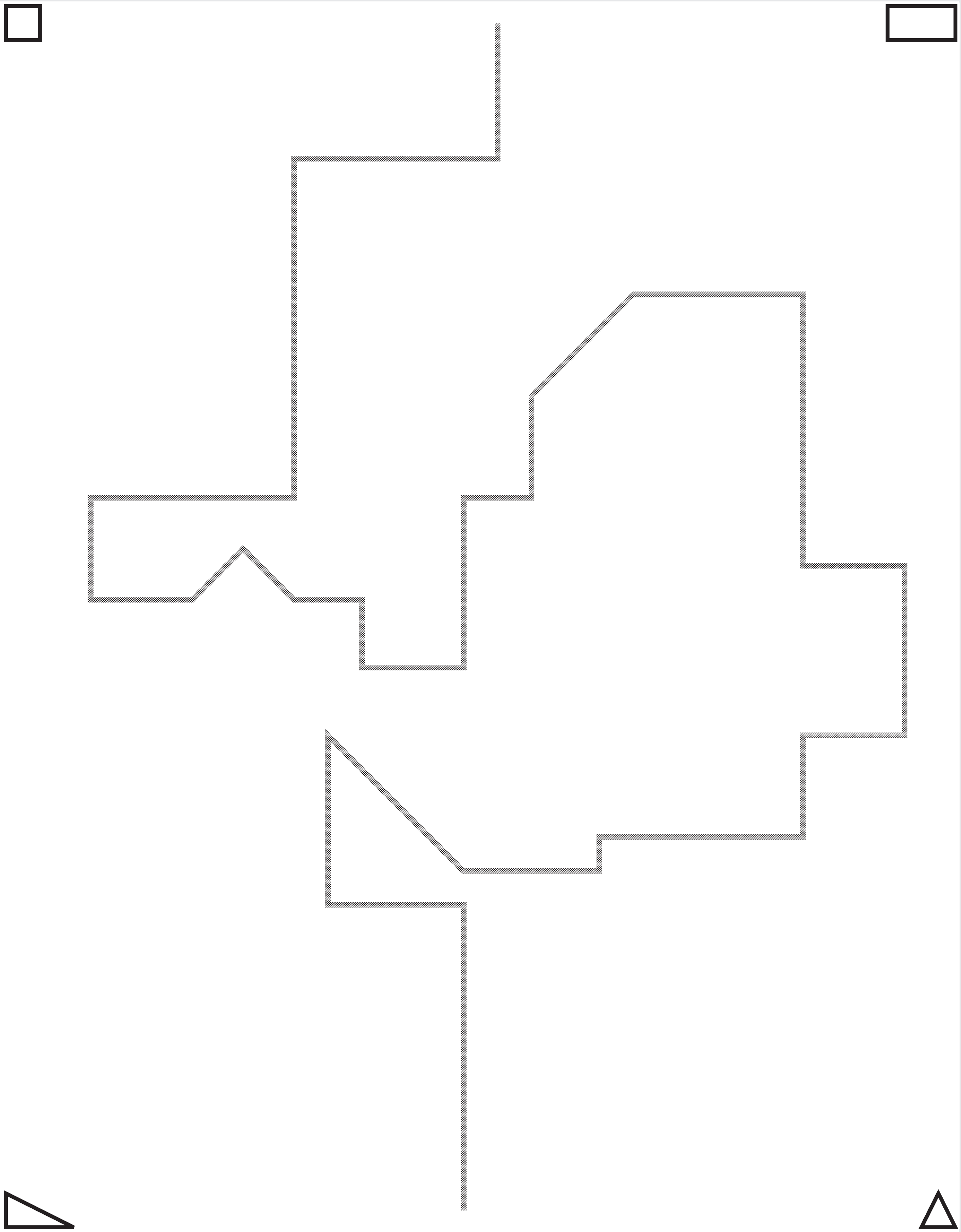


MAZE 2A

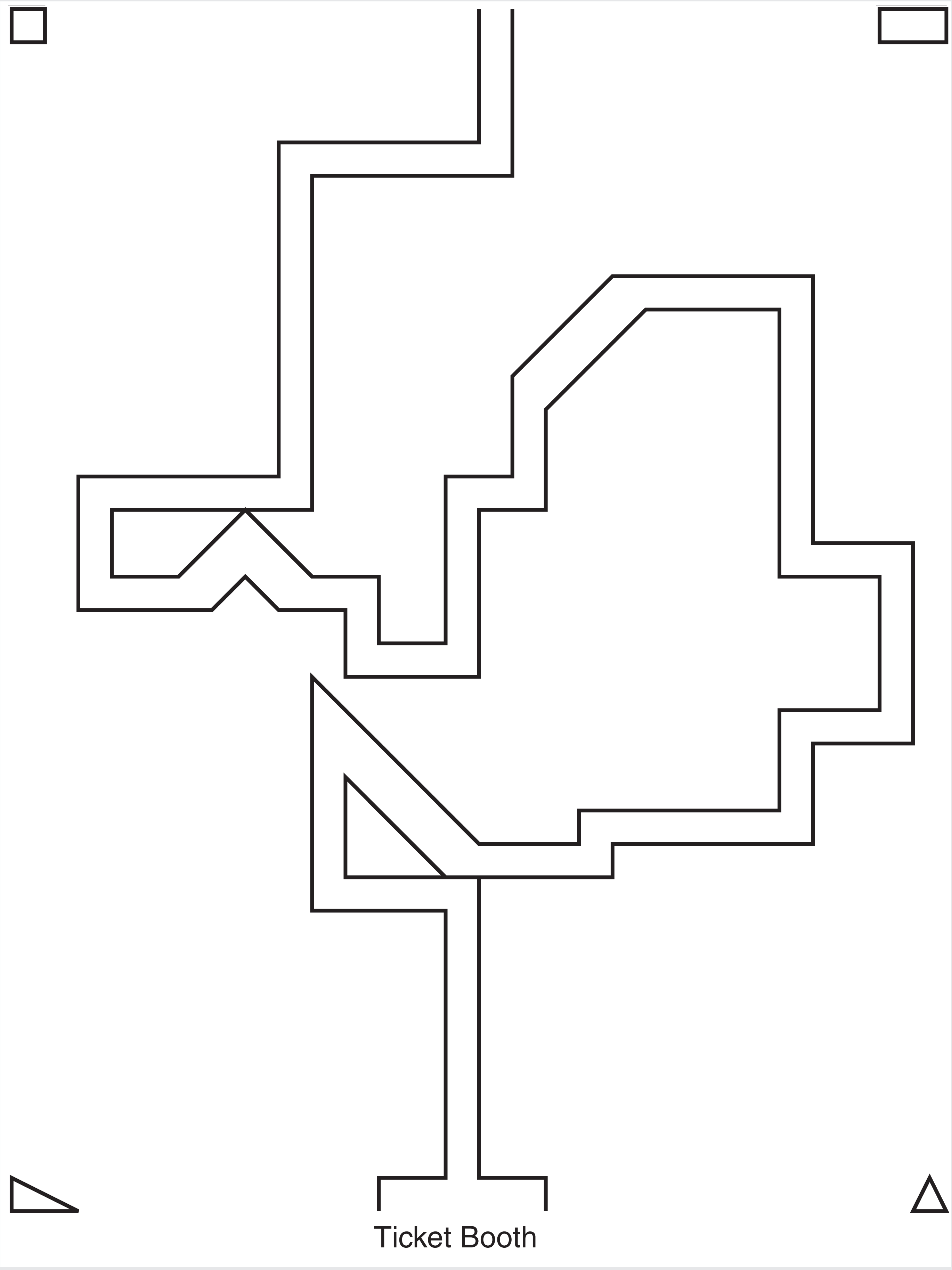


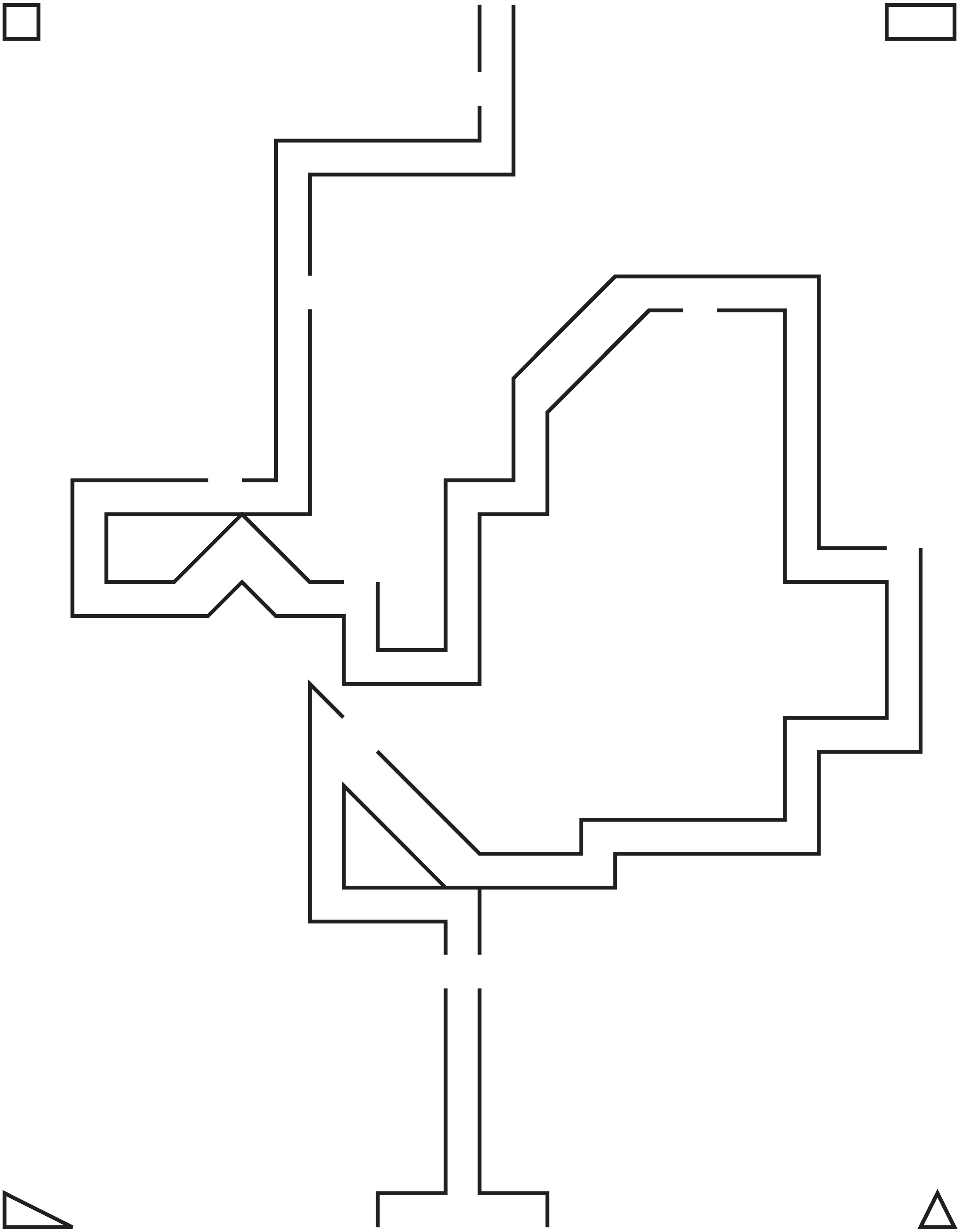


Ticket Booth



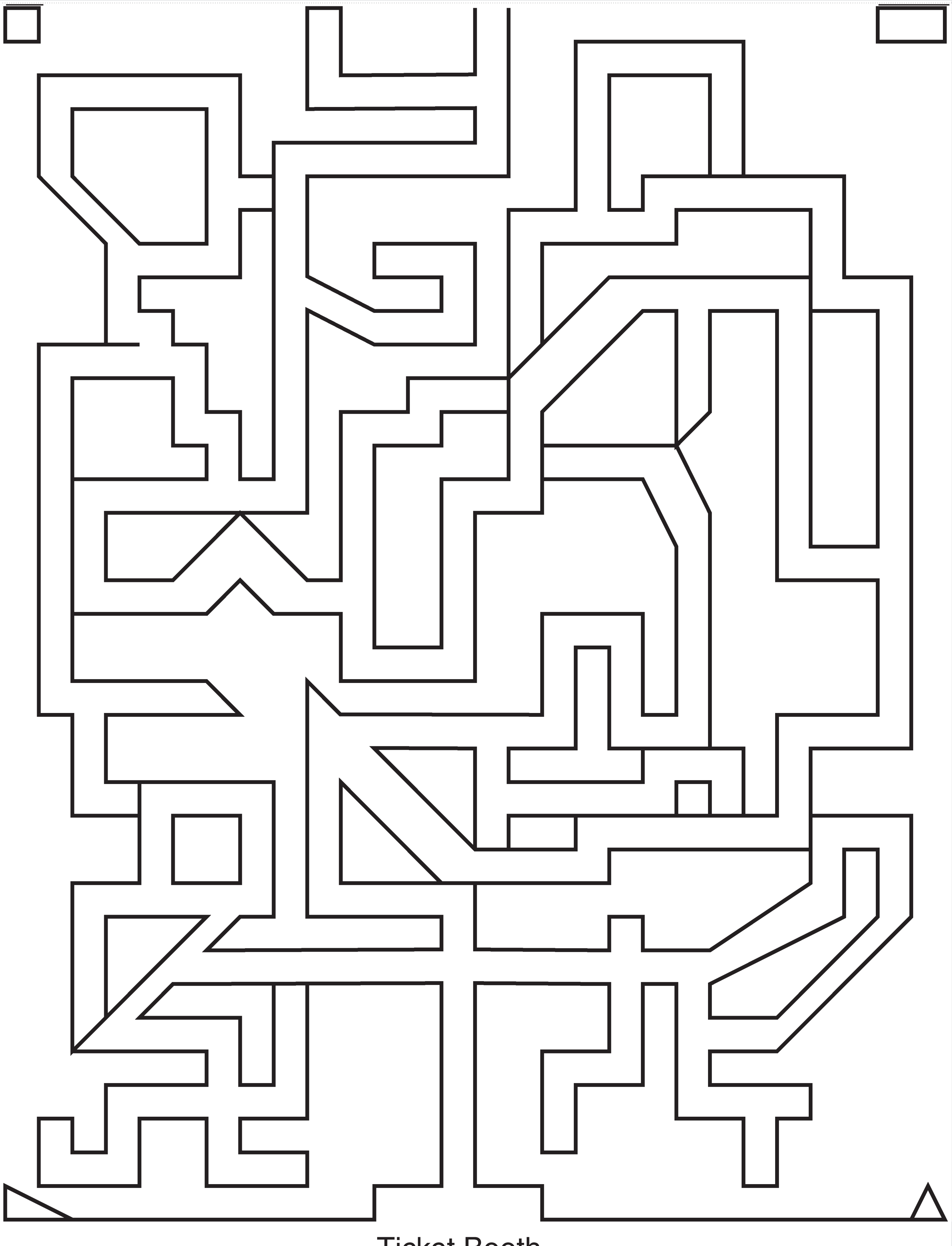
Ticket Booth


Ticket Booth



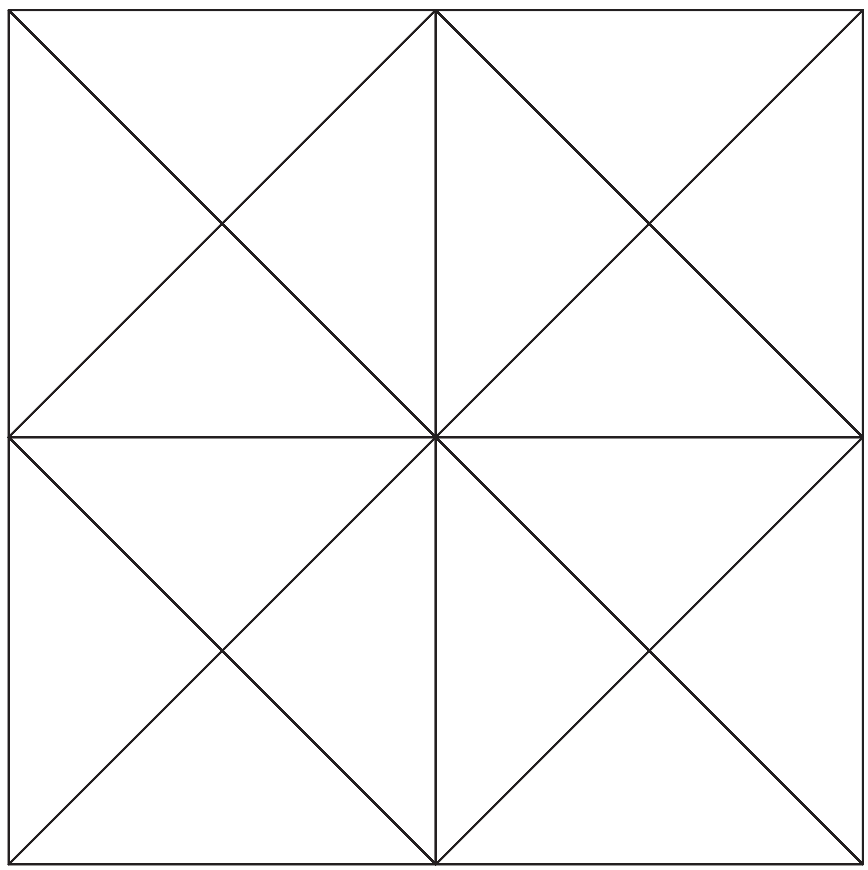
Ticket Booth

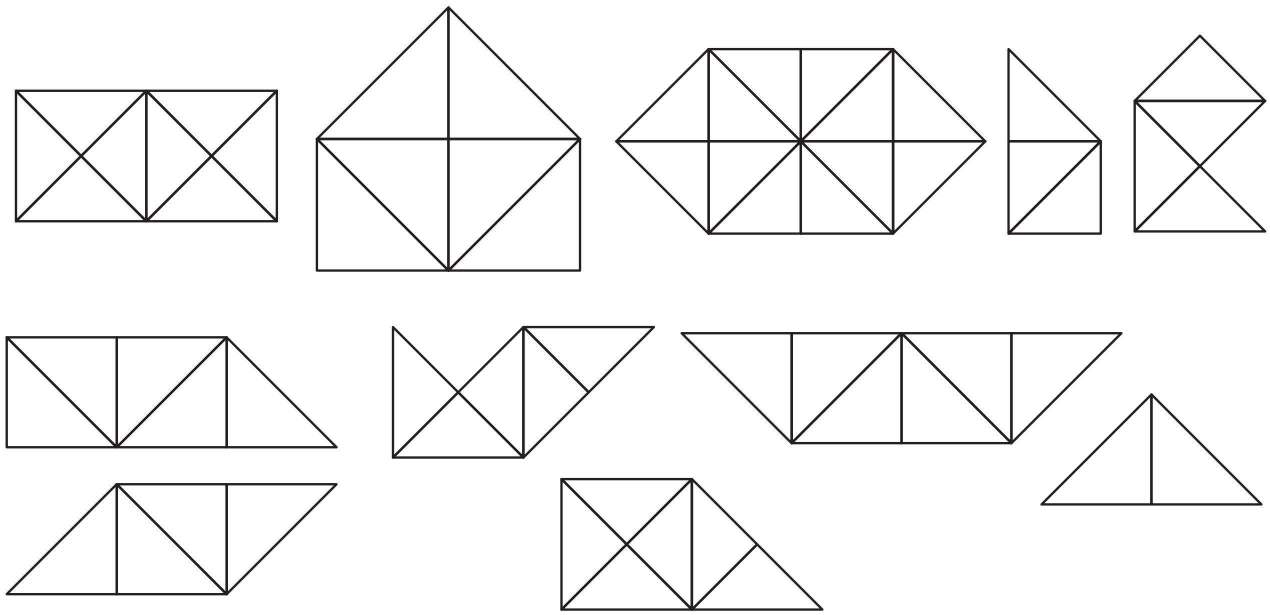
MAZE 2F



EXTENSION DAY 1: **Fold A Shape**

Cut out the square figure. Trace the lines on the back of the square. Fold on each line. Then use your Fold A Shape to make the shapes at the bottom of the page.



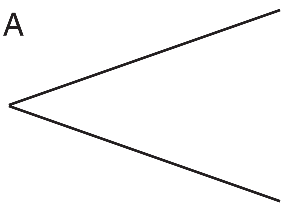


EXTENSION DAY 8: **Angle Line Designs**

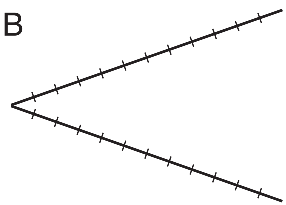
Materials: Blank paper, rulers, pencils
Key vocabulary: Acute angle, Ray, Vertex

1. Using your ruler, draw an acute angle with rays 12 centimeters long that looks similar to drawing A. Next make 1 centimeter spaces notched off along each side like drawing B.

A

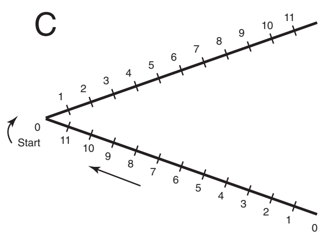


B

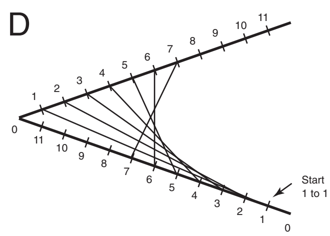


2. Be sure that each ray has the same number of notches. Put a zero at the vertex and number along one side of the angle. Begin to number the notches along the other side of the angle. However, put zero at the end of this ray, and start to number toward the vertex. Now, connect each pair of the same numbers. Draw a line from one to one, a second line from two to two, and so on.

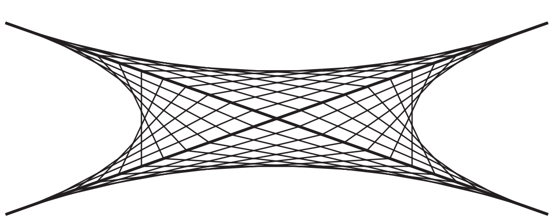
C

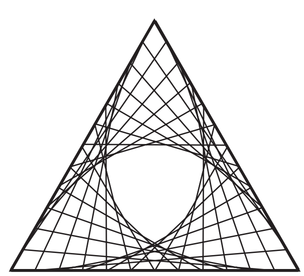


D



3. Now you can make a more elaborate intersecting line design and/or geometric figure design.

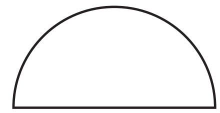




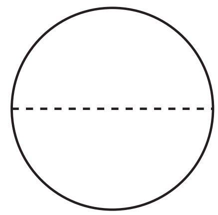
EXTENSION DAY 9: **Make A Protractor** - 1

Materials: Circular coffee filters (ironed flat), string, tape
Key Vocabulary: Chord, Degree, Diameter, Protractor, Radius
Key Concepts: The unit of measure for an angle is the degree.
A degree measures the angles that make up a circle.
There are 360 degrees in a circle.

Procedure and Discussion Questions:



1. Fold the coffee filter in half creating two congruent pieces.

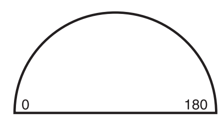


2. Unfold the filter, and observe a line segment.

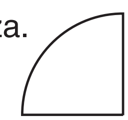
   What do we know about the line segment?

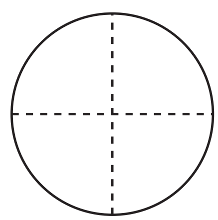
   (**Possible answers**: It is straight. It is the longest line segment in the circle. It goes through the center of the circle. It divides the circle into two equal areas. It is a line of symmetry. It is a chord. Each area is half of the circle. The two halves of the circle are congruent. The line segment is called a diameter.)

   We know there are 360 degrees in a whole circle. If the diameter cuts a circle in two congruent parts, how many degrees must there be in half a circle? (180°) How do you know? Can you prove it a different way? Fold your circle back into two congruent half circles and write 0° and 180° at either end of the half circle.



3. Fold the circle again into four congruent pieces that resemble cuts of a pizza.

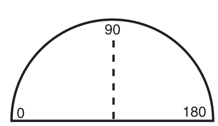




4. Unfold the filter.

   What can you tell me about these line segments?

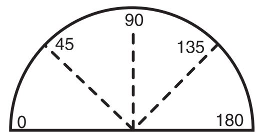
   (**Possible answers**: Half the diameter. Called a radius. Each radius is congruent to every other radius. Starts at the center and continues to a point on the circle. Divides the circle into four equal areas. Each section is 1/4 of the circle.)



5. Fold the filter in half to make a half circle again.
   Start at zero degrees and move around the edge of the half circle to the right.
   How many degrees will you find half of the way around the half circle? (90°)
   How do you know? Can you prove it a different way?
   Write 90° on the coffee filter at the 1/4 fold line.

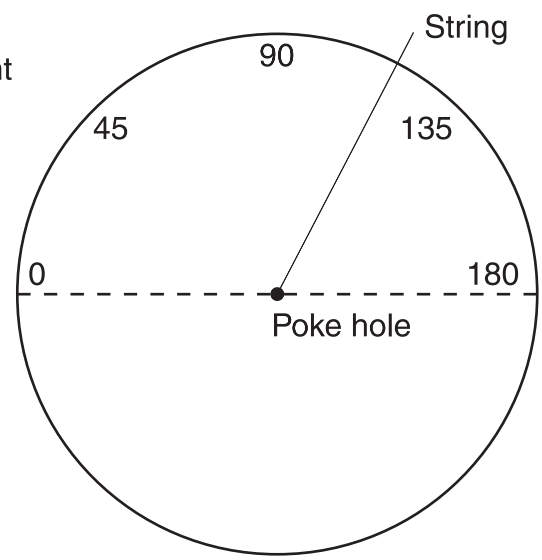
EXTENSION DAY 9: **Make A Protractor** - 2

6. Fold the circle one more time and write in the number of degrees on the coffee filter.

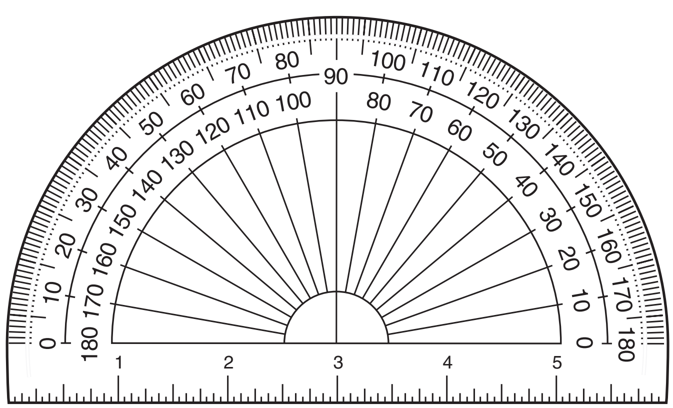


You will not have a complete protractor because after a few folds you will not be able to continue folding. There will be enough folds to give the idea of what a protractor is and how it works.

7. Poke a hole in the center of the filter.
   a. Push a string through the hole.
   b. Tape it on the back of the filter.
   c. Using the string as a radius, experiment with angles, always measuring from 0°.



8. A second set of numbers can be added so that angles can be measured from the right as well as from the left. For example, 180° should also show 0°, as in a protractor.

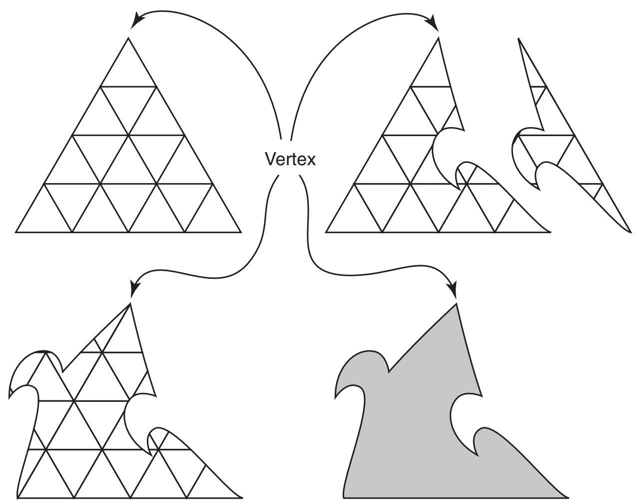


Adapted from activity developed by Brian Tash, Oceanside Unified School District, Oceanside, California

EXTENSION DAY 12: **Triangle Tessellations** - 1

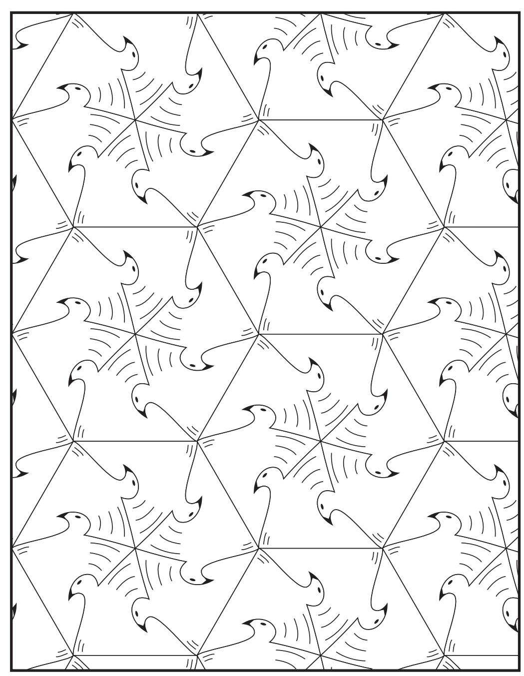
Materials: Equilateral triangle paper, blank paper, scissors, tape.
Key Vocabulary: Tessellation: a design that covers a plane with no gaps or overlaps.

1. Cut an equilateral triangle shape with 4 units on each side. Cut a figure from one side of the triangle. (Sometimes this is called a nibble.)
   a. Cut into only one side of the triangle.
   b. Tape the cutout portion to one of the uncut sides.
   c. The piece must be taped with the two pieces showing the printed side, with the lines continuing together.
   d. The tape shouldn't overlap at the edges of the pattern.
   e. The cut out piece should be the same distance from the vertex as it was on the cut out side.
   f. Trace around the figure.

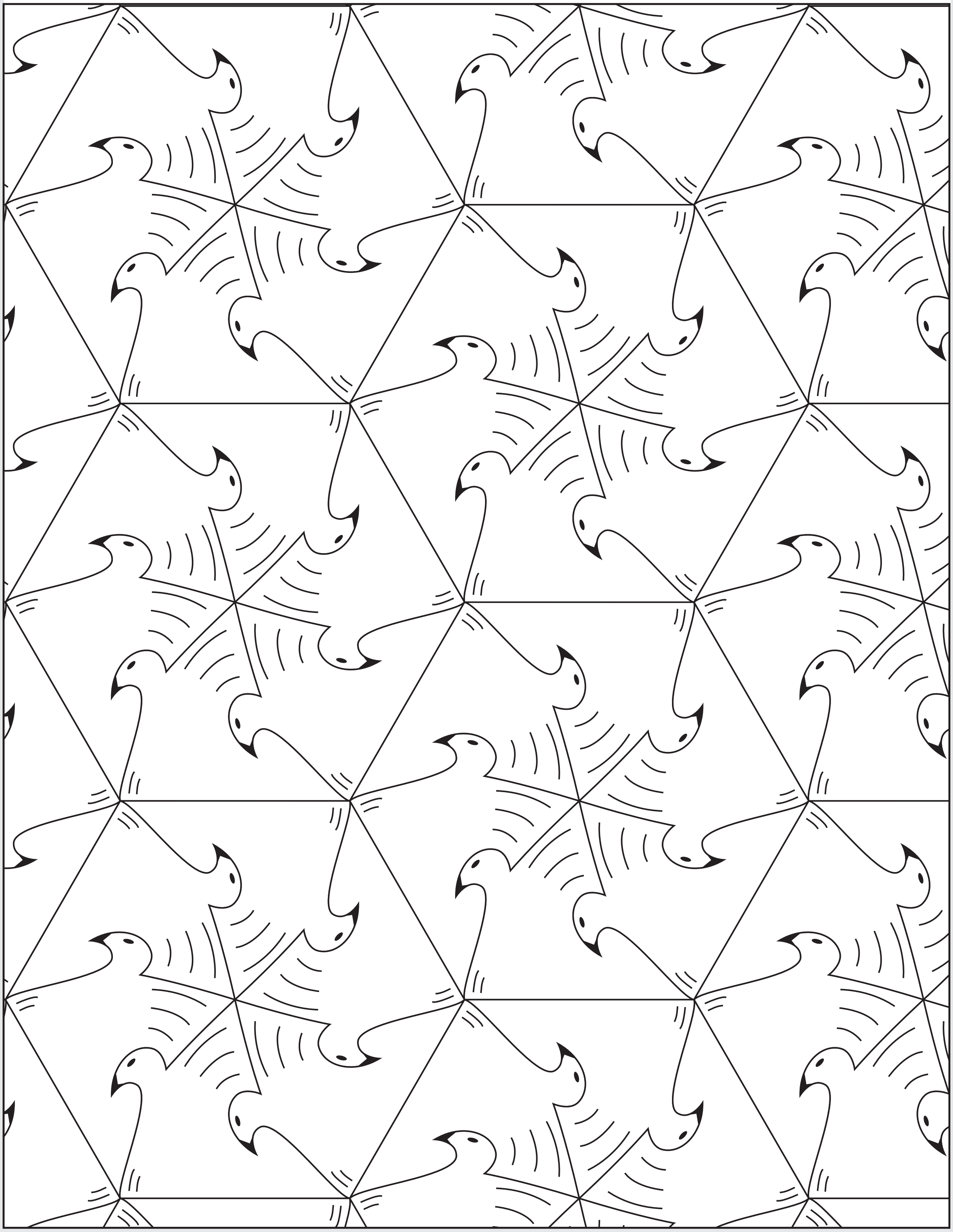


2. When you have designed a shape you like, trace the shape on tag or other heavy paper and cut it out.

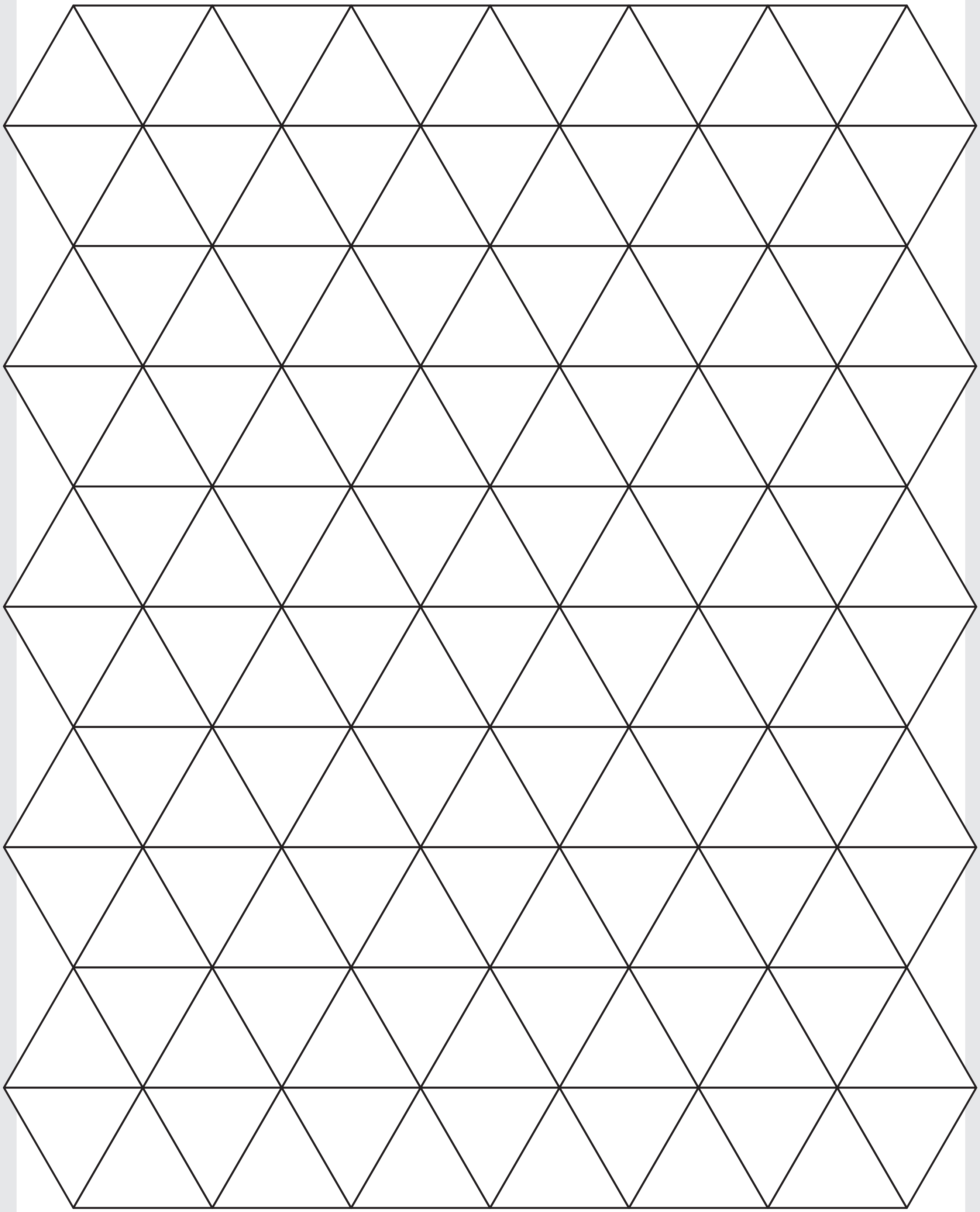
3. Draw around the shape several times to make the tessellation.
   a. Place the pattern in the center of a piece of paper with the uncut leg of the triangle facing the edge of the paper.
   b. Trace around it.
   c. Rotate the pattern around the vertex between the cutout side and the taped side until the pattern fits exactly into the shape of the tracing.
   d. The uncut leg of the triangle forms the outside edge of the tessellation design.
   e. Continue tracing the figure to complete your tessellation design. Notice how one figure melts into the next.
   f. Repeat your entire tessellation design several times in every direction, matching bases of the triangles to begin the next rotated design. The design should seem to continue beyond the edge of the paper.
   g. It is important that your tessellation design fills the entire area, with no gaps or overlaps.



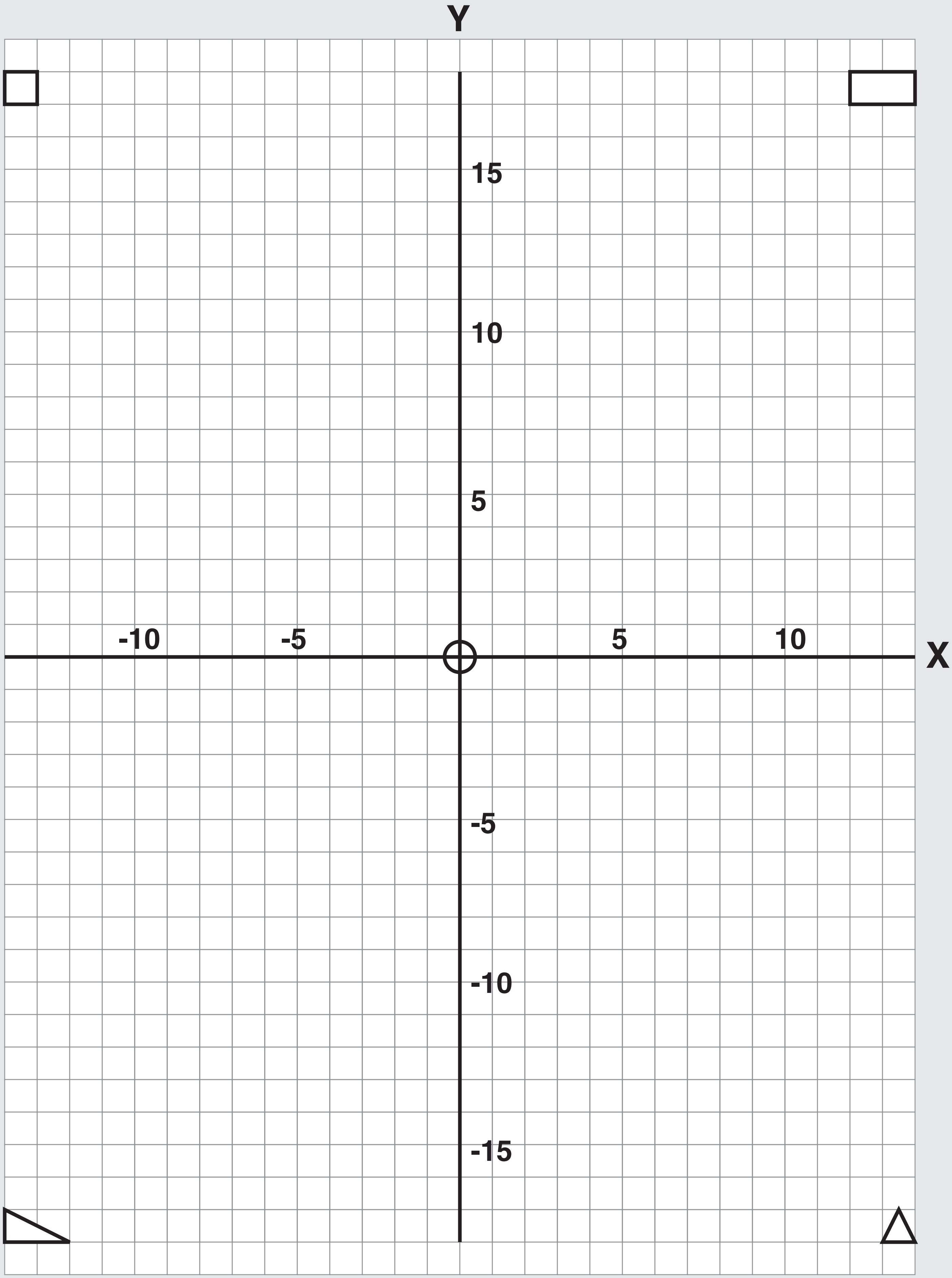
EXTENSION DAY 12: **Triangle Tessellations** - 2



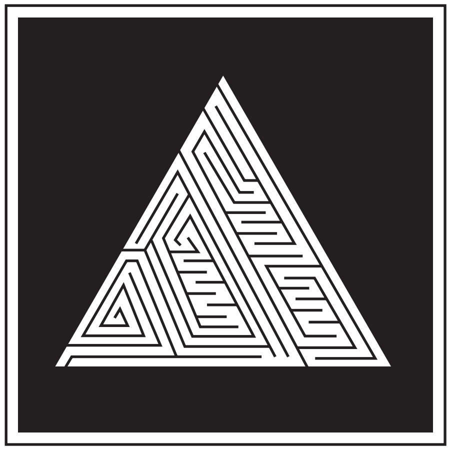
EXTENSION DAY 12: **Triangle Tessellations** - 3



EXTENSION DAY 14: **Cartesian Coordinate GRID**



A-Maze-Ing Shapes



Certificate of Participation

This certifies that

____

has learned about
geometric shapes and mazes

_____________________________ __________________

Teacher Date

NOTES

Teacher Feedback Form

At Interact, we constantly strive to make our units the best they can be. We always appreciate feedback from you—our customer—to facilitate this process. With your input, we can continue to provide high-quality, interactive, and meaningful instructional materials to enhance your curriculum and engage your students. Please take a few moments to complete this feedback form and drop it in the mail. Address it to:

Interact • Attn: Editorial
10200 Jefferson Blvd. • P.O. Box 802
Culver City, CA 90232-0802

or fax it to us at **(800) 944-5432**

or e-mail it to us at **access@teachinteract.com**

We enjoy receiving photos or videotapes of our units in action!
Please use the release form on the following page.

Your Name: ____

Address: ____

____

E-mail: ____

Interact Unit: ____

Comments: ____

____

____

____

____

____

____

____

____

Release Form for Photographic Images

To Teachers:

To help illustrate to others the experiential activities involved and to promote the use of simulations, we like to get photographs and videos of classes participating in the simulation. Please send photos of students actively engaged so we can publish them in our promotional material. Be aware that we can only use images of students for whom a release form has been submitted.

To Parents:

I give permission for photographs or videos of my child to appear in catalogs of educational materials published by Interact.

Name of Student: ____ (print)

Age of Student: ____ (print)

Parent or Guardian: ____ (print)

Signature: ____ Date: ___________________

Address:

____

____

____

____

Phone: ____

Interact
10200 Jefferson Blvd.
Culver City, CA 90232-0802
310-839-2436

A-MAZE-ING SHAPES

An interaction unit exploring plane geometry while designing mazes

Student Guide

INTRODUCTION

Geo World, a geometry theme park, will open next fall. Incorporating state-of-the-art computer generated thrills, each ride and exhibit will offer experiences found no where else on earth. Paul and Pamela Pythagoras, descendants of the great mathematician, are building the theme park using the royalties earned from his famous theorem.

Paul plans four entrances, with each entrance offering a different maze to challenge park visitors. All visitors must find their way through one of these mazes in order to enter the park.

Paul and Pamela are asking your help to design the mazes for each of the entrances. Use your creativity and design a maze, making it as challenging as possible. The maze will, of course, mainly be a pathway into the park.

This entrance maze must also contain the following design elements:

- A rectangular perimeter of less than or equal to 640 meters
- An area of less than or equal to 26,000 square meters
- A minimum of four different types of quadrilaterals
- A minimum of one isosceles triangle
- A minimum of one equilateral triangle
- A minimum of one scalene triangle
- A minimum of one right triangle
- A minimum of two right angles
- A minimum of two acute angles each measuring less than 60 degrees
- A minimum of two obtuse angles each measuring more than 100 degrees
- The solution path through the maze must touch each of the required polygons.

In order to design the entrance mazes designers need to understand all of these geometric requirements and how they can fit together.

Beginning today you will meet with a study group of three other students. Together you will study and discuss the polygons as they are introduced. After learning about the polygons, each individual will be responsible for designing and building your own maze. Then your study group will select one design to submit for final approval. Your group will write a proposal to Pamela and Paul proving that your selected maze contains all of the required design elements. Your group will also present your maze to the whole class and will demonstrate how it contains the required design elements. Good luck.

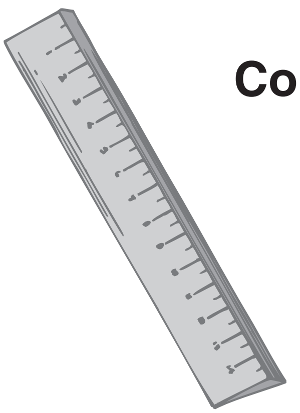
978-1-56004-452-9

9 781560 044529

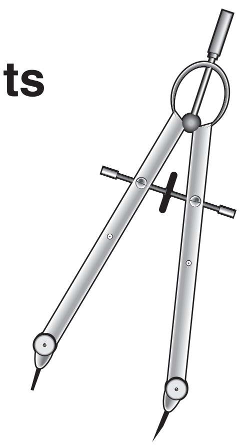
INT941SG

STEPS TO CREATE A MAZE

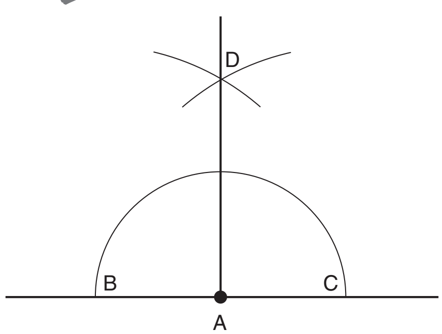
1. Decide what scale you will use. If you use 1 square = 5 meters you will have a larger area for the maze. If you prefer a smaller maze you can use 1 square = 10 meters.
2. On the grid paper, mark the boundary corners of the rectangular maze. Remember the perimeter cannot be larger than 640 meters.
3. Draw all of the required shapes on grid master paper.
4. Draw the maze pathway (solution) on the grid, being sure it includes or touches each shape. Be sure to draw it very lightly so you can erase it when you are finished.
5. Draw the sides of the pathway very lightly so that you can erase portions to create dead-ends.
6. Check that your solution path touches all of the required shapes and angles.
7. Add some dead-ends, being sure not to cross over the original solution path.
8. Transfer the maze to plain paper. All the work should be done in pencil first; then, when the maze is completed, you can erase the solution path and darken the barriers using felt pen.



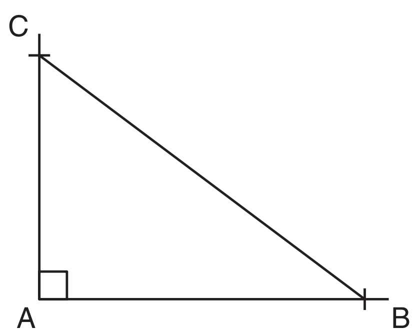
Constructing Geometric Design Elements



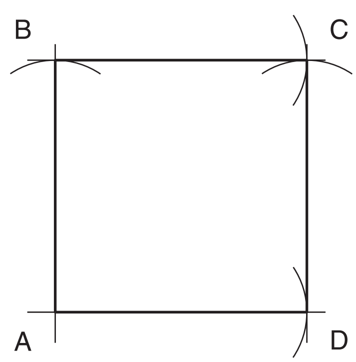
Using a ruler and a compass you will construct various geometric shapes to build into your maze.



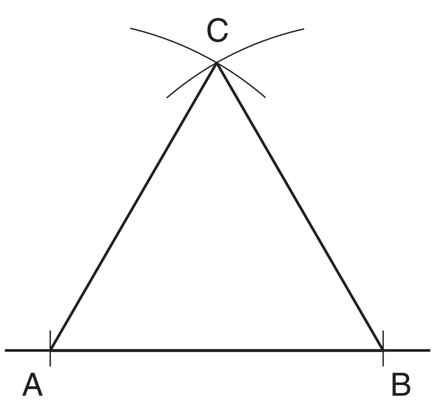
Constructing a right angle



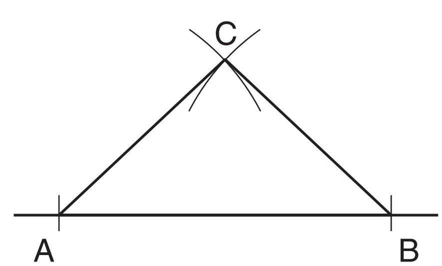
Constructing a right triangle



Constructing a square



Constructing an equilateral triangle



Constructing an isosceles triangle

VOCABULARY

As you are working to complete the maze entrance, you will learn many new words and ideas. You will learn to identify the following geometry shapes, ideas and terms:

DRAWINGS	DEFINITIONS
	Acute angle:
	Acute triangle:
	Angle:
	Area:
	Bisect:
	Chevron:
	Compass:
	Congruent:
	Degree:
	Diagonal:

DRAWINGS	DEFINITIONS

Diameter:

Equilateral triangle:

Isosceles triangle:

Kite:

Line segment:

Line of symmetry:

Maze:

Midpoint:

Obtuse angle:

Obtuse triangle:

Parallel:

Parallelogram:

DRAWINGS	DEFINITIONS

Perimeter:

Perpendicular:

Plane:

Polygon:

Point:

Protractor:

Quadrilateral:

Radius:

Ray:

Rectangle:

Rhombus:

Right angle:

DRAWINGS	DEFINITIONS

Right triangle:

Scale:

Scalene triangle:

Square:

Symmetrical figure:

Topology:

Trapezoid:

Triangle:

Vertex:

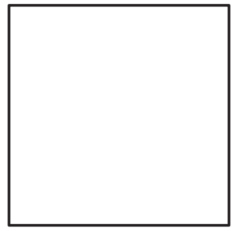
________________________ :

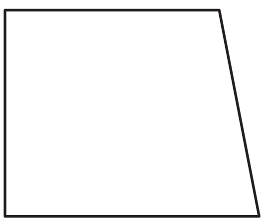
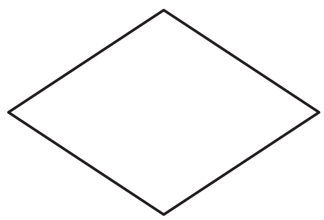
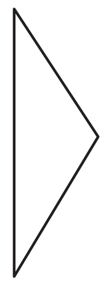
________________________ :

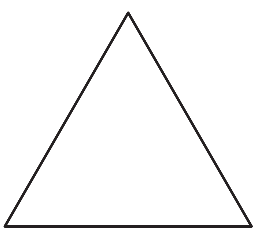
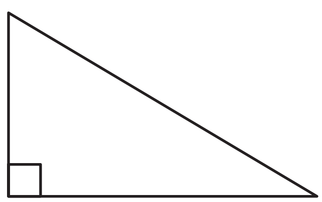
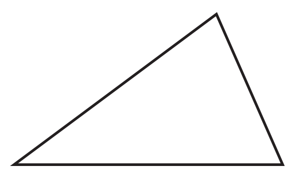
________________________ :

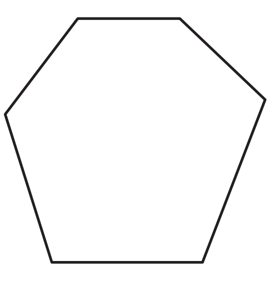
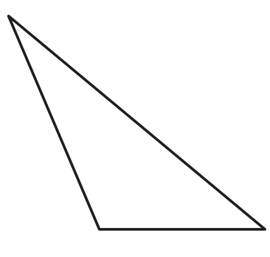
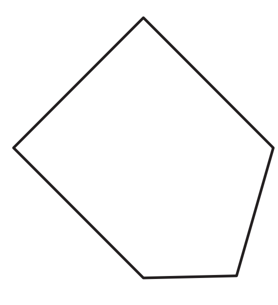
REFERENCE SHAPES

Label the polygons on the lines provided. Some may have more than one name.

QUIZZICAL QUADRILATERAL